II.

MATHEMATICS:
A BRIDGE TO THEOLOGY

Carlos Steel

PROCLUS ON THE USEFULNESS OF MATHEMATICS FOR THEOLOGY

How Can Geometry Prepare for Theology?

In the first prologue of his *Commentary on Euclid*, Proclus discusses at length why it is useful for philosophers to study mathematics[1]. He draws upon a long-standing tradition in defence of mathematics, beginning with Plato, who emphasised the importance of mathematics, particularly geometry, in the advanced education of philosophers, the future leaders of the state. However, not all philosophical schools accepted this Platonic prejudice in favour of mathematics. Proclus refers to Zeno of Sidon, an Epicurean who attacked the scientific nature of geometric proofs[2]. Neither the Epicureans nor the Stoics considered the study of mathematics to be a valuable pursuit for a philosophical life. They observed that advanced mathematics with demonstrations of general theorems offered little practical benefit. Applied sciences that dealt with specific problems, such as mensuration,

1. For some arguments, this contribution draws upon the introduction I composed together with Alain Lernould for our new edition with a French annotated translation of Proclus' commentary on Euclid. For various reasons, the publication of this edition by Vrin has been delayed and is now scheduled for early 2026. To avoid complications with other contributions in this volume, references will not be to the new edition, but to Friedlein 1873. Fortunately, the old edition is in most cases reliable. When I diverge from Friedlein's edition, a variant reading will be noted. For quotations and paraphrases in English I rely on Morrow's 1992 excellent translation, adapting it freely.
2. On the Epicurean criticism of geometry, and in particular of Zenon of Sidon, see Benatouïl 2010.

Proclus and the Sciences. Approaches to Understanding the Divine. Edited by A. Motta and D. P. Taormina, Firenze, SISMEL – Edizioni del Galluzzo, 2026, pp. 123-147
ISBN 978-88-9290-508-5 • e-ISBN 978-88-9290-484-2 •
https://doi.org/10.36167/ML136/SteelPDF •

were considered to be more useful in everyday life. Even within the Platonic school, as Proclus mentions, some people questioned the value of mathematics as a science. They referred to Plato's criticism of mathematics in comparison to dialectic in the *Republic*. However, Proclus offers an extensive response to these arguments, defending the essential role of mathematics in intellectual formation alongside Plato, though he acknowledges that it is not first philosophy[3]. He refers to mathematics with an expression coined by Plato in the *Timaeus*: «the paths of education» (τῶν κατὰ παίδευσιν ὁδῶν). The phrase is used at a pivotal moment in the exposition. Before explaining how the four elementary bodies are formed from a combination of triangles, Timaeus warns his audience that his account will be «unfamiliar»: «but since you have been schooled in the 'paths of education' through which I must demonstrate what I am saying, you will follow me»[4]. The context makes it clear that Timaeus assumes that his audience is well trained in geometry, as all his arguments start from geometrical figures (triangles) and their properties. As Proclus explains, Timaeus uses the phrase κατὰ παίδευσιν ὁδοί because in his view mathematics is related to philosophy in the same way that education leads to a virtuous life (*In Eucl.* 20.14-17):

> As education prepares the soul (προευτρεπίζει) for a perfect life through infallible dispositions, mathematics makes ready our understanding and the eye of our soul for turning towards the upper world[5].

As he explains later, one should practise mathematics for its own sake and not for some *external* utility (*In Eucl.* 28.1-7):

> However, if one must relate its usefulness to something outside of itself, it is to intellectual insight that it must be said to contribute. For to that it leads the way and prepares us (ποδηγεῖ καὶ προευτρεπίζει) by purifying the eye of the soul and removing the hindrances from the senses for the knowledge of the whole of being.

3. See on this discussion within the Platonic school: *In Eucl.* 29.14-32.20.
4. Pl. *Ti.* 53c1-4: ἀήθει λόγῳ πρὸς ὑμᾶς δηλοῦν, ἀλλὰ γὰρ ἐπεὶ μετέχετε τῶν κατὰ παίδευσιν ὁδῶν δι' ὧν ἐνδείκνυσθαι τὰ λεγόμενα ἀνάγκη, συνέψεσθε.
5. [15 ἤθεσιν correxi (cf. 23.24 and *In R.* I 161.28): ἔθεσιν Friedlein with M].

Just as the purifying virtues are not useful for the necessities of life but prepare the soul for the contemplative life, in the same way one must relate the study of mathematics to the achievement of ultimate wisdom (*In Eucl.* 49.9-12).

We say that geometry releases us from the sensible world and leads us to the incorporeal being and that it habituates us to the view of the intelligibles and prepares us for activity according to the intellect[6].

Proclus adopts the term «habituation» (συνεθισμόν) from Plotinus, whom he quoted earlier. It is not enough to have a soul that is «by nature» philosophical; it must be educated (*In Eucl.* 21.19-24):

Certainly, someone of this nature has already from himself awakened and taken flight towards being, but one must give him, says Plotinus, mathematics to accustom him with the incorporeal nature, and, while he uses mathematics as steps, it is necessary to lead him to the reasonings of dialectic and, in a general way, to the study of beings[7].

Proclus also uses the phrase «paths of education» in his introduction to the *Platonic Theology*, when he discusses the necessary preparations and dispositions for studying theology. First, the student of theology must be purified by all moral virtues. Next, he should be trained in logic. Thirdly, he should have studied physics in order to investigate «the nature of the separate and primordial substances» and become acquainted with «the truth in appearances». Finally, he should not be ignorant neither of the «paths of education»: «for through them we know the divine being more immaterial»[8]. Proclus is himself an excellent example of such an education, which he received from Heron in Alexandria. His teacher is said to have had a perfect training in the mathematical «paths of education»[9].

6. τὴν γεωμετρίαν τῶν αἰσθητῶν ἡμᾶς ἀπολύειν φήσομεν καὶ περιάγειν εἰς τὴν ἀσώματον ὑπόστασιν καὶ συνεθισμὸν εἶναι πρὸς τὴν θέαν τῶν νοητῶν καὶ προευτρεπίζειν εἰς τὴν κατὰ νοῦν ἐνέργειαν.

7. Cf. Plot. *Enn.* I 3 [20], 3.5-6 and 9-10.

8. Procl. *Theol. Plat.* I 11.4-7: Μήτ' οὖν ταύτης, ὅπερ εἴπομεν, τῆς ἐν τοῖς φαινομένοις ἀληθείας, μήτε αὖ τῶν κατὰ παίδευσιν ὁδῶν καὶ τῶν ἐν αὐταῖς μαθήσεων ἀπολελείφθω· διὰ γὰρ τούτων ἀυλότερον τὴν θείαν οὐσίαν γινώσκομεν.

9. Marin. *V. Pr.* § 9.15: ἐπὶ δὲ μαθήμασιν Ἥρωνι ἐπέτρεψεν ἑαυτόν, ἀνδρὶ θεοσεβεῖ καὶ τελείαν παρασκευὴν ἐσχηκότι τῶν κατὰ παίδευσιν ὁδῶν.

We may, therefore, conclude with Proclus that the utility of mathematics should not be measured by considering human needs. As with all theoretical activities «it is worthy of pursuit for its own sake and not merely because it satisfies human needs» (27.28-28.1). If we are to continue discussing its usefulness, we must consider its essential role in the formation of a philosophical mind and its ascent to higher immaterial reality. Proclus also makes this point in his introduction to the commentary. He wants:

> to emulate the Pythagoreans whose byword (σύμβολον) was «a figure and a stepping-stone» (βᾶμα), not «a figure and three obols». By this they meant that we must cultivate that (ἐκείνην) sort of geometry that, with each theorem, lays the basis for a step upward and elevates the soul to the higher world[10].

«Prefer the figure and step instead of the figure and three obols» is listed as the 36th Pythagorean *symbolon* in Iamblichus' *Protrepticus*. Iamblichus explains it as an exhortation to study mathematics with the goal of contemplating the incorporeal beings (150). A later commentator, Elias, interprets this goal as a speculation about the divine: «let us make on each figure a step up to theology»[11]. Three obols (a day pay given to members of the assembly or tribunal in Athens) stands for the ordinary applied mathematics, for calculation involved in mensuration and commerce. In the *Vita Pythagorica* (§ 5.21-24), Iamblichus recounts how Pythagoras, having observed a young man in Samos wasting his time on physical training, convinced him to study mathematics with him, promising to pay for his training. Pythagoras paid him three obols for each figure he learnt. After some time, Pythagoras said he had no more money to pay him. However, by that time, the young man had become so highly motivated that he continued his study of mathematics without any financial reward. A similar anecdote is told about a disciple

10. *In Eucl.* 84.15-20: ζηλοῦντες τοὺς Πυθαγορείους, οἷς πρόχειρον ἦν καὶ τοῦτο σύμβολον «σχᾶμα καὶ βᾶμα, ἀλλ' οὐ σχᾶμα καὶ τριώβολον» ἐνδεικνυμένων, ὡς ἄρα δεῖ τὴν γεωμετρίαν ἐκείνην μεταδιώκειν, ἣ καθ' ἕκαστον θεώρημα βῆμα τίθησιν εἰς ἄνοδον καὶ ἀπαίρει τὴν ψυχὴν εἰς ὕψος.

11. El. *In Porph.* 28.21-23: οὕτω γάρ φασι καὶ οἱ Πυθαγόρειοι «σχᾶμα καὶ βᾶμα» ἀντὶ τοῦ «καθ' ἕκαστον σχῆμα βαθμὸν ἄνιμεν ἐπὶ θεολογίαν».

of Euclid who once asked him, after having studied the first proposition, what profit he will obtain from having studied it. Euclid said, «give him a threeobol» for he must have some profit from what he learned[12]. Proclus shares the Pythagorean view. According to him, the only benefit of studying mathematics is «elevating the soul to the higher world, instead of letting it descend among sensible things to satisfy the common needs of mortals» (84.20-23).

However, this emphasis on mathematics as a study worthy of choice for its own sake should not make us forget that it also has a utility that extends to all sciences and all technical arts. Drawing inspiration from Iamblichus' arguments in *On common mathematics* Proclus discusses how mathematics contributes to the study of theology, physics, ethics, and politics, and to the different applied arts[13]. I will focus in this contribution on the utility of mathematics for theology, which Proclus formulates as follows (*In Eucl.* 22.1-16):

Mathematics prepares for theology intellectual insights (προευτρεπίζει τὰς νοερὰς ἐπιβολάς). Mathematical arguments reveal through images (εἰκόνων) truths concerning the gods that are difficult for imperfect (minds) to discover and (too) elevated to discern, making them trustworthy, evident, and irrefutable. For they show, reflected in numbers, the properties of that which transcends being and reveal in the objects of discursive thought the powers of intellectual figures. That is why Plato teaches us many wonderful doctrines about the gods by means of mathematical forms and the philosophy of the Pythagoreans also uses such veils to cover its mystical initiation into the divine doctrines. So is the entire *Sacred Discourse*[14], so does Philolaos in the *Bacchae*[15] and so is

12. Stob. *Anth.* II 31.114 Wachsmuth: Παρ' Εὐκλείδῃ τις ἀρξάμενος γεωμετρεῖν, ὡς τὸ πρῶτον θεώρημα ἔμαθεν, ἤρετο τὸν Εὐκλείδην· «τί δέ μοι πλέον ἔσται ταῦτα μαθόντι;», καὶ ὁ Εὐκλείδης τὸν παῖδα καλέσας «δός», ἔφη, «αὐτῷ τριώβολον, ἐπειδὴ δεῖ αὐτῷ ἐξ ὧν μανθάνει κερδαίνειν».

13. See on the contribution of mathematics to the different sciences O'Meara 1989, 160-64.

14. On the ἱερὸς λόγος attributed to Pythagoras, see Thesleff 1965, 164-68, and the recent update Huffman 2024.

15. On (pseudo-)Philolaos see Huffman 1993, on Proclus' use of Philolaos in his commentary see Steel 2007.

the whole way in which Pythagoras teaches (τρόπος τῆς ὑφηγήσεως) about the gods[16].

The way of teaching of Pythagoras

As is well known, Proclus distinguishes four *tropoi* of theology, one through oracular language (Chaldaean way), one through myths (Hesiod, Orphic hymns), one through (mathematical) images (Pythagorean way), one through dialectic[17]. The privileged Platonic way of theology is the rational scientific approach, as displayed in the dialectic of the *Parmenides*. Yet Plato occasionally uses the three other modes; he inserts in the dialogues myths, he writes an almost oracular speech on Love in the *Phaedrus*, and uses arithmetic and geometry to explain the structure of the divine world soul. However, the Pythagorean theology, as exemplified by Philolaos and the Sacred Discourse, is entirely in the mathematical mode (Procl. *Theol. Plat.* I 4.20.8-11):

> The mode of exposition that makes use of images is Pythagorean, since the discovery of mathematics was made with a view to the recollection (ἀνάμνησιν) of divine principles, and by means of these entities as images they endeavoured to gain access to the principles there; hence they consecrated numbers and figures to the gods.

When discussing the utility of mathematics to theology Proclus draws upon Iamblichus, who even used the formula προευτρεπίζει παρασκευὴν τῇ θεολογίᾳ[18]. Among the different ways to study mathematics Iamblichus puts the theological first (*Comm. Math.* 63.24-30):

16. ὅτι θεολογίᾳ μὲν προευτρεπίζει τὰς νοερὰς ἐπιβολάς. ὅσα γὰρ τοῖς ἀτελέσι δυσθήρατα καὶ ἀνάντη φαίνεται τῆς περὶ τῶν θεῶν ἀληθείας εἰς διάγνωσιν, ταῦτα οἱ τῆς μαθηματικῆς λόγοι πιστὰ καὶ καταφανῆ καὶ ἀνέλεγκτα διὰ τῶν εἰκόνων ἀποφαίνουσι. τῶν μὲν γὰρ ὑπερουσίων ἰδιοτήτων ἐν τοῖς ἀριθμοῖς τὰς ἐμφάσεις δεικνύουσι, τῶν δὲ νοερῶν σχημάτων ἐν τοῖς διανοητοῖς τὰς δυνάμεις ἐκφαίνουσιν. διὸ καὶ ὁ Πλάτων πολλὰ καὶ θαυμαστὰ δόγματα περὶ θεῶν διὰ τῶν μαθηματικῶν εἰδῶν ἡμᾶς ἀναδιδάσκει καὶ ἡ τῶν Πυθαγορείων φιλοσοφία παραπετάσμασι τούτοις χρωμένη τὴν μυσταγωγίαν κατακρύπτει τῶν θείων δογμάτων. τοιοῦτος γὰρ καὶ ὁ ἱερὸς σύμπας λόγος καὶ ὁ Φιλόλαος ἐν ταῖς Βάκχαις καὶ ὅλος ὁ τρόπος τῆς Πυθαγόρου περὶ θεῶν ὑφηγήσεως.

17. See on the four modes of theology, Steel 2005, Steel 2007, Hoffmann and Gavray 2024.

18. Iambl. *Comm. Math.* 55.8-9 (ch. 15). On Iamblichus' scientific theology, see Lecerf - Taormina 2024.

The first way to study any mathematical object, and whatever is peculiar to it, is the theological approach, which harmonises (συναρμοζόμενον) it with the being and power of the gods, their rank and acts, through a suitable analogy. This is indeed what deserves special attention from the Pythagoreans [τοῖς ἀνδράσιν]; for instance, in the case of numbers, which sorts of numbers are of like nature and akin to which sorts of gods, and in the case of other branches of mathematics they are used to think the same[19].

Theological mathematics consists in finding similitudes, analogies between mathematical objects such as numbers or figures and different classes of gods and their respective powers and acts. Like Iamblichus, Proclus recognises two modes of images in the Pythagorean theological tradition: either numbers or geometrical forms. The former is found in the tradition of theological arithmetic, best known by the *Theologoumena arithmeticae*[20]. The second method involves the use of geometric figures such as lines, triangles and circles. Philolaos, quoted by Proclus, provides an interesting example of this method. According to Proclus, geometrical arguments may reveal «the powers of intellectual figures in the objects of discursive thought» (τῶν δὲ νοερῶν σχημάτων ἐν τοῖς διανοητοῖς τὰς δυνάμεις, *in Eucl.* 22.8-9). Τὰ διανοητά is the realm of being that corresponds to discursive thought (διάνοια) and is distinguished both from the intelligible and the sensible. As Proclus shows in the opening paragraph of the prologue (following Plato's analogy of the divided line in *Resp.* VI), the mathematical objects and the knowledge related to them occupy that intermediate region. In this realm one may study triangles and other figures in their essence and powers (and the effects they have in the construction and demonstration of theorems). However, when considered from a theological perspective, these geometrical figures can reveal properties and powers of gods. For certain classes of gods, as we shall see, are characterised by «intellectual figures» that transcendently anticipate the powers of geometrical figures[21]. Thus, the study of geometry can offer, opportunities to grasp through images the divine paradigms of its objects.

19. Cf. Dillon - Urmson 2020, 73 (modified).
20. See on this work the recent study by Salerno 2024.
21. See below, 139-40 and Steel 2007.

In fact, as Proclus says, mathematics is a preparation for theology because it offers through images intuitive insights (νοεραὶ ἐπιβολαί) that transcend the multiplicity of discursive argumentation[22]. These insights are necessary in order to understand divine beings, which are themselves beyond all multiplicity[23]. We can only grasp their nature by an act of the intellect. Mathematics prepares us for theology, helps us to reach this intellectual intuition of divine orders not directly by its definitions and demonstrations but because some of its basic notions (not only figures, but also propositions) can be used as images (*eikones* or *sumbola*) to «indicate» a truth that is hard to reach by demonstrations. As Proclus says, a certain argument «may offer an indication»: ἴσως ἔνδειξιν ἂν φέροι (102.4; 110.4) ἔχοι ἂν ἔνδειξιν (291.15) εἰς ἔνδειξιν (91.11)[24]. Some examples of geometrical symbols are obvious, and they belong to a long tradition of Pythagorising reading of geometry. Points, lines, surfaces, triangles, cubes, circles, spheres can be used as symbols to reveal fundamental metaphysical principles. Take 108.10-15: the straight line is a symbol of the inflexible providence, and the circumference a symbol of the activity that returns to itself; or 115.10: «if we take this [i.e. definition VI of lines as limits of a surface] as image (ἀπὸ τούτων ὡς εἰκόνων ληπτέον) we can understand that every simpler being supplies a limit to what follows»; 164.8: the straight line is symbol of procession and movement and infinity; 290.17: the line at right angles is symbol of life lifting itself to the upper world, the perpendicular is an image of life following the path downwards, the right angle is symbol of undeviating activity.

However, in some cases, the supposed images do not actually help us to understand the superior divine beings. In fact, one

22. Proclus uses a similar expression for Cratylus' attempt at intuitive knowledge of the gods through images of «names»: τὴν ἐκεῖ διὰ τῶν εἰκόνων ἐπιβολήν (*Theol. Plat.* VI 12.60.3-4).

23. See *Theol. Plat.* II 12.73.9-11.

24. Simplicius (*In Cael.* 641.23-28) notices that Timaeus uses the verb ἐνδείκνυσθαι in his remarks on the «unusual (geometrical) account he will give to explain the elementary bodies». In Simplicius' view, this is an indication that we should not take this hypothesis literally: «Plato makes clear that these things are like the hypotheses used by the astronomers with which it is possible to save the phenomena».

must be well versed in a theological system, such as Chaldean, Orphic or Proclus' own Platonic theology, to understand how a certain element of a gometrical argument may function as an image of a paradigmatic reality. Take, for instance, the commentary on the first Proposition (the construction of an equilateral triangle on a straight line 213.14f.). Proclus noticed that Euclid in his demonstration constructs two circles that enclose this triangle (each of them in part). «This seems to indicate in a likeness (ὡς ἐν εἰκόσιν)», he notices, «how the things that proceed from first principles receive perfection, identity and equality from these principles» and «how the things that move in straight line are carried about in a circle through the eternal world-process, and souls are likeness of the immovable activity of Nous because of their periodic return» (213.21-26). So far, we may understand the analogies. But what follows is a more difficult step: «It is said also that the live-giving source of souls is bounded by a twofold Intellect» (214.2-3). Specialists in Proclus' theology may identify the «source of souls» with Rhea, who occupies the second place in the first triad of intellectual gods between two intellects, Kronos and Zeus. Proclus integrated this Orphic-Chaldean doctrine in his Platonic theology[25]. Only in that context one may understand how Euclid's demonstration of the first proposition can function as «image» of a theological doctrine. But how could Proclus still claim that Euclid's proof of Proposition 1 reveals truths about the gods that are difficult for imperfect minds to discover, and makes them «trustworthy, evident, and irrefutable» through images? In this case, the geometrical demonstration is not an image helping us to understand a divine truth, it is the inverse: unless one is familiar with a particular theological system it is not possible to recognise images in the argument. For the non-initiated it is «obscura per obscuriora». And yet Proclus concludes this digression in a Pythagorean manner: «But all this reminds us, as through images, of the nature of things» (Ἀλλὰ ταῦτα μὲν ὡς ἐξ εἰκόνων ἡμᾶς ἀναμιμνησκέτω τῆς τῶν πραγμάτων φύσεως 214.12-13). *Anamnesis* of the divine was, as we have seen, the main motivation for the Pythagoreans to study mathematics.

25. See *Theol. Plat.* V 11.36.12-38.9.

Rather than being images that evoke divine beings, the geometrical definitions and arguments serve as triggers for theological digressions on topics dear to Proclus. Take 91.1-3: «if I may give my own view (εἴ με δεῖ τοὐμὸν εἰπεῖν), the centres of all the spheres and poles are symbols of the wry-necked gods (τῶν ἰυγγικῶν)». The epitheton ἰυγγικός used to designate a particular class of gods originates from the Chaldean oracles (see fr. 77). In his *Platonic Theology* (IV 1.6.10-18) Proclus makes this class correspond to what is called in the *Phaedrus* the «supercelestial place» (247c), which represents the first triad of intelligible-intellectual gods. When introducing «his own view» in the Euclid commentary, Proclus gives a summary of his doctrine of the three triads of the intelligible-intellectual gods showing how they are represented in this world. As is evident again in this case, Proclus uses the argument to find a mathematical confirmation of a doctrine from his Platonic theology. Another example of this «exegetical» practise can be seen in his speculations about a «triadic god who proceeds from 'the hidden world' (= *Or. Chald.* fr. 198) and comprehends in himself the primary cause of the rectilinear figures» (155.19-21). Glenn Morrow suggested that that this name 'triadic' «given by the most mystic theology» might be a reference to Hermes Trismegistos, what in this context is implausible[26]. It is again a reference to the first triad of the intelligible-intellectual gods.

In sum, one gets the impression that Proclus wants to confirm his complex Platonic theology, and above all the doctrine on the triads of intelligible-intellectual gods, with references to Euclid's geometry. In that way, the Platonic theology, which finds its ultimate scientific foundation in the first and second hypothesis of the *Parmenides*, not only integrates the oracular wisdom of the Chaldean tradition and of the Orphic mythology but also allows us to interpret symbolic elements of geometry. The idea that we have intuitive access (ἐπιβολή) to this complicated theology through geometrical figures serving as images is wishful thinking. In fact, we should first study Proclus' Platonic theology in order to understand which gods the images represent.

26. See Morrow 1970, 123 note 120, see also 38, note 88 and Goulding 2022, 382 note 35.

Text and Theoria

Proclus' commentary on Euclid's work is clearly different from those of his predecessors, Heron and Pappus (which, unfortunately, we only know partially). Although he does not neglect a technical-mathematical approach to Euclid's text, he is not primarily interested in it. His main interest lies in *theoria*: a speculative approach that investigates the reality (τὰ πράγματα) corresponding to mathematical objects, as well as the first causes and principles. This approach is evident in the way he presents his commentary after the prologues. He urges his readers not to expect a technical analysis from him, the kind usually found in commentaries on Euclid. As he says, there are already enough explanations of the technical aspects of Euclid's *Elements*. «We are surfeited with those topics and shall touch on them only sparingly [...]. But whatever involves a speculation more about reality (πραγματειωδεστέραν θεωρίαν) and contributes to philosophy as a whole, we shall make it our chief concern to mention» (84.11-15). Not only in this introduction, but also in the conclusion of his commentary Proclus expresses the same view, as an advice to those who might consider continuing his commentary beyond book One (432.15-19):

> For the commentaries (ὑπομνήματα) which are now in circulation present much confusion of every kind: they do not expose the causes, and we see in them neither dialectical judgment (κρίσιν διαλεκτικήν) nor philosophical speculation (θεωρίαν φιλόσοφον).

Throughout the commentary we find remarks with the same emphasis (200.15-16 and 294.13-14):

> we should give greater attention to the examination of reality (τῇ τῶν πραγμάτων ἐπεξεργασίᾳ) than to the variety of cases.

> so much can we derive from this theorem for understanding the whole of things (εἰς τὴν τῶν ὅλων φύσιν).

From all these passages the same interest emerges: one must approach Euclid's text as a philosopher, try to understand how the definitions and propositions relate to reality (τὰ πράγματα), seek to establish the causes in the demonstrations. By proceeding in this way, says Proclus, geometry comes close to the «first science» (243.18), i.e. metaphysics, and even theology, since the first principles of beings are divine.

Proclus often indulges in speculative considerations of first principles. For example, after his explanation of the definition of a line (I 2), he provides an extensive exposition of the Pythagorean doctrine on points, lines, surfaces and solids, and their respective correspondences with the monad, dyad, triad and tetrad, and finds confirmation of this doctrine in the *Parmenides*. Proclus concludes (100.4-5): «So much can be said about the line according to more speculative views» (κατὰ τὰς θεωρητικωτέρας ἐπιβολάς). Another example is the *theoria*, which comes after the explanation of propositions 11 and 12. «If we must now add a speculative consideration (θεωρίαν) to these last two problems, it seems that the straight line raised at right angles imitates Life lifting itself up to the upper world [...] whereas the perpendicular is a likeness of life following the path downwards [...] For the right angle is a symbol of undeviating energy [...]» (290.15-22). Or after having discussed what the circle and its circumference are (definition 15 /16): «let us once more ascend from these to the consideration of their paradigms (εἰς τὴν τῶν παραδειγμάτων ἀναδράμωμεν θεωρίαν)» (153.12-13).

In many cases the theory offers theological speculations about different classes of gods. As I mentioned earlier, the commentary on Proposition I,1 demonstrates that the equilateral triangle and the two circles used to construct it are images of the first triad of intellectual gods. However, the conclusion does not make any explicit theological claims: «Let these remarks bring us to reminiscence, as from images, of the nature of things» (214.14-15).

Definition 9 offers Proclus the occasion for a long speculation on how the angles are symbols of divine connective powers, with reference to Philolaos. Proclus concludes: «Thus these [features of the angles] bring us around to the consideration of beings (τὴν τῶν ὄντων θεωρίαν)» (130.22). The commentary on the definition of trilateral figures (24-29) leads to a long speculation (θεωρεῖν 168.3) on how and why the Pythagoreans, and in particular Philolaos, dedicated the angles of triangles to particular gods, Kronos, Hades, Ares and Dionysos[27]. Similarly, when commenting on the definition of quadrangular figures (30-34), Proclus draws upon Philolaos to develop the analogies between the angles of these figures and certain divinities. He concludes (174.17-21):

> This is all we could say about quadrilaterals to explain the thought of the author of Elements and to give those who wish to know the intelligible and invisible beings (τῶν νοητῶν καὶ ἀφανῶν) starting points (ἀφορμάς) towards more speculative intuitions (θεωρητικωτέρας ἐπιβολάς).

This conclusion highlights two objectives of the previous commentary: (1) to explain Euclides' argument and (2) to provide a starting point for «a speculative contemplation (intuition)» of invisible (divine) beings. As we have seen, Proclus used the term ἐπιβολή when examining the contribution of mathematics to theology. Mathematics can prepare for theology by providing intuitions.

Proclus clearly likes to use the term θεωρία for these speculative considerations, often connecting it with πράγματα or πραγματειώδης, once with φιλόσοφος but never with θεολογικός. The πραγματειώδης θεωρία[28] is distinguished from, and opposed to, the λέξις which offers a technical exegesis of the argument, as is the practice in Proclus' commentaries on Plato[29]. As we have seen,

27. See on this interpretation Steel 2007.

28. Cf. Procl. *In Ti.* III 260.25 Van Riel (= II 193.9 Diehl): πραγματειώδης ἐξήγησις. On the meaning of πραγματειώδης see Festugière 1963, 94-100, and in this volume, see Sheppard 48-49, note 20.

29. See, for instance, *In Prm.* III 784.12: Ἀλλὰ περὶ μὲν τῶν λέξεων τοσαῦτα εἰρήσθω· μεταβατέον δὲ ἡμῖν ἐπὶ τὴν τῶν πραγμάτων θεωρίαν. VI 1092.14: αὐτὴν δὲ τὴν λέξιν λοιπὸν ἐπισκεπτέον, ἵνα καὶ ταύτην ἐπὶ τὴν τῶν πραγμάτων θεωρίαν ἐπαναγάγωμεν. VII 1146.23: Ταῦτα μὲν οὖν εἰρήσθω περὶ τῆς πραγματειώδους τῶν προκειμένων θεωρίας. περὶ δὲ τὴν λέξιν ἐπισημαντέον ὅτι. *In R.* I 164.9: Εἰ δὲ τὴν περὶ τὴν λέξιν πολυπραγμοσύνην ἄλλοις ἀφέντες ἐπὶ τὴν τῆς θεωρίας ἀναδράμοιμεν. *In Alc.* 30.5-6: Ταῦτα μὲν οὖν τῆς περὶ τὴν λέξιν θεωρίας ἀντεχόμενα μέχρι τούτων ἐξητάσθω, τὸ δὲ ἐντεῦθεν ἐπ' αὐτὴν τὴν τῶν πραγμάτων τραπόμενοι θεωρίαν. *In Ti.* II 138.10-12 Van Riel (= I 299.20 Diehl): ἡμᾶς δὲ πρῶτον χρὴ τὴν λέξιν αὐτὴν καθ' αὑτὴν ἐξετάσαντας ἔπειτα οὕτω πρὸς τὴν ὅλην θεωρίαν ἀναδραμεῖν. On the distinction between λέξις and θεωρία in Proclus' commentaries see Festugière 1963 and Saffrey - Westerink 1987, 83 note 3 (note compl. 142).

most of these speculative considerations deal with metaphysical questions that are beyond the scope of mathematics. Some of them are explicitly theological, making connections with deities. This raises the question of this commentary's overall purpose, in addition to its immediate purpose of explaining Euclid's text. Do we find this overall purpose in de speculative *theoriai*? As is well known, Proclus conceived of his Platonic theology as an endeavour to unify all interpretations of Plato's texts (τὴν τῶν λέξεων ἔφοδον), as set out in his commentaries, into a comprehensive and holistic theory (πραγματειώδη θεωρίαν) applicable to all classes of gods[30]. Should we also include in this overall project his commentary on Euclid with its distinction between *lexis* (the technical-mathematical) and *theoria* (philosophical/theological)?

Listening to Anne Sheppard's presentation I was struck by a passage in Proclus' *Commentary on the* Timaeus (*In Ti.* III 236.20-237.6 Van Riel = II 174.15-22 Diehl).

Mathematical theory ought neither to be entirely scorned, nor sought after for its own sake. The latter option means that Plato will not be able to indicate to us, as he wishes to, the things in their images (τὰ πράγματα ἐν ταῖς εἰκόσι), while the former option makes the whole exegesis of the text as unstable as a ship without ballast, for it is necessary to be firmly anchored, as it were, when one is heading off after the essence of those realities with which the dialogue is concerned (τῆς τῶν πραγμάτων περὶ ὧν ὁ λόγος ἐστὶν οὐσίας). We, however, will steer a middle course, as we said before, having first set out the text mathematically[31].

«Mathematics is not sought after for its own sake». Although this advice may seem relevant to interpreting the Euclid commentary, it does not apply. In fact, the interpretation of the *Timaeus* is a quite different case. After all, the *Timaeus* is about physical matters, not about mathematics, which in this dialogue is subservient to the goal of explaining nature. Proclus rightly insists that the mathematical sections in the *Timaeus* should be

30. See Procl. *Theol. Plat.* III 23.83.7-10: Δεῖ δὲ ἡμᾶς τὴν πραγματειώδη καὶ συνοπτικὴν περὶ ἑκάστης τάξεως συνάγειν εἰς ἓν θεωρίαν, ἐπειδὴ καὶ τὴν τῶν λέξεων ἔφοδον ἐν τοῖς ὑπομνήμασι ποιησαμένους τὰ αὐτὰ καὶ νῦν ἀνακυκλεῖν οὐκ ἂν ἔχοι λόγον.

31. Quoted and discussed by Anne Sheppard in this volume, 47-48.

studied seriously. For they are essential for understanding the text's main purpose: to explain the physical world and its causes. However, mathematical constructions only offer «things in images»[32], as they are not themselves the things they represent. Therefore, although the mathematical arguments must be examined seriously, they should not be considered the ultimate explanation of the *Timaeus*. That is why the interpreter must be steering «a middle course» between neglecting mathematics and making it the main purpose.

But what if we turn to the commentary on Euclid? Is there a «middle course» to follow? Should we study geometry seriously, but not make it our ultimate purpose? What, then, could be the overall purpose of studying geometry with Euclid? Proclus discusses the σκοπός of the *Elements* in his prologue (70.20-71.24). He distinguishes between the treatise's purpose, as determined by the things (πράγματα) investigated, and its purpose with regard to the student. With regard to the student the *Elements* offer a training in the elements of geometry that makes the student capable to apply its reasoning to the whole of geometry. With regard to the matters it investigates it helps us understand the geometrical structure of the four elementary bodies and of the world as a whole, as set out by Plato in the *Timaeus*[33]. Some interpreters, Proclus notes, thought that each of the books had a purpose related to the understanding of the cosmos, and that its «usefulness» (χρεία) lay in its «contribution to a theory of the universe» (πρὸς τὴν τοῦ παντὸς θεωρίαν). This corresponds to what Proclus said about the importance of mathematics for understanding the natural world in the above-quoted text from the *Commentary on the* Timaeus.

So far for Euclid's *Elements*. But what is the purpose of Proclus' commentary, and what place did it have in the school curriculum? For the students, it is undoubtedly an advanced training in geometry, which will help them considerably when studying

32. «Things in images» to be compared with what Proclus says in *Theol. Plat.* I 2.11.5 of physics: the study of «truth in appearances» (τῆς ἐν τοῖς φαινομένοις ἀληθείας).

33. The same purpose is mentioned in *In Eucl.* 82.13-83.2 (συντελεῖ πρὸς ὅλην τὴν τῶν κοσμικῶν στοιχείων θεωρίαν); 384.3-4; 423.13-18.

dialogues in which Plato indulges in geometrical arguments. But what about the «things examined»? Should it have a more overarching, sublime goal? Dominic O'Meara writes: «The purpose of Proclus' *Commentary on Euclid* is [...] to use mathematical science as a bridge, as a fore-shadowing of and initiation to metaphysics. The overall purpose of the Commentary [...] is Pythagorean»[34]. The Pythagorean-Platonic emphasis on the essential role of geometry in the education of philosophers may well have been the main justification for including this treatise in the curriculum. However, despite the theological nature of much of its speculative theory, the *Commentary on Euclid* is certainly not an example of theological geometry, a kind of *theologoumena geometriae*[35]. After all, we should not exaggerate the theological scope of this commentary. The Pythagorean approach is most evident in the section on definitions, as speculations on points, lines, circles and angles fit well here. There is already a long tradition of symbolic interpretation of these definitions. However, we should not forget that the most important section of the commentary, which constitutes half of the volume, concerns the propositions and contains very few speculations that go beyond the text. The commentaries on the propositions are often very technical for non-specialists. Furthermore, some of the longer speculations focus on «logical and methodological issues closely related to Euclid's text» rather than on theological or metaphysical questions[36]. Above all, the philosophical-theological approach in the *theoria* does not cause Proclus to neglect the technical exegesis of the text. He carefully analyses the Euclidean text, addresses the objections raised against some of his proofs, and presents alternative proofs, including his own[37]. His explanation of the distinction between problems and theorems, and his discussion on the fifth postulate (also known as the parallel postu-

34. O'Meara 1989, 174.

35. See Mueller 1987, 309: «In the Euclid commentary Proclus blends these two approaches to mathematics (that is the Pythagorean approach and the technical) in an interesting and unique way». Even 'blending' may be too much said, as both approaches are clearly distinguished.

36. Harari 2022, 64.

37. On Proclus' own mathematical contributions see Vitrac 2004 and Maronne - Rabouin 2022, ch. 1, 9-12.

late), are remarkable[38]. Clearly, Proclus was interested in discussing technical geometrical problems, despite his stated intention not to devote too much time to them.

In conclusion, the Pythagorean speculative *theoriai* do not provide a perspective on an overarching purpose of the commentary different from the textual and technical explanations it offers. Rather, they are digressions triggered by certain arguments, in which the author indulges his theological interests. Fortunately, they do not interfere with the textual explanation. On the contrary, as Iamblichus already observed, it is only when knowing the «common elements of mathematics» that one may also understand «the symbolic and outlandish use of mathematical expressions» in theological speculations[39].

Mathematics and Theurgy

The most significant theological digression is found in the commentary on the definition of figure (σχῆμα) (136.20 ff.). This should come as no surprise, given that we have seen that «figure» is the central concept for making symbolic analogies to divine powers. Proclus concludes this lengthy section: «we have drawn out at great length these matters of Pythagorean doctrine»[40].

To explain how figures could be symbols of gods, Proclus first gives a survey of the various levels where figures may be found: figures produced by art, figures in nature, in the soul, in the intellect, and «transcending all these figures, the perfect, uniform, unknowable and unutterable figures of the gods» (138.6-7). Theurgy, Proclus claims, may represent the properties of these divine figures by creating statues of the gods and assigning different forms to each deity. It symbolises some of the divine figures by means of magical «characters» upon the statues; it «imitates other through forms and shapes making some [statues] standing, some sitting; and some spherical, some heart-shaped and some fashioned still otherwise, etc.» (138.12-18).

38. Cf. Salerno 2021.

39. Iambl. *Comm. Math.* 67.16-17 (ch. 22): ἀκοῦσαι τὴν συμβολικὴν καὶ ἀπεξενωμένην χρῆσιν τῶν μαθηματικῶν λέξεων.

40. *In Eucl.* 142.8: Ἀλλὰ ταῦτα μὲν κατὰ τὸ Πυθαγόρειον ἀρέσκον ἐμηκύναμεν.

That the gods can be characterised by figures can also be deduced «dialectically» from the second hypothesis of the *Parmenides*. «If the One is of this sort, it would partake of some figure, as it seems, either straight or round, or some figure mixed from both» (145b2-6). Proclus argued that in this deduction Parmenides refers to the third intelligible-intellectual triad, which corresponds to the Teletarchs (τελετάρχαι) in the Chaldean Oracles. These divinities play an important role in the theurgical rituals, and it is plausible that certain geometrical forms as triangles and circles were used[41].

The association between divine figures and theurgy has been examined in a recent study by Robert Goulding[42]. Building upon the studies of Gregory Shaw[43], the author distinguishes between *external* theurgy, which uses statues and other magical objects in rituals to purify the soul and draw it towards the divine, and a superior *inner* theurgy, which is «conducted within the contemplative space of the mind to raise the soul to the highest reaches of the intelligible». Goulding (2022, 383) radicalises this view arguing that «geometry, practiced in a certain way, is not just a preparation but constitutes for Proclus the core of the higher theurgy».

> Geometrical objects are tokens of the gods in the *phantasia*, which are employed in an internal ritual. The efficacy of this inner theurgy depends on the practitioner being aware of the objects just as symbols and recognizing the inner rituals just as rituals (376). The propositions themselves of the *Elements* constitute the theurgic rituals – that is, the whole of each proposition, including the instructions for making the construction and the demonstration, as well as the geometrical truth stated in the proposition, and the finished diagram that accompanies it. The process of inscribing the diagram in the *phantasia*, line by line and circle by circle, is a ritual act, as is the movement of the *dianoia* through the steps of the demonstration; theurgy differs from contemplation precisely in the fact that something is done (383).

41. For a study of Proclus' doctrine on divine figures and the connection with the *Parmenides* see Steel 2007.

42. Goulding 2022.

43. Shaw 1995.

In short, Proclus' commentary does not merely contain theological digressions (*theoriai*) and analogies with theurgical practises; the Euclidean demonstrations themselves become «rituals of inner theurgy». Goulding has two main arguments (1) The role of *phantasia* in geometry: Proclus held that the same organ that was the vehicle and receptacle of theurgy was also the location of the mathematical objects that are the subject of Euclid's geometry. (2) Proclus observed «connections between geometrical objects and the gods» and structured them «into σειραί, chains of correspondences, which formed the basis of theurgy» (380). I will not discuss the first argument, on the status of *phantasia*, which is as such not convincing[44]. Besides, *phantasia* is certainly not an «organ» that is the «location of mathematical objects». Mathematical objects belong essentially to the realm of dianoetic knowledge though they can be projected within the sphere of imagination. Concerning the second argument it is correct that Proclus (and his Pythagorean predecessors) found connections between geometrical objects and divine principles. As Proclus says: «The figures first appear in the successive classes of the (intellectual) gods» (156.2-5). Because of these connections with the gods it is possible to use a geometrical notion as an image to grasp indirectly the divinity that is beyond understanding. However, to develop those symbolic relations is in no way an act of theurgy, but is, as we have seen, one of the *tropoi* of theology. Moreover, not all notions or elements of a geometrical demonstration can be seen and experienced as images of the divine. Geometrical arguments have a technical aspect that cannot be reduced to theological speculation. Claiming that practising Euclidean demonstrations is an act of theurgy removes the intrinsic rational finality of those arguments.

However, one might not focus on the technicalities of the demonstrations, but rather defend the theurgic claim from Proclus' perspective on mathematical objects as projections originating from the *dianoia* in the imagination[45]. Although the soul may find in its own being the geometrical reasons (*logoi*), it is too

44. One could also use this argument to make the role of *phantasia* in sense-perception theurgical.

45. On this well-studied doctrine see *inter multos* Nikulin 2008, 2.

weak to consider them in a unified way within itself, it has to project them in its imagination. This projection gives extension and plurality to figures that have in their ideal state neither extension nor place or plurality: the ideal circle, the ideal triangle. In the compositions and divisions of the figures as well as in the related arguments discursive thought makes use of imagination as an intelligible matter. With his projection doctrine, Proclus attempts to explain the tension within geometry between, on the one hand, the understanding of its ideal content, and, on the other hand, the discursivity of its argumentation. This is the type of geometrical knowledge, as we practice it in explaining Euclid's *Elements*. However, as Proclus claims, this discursive geometrical reasoning is not an end in itself, it should be «a path» to go back up, to rediscover the richness of the soul's thought (*In Eucl.* 55.13-18).

> If [the soul] should be able to roll up the extension and view the multitude of the figures in a non-figurative and unitary way, then in turning back to itself it would see in a superior way the undivided, unextended, and essential geometrical ideas (λόγους) of which it is the fullness.

Such an activity would be «the supreme end of studies in geometry». «It is to this exercise», says Proclus, «that he who is truly a geometer must devote himself, and this should be the goal he must set for himself: to awaken and turn away from the imagination towards thought alone in itself»[46].

Goulding (2022) comments on this passage: «Thus, simple visualization of geometrical objects and reversion to their dianoetic models must have been part of the practice of Proclus' inner theurgy» (381). However, what has this reversion from imagined figures to their *logoi* in the soul to do with theurgy? The projection of its content in imagination and rolling it up when returning to itself is a relation within the soul. The dianoetic *logoi* are not the ideal paradigms in the intellect, and a fortiori not the secret figures of the intelligible-intellective gods: they are *logoi* in the soul. Proclus is right in saying that no true

46. *In Eucl.* 55.20-23.

geometer would like to remain in endless argumentation and demonstration; rather, he would want to return to the state of knowledge that the soul has always possessed in its essence. However, Proclus never considered the projection from the essential *logoi* to the realm of imagination, and their subsequent return to an ideal status as an internal ritual of theurgy. One might argue that the geometrical approach is analogous to the aims of theurgic practice. However, even this more modest assertion is not put forward by Proclus.

Rather than relating geometry to an esoteric practice, Proclus' doctrine of projection, which situates the practice of Euclidean geometry within the movement of the soul – outgoing in imagination and returning to its innate content – provides geometry with its ultimate philosophical foundation. Moreover, even his Pythagorean-Platonic approach does not remove geometry from its natural location, discursive reasoning (διάνοια). To be sure, Proclus likes to show how geometry is capable of extending its reach to all levels of reality, including the divine. However, in this range to all beings, it remains within its own level and its own capabilities. Even when revealing through images sublime insights about the gods, the soul is not transported to a higher sphere but remains on its own level. This is evident in a beautiful text wherein Proclus demonstrates the extension of geometry to all things.

Geometry is coextensive with all things; it applies its reasoning to all things and contains in itself the forms of all things (*In Eucl.* 62.1-22).

At its highest intellectual level, it thoroughly inspects real beings and, through images, teaches the properties of the divine orders and the powers of the intellectual forms. For it also contains within its own speculations the *logoi* of these entities and shows which figures are proper to the gods, which to primary substances, and which to the natures of souls.

In the middle range of knowledge, it develops and unfolds its dianoetic *logoi*, considering the diversity within them and revealing their essences, properties, commonalities, and differences. Starting from these *logoi*, it encompasses within defined limits the figures shaped in imagination referring them to ther essential *logoi*.

At the *third* stage in the development of its reasoning (διανοήσεως) it examines nature and the species of the sensible elements and their powers and explains how they are contained in a causal manner in its own *logoi*[47].

The three levels correspond to 1) the use of geometry in theological speculations, 2) the level of ordinary mathematics as Proclus understands it, 3) the use of mathematics in explaining the physical world.

What Proclus says about the superior dimension of geometry («it teaches through images the properties of the divine orders and the powers of the intellectual forms»), corresponds to what he said earlier about its contribution to theology: mathematical *logoi* show through images «the properties of that which transcends being and reveal the powers of intellectual figures»[48]. Proclus insists that geometry finds these *logoi* of the divine beings, «in its own speculations», that is in discursive thought.

The second level, which lies between the intelligible and the sensible, is characteristic of the rational soul when it is engaged in discursive thinking (*dianoia*). This is the level of Euclidean geometry, as explained by Proclus in his commentary. It develops the notions (*logoi*) corresponding to the different figures, studies their properties, shows what they have in common and in what they are different. Starting from those *logoi*, which belong to the very essence of the human soul, geometry shapes the figures in imagination but keeps them within their defining limits by connecting them with the essential *logoi*.

The third level is the extension of geometry in its application to the physical world. It examines in particular, as in the *Timaeus*, the geometrical structure of the elements.

Proclus concludes (*In Eucl.* 62.22-26):

Geometry has images of the total intelligible kinds and paradigms of sensible ones *but it has its essence in the dianoetic forms* and through this middle region it ranges upwards and downwards to everything that is or comes to be.

47. Mueller 1987, 317-18 quotes this text to show how Proclus used his doctrine of projectionism as a foundation of ordinary geometry. See on this text also also O'Meara 1989, 173.

48. See text quoted above note 16.

The conclusion recalls an important principle of Proclus' psychology. The soul is everything, in its own manner: what is above the soul, it knows as image, what is inferior, it knows in a paradigmatic way. This principle figures as proposition 195 in the *Elements of Theology*: «Every soul is all things, in a paradigmatic way (παραδειγματικῶς) the sensible, in an iconic way (εἰκονικῶς) the intelligible». Unlike Plotinus and Porphyry, Proclus maintains that, in its attempts to ascend or descend, the soul always remains at its own intermediate ontological level, characterised by discursive thinking (*dianoia*). Even in its theological speculations the soul keeps a mode of vision proper to it (ἐν τοῖς οἰκείοις θεάμασι). Rather than a theurgic esoteric transformation, there is a reasoning that enables us to understand, through images, that which we cannot directly grasp. This is also what Proclus had said earlier in the prologue (20.4-5):

> Geometry reveals in its own arguments (ἐν τοῖς οἰκείοις λογισμοῖς) the truth about the gods and a theory of being (καὶ τὴν περὶ θεῶν ἀλήθειαν καὶ τὴν περὶ τῶν ὄντων θεωρίαν).

Bibliography

T. Benatouïl, *Les critiques épicuriennes de la géométrie*, in P.-E. Bour - M. Rebuschi - L. Rollet (eds), *Construction. Festschrift for Gerhard Heinzmann*, London, College Publications, 2010, 151-62.

J. M. Dillon - J. O. Urmson (eds), *Iamblichus: On the General Science of Mathematics*, London, Bloomsbury, 2020.

A.-J. Festugière, *Modes de composition des Commentaires de Proclus*, «Museum Helveticum» 20.2 (1963), 77-100.

R. Goulding, *Geometry and the Gods: Theurgy in Proclus's Commentary on the First Book of Euclid's* Elements, «Perspectives on Science», 30.3 (2022), 358-406.

O. Harari, *Philosophical Commentaries on Mathematical Texts: The Case of Proclus' Commentary on the First Book of Euclid's Elements*, in K. Chemla - G. W. Most (eds), *Mathematical Commentaries in the Ancient World*, Cambridge, Cambridge University Press, 2022, 53-95.

P. Hoffmann - M.-A. Gavray, *Modes d'exposition de la théologie dans le néoplatonisme athénien (Proclus, Damascius, Simplicius)*, in O. Boulnois – Ph. Hoffmann - C. Lafleur - J.-M. Narbonne (éd.), *L'Esprit critique dans l'Antiquité*, II. *La Naissance de la théologie comme science*, Paris, Les Belles Lettres, 2024, 219-58.

C. Huffman, *Philolaus of Croton: Pythagorean and Presocratic*, Cambridge, Cambridge University Press, 1993.

C. Huffman, *Pythagoras*, in *The Stanford Encyclopedia of Philosophy*, https://plato.stanford.edu/archives/spr2024/entries/pythagoras/. (Seen July 30 2025), 2024.

A. Lecerf – D. P. Taormina, *Jamblique et la théologie scientifique*, in O. Boulnois – Ph. Hoffmann – C. Lafleur – J.-M. Narbonne (éd.), *L'Esprit critique dans l'Antiquité*, II. *La Naissance de la théologie comme science*, Paris, Les Belles Lettres, 2024, 167-217.

S. Maronne – D. Rabouin, *La philosophie des mathématiques dans son histoire de l'Antiquité à la période moderne*, in A. Arana – M. Panza (éd.), *Précis de philosophie de la logique et des mathématiques*, Vol. 2: *Philosophie des mathématiques*, Paris, Ed. Sorbonne, 2022, 4-47.

G. R. Morrow (ed.), *Proclus. A Commentary on the First Book of Euclid's* Elements, Princeton, Princeton University Press, 1970.

I. Mueller, *Mathematics and Philosophy in Proclus' Commentary on Book I of Euclid's Elements*, in J. Pépin – H. D. Saffrey (éd.), *Proclus lecteur et interprète des anciens*, Paris, Ed. CNRS, 1987, 305-18.

D. Nikulin, *Imagination and Mathematics in Proclus*, «Ancient Philosophy», 28 (2008), 153-72.

D. J. O'Meara, *Pythagoras Revived. Mathematics and Philosophy in Late Antiquity*, Oxford, Clarendon, 1989.

H. D. Saffrey – L. G. Westerink (éd.), *Proclus.* Théologie Platonicienne. *Livre III*, Paris, Les Belles Lettres, 1987.

L. Salerno, *Il quinto postulato di Euclide nel* Commento *di Proclo al primo libro degli* Elementi, «Studia graeco-arabica», 11.1 (2021), 19-81.

L. Salerno, Theologumena Arithmeticae *dello ps.-Giamblico: alcune note testuali al capitolo sulla Tetrade*, «Studia graeco-arabica», 14 (2024), 39-57.

G. Shaw, *Theurgy and the Soul: The Neoplatonism of Iamblichus. Hermeneutics, Studies in the History of Religions*, University Park, Penn. State University Press, 1995.

C. Steel, *Le jugement de Proclus sur le style du* Parménide, in J. M. Dillon – M. Dixsaut (eds), *Agonistes. Essays in Honour of Denis O'Brien*, London, Routledge, 2005, 209-25.

C. Steel, *Divine figures. An essay in Platonic-Pythagorean Theology*, in M. Bonazzi – C. Lewy – C. Steel (eds), *A Platonic Pythagoras. Platonism and Pythagoreanism in the Imperial Age*, Turnhout, Brepols, 2007, 215-42.

H. Thesleff, *The Pythagorean Texts of the Hellenistic Period*, Åbo, Åbo Akademi, 1965.

B. Vitrac, *À propos des Démonstrations Alternatives et autres Substitutions de Preuves dans les* Éléments *d'Euclide*, «Archive for History of Exact Sciences», 59 (2004) 1-44.

Carlos Steel, *Proclus on the Usefulness of Mathematics for Theology*

Despite Proclus' programmatic declarations in the introduction of his *Commentary on Euclid* that mathematics «prepares intellectual insight for theology» and should be used as a «stepping-stone» to a higher world, this chapter cautions against overemphasizing the theological purpose of his geometrical arguments. Such declarations have led to a neglect of the real import of Proclus' technical geometrical discussions, with some arguing that geometry for him constituted the core of higher theurgy. This chapter argues that scholars should not be misled by Proclus' Neo-platonic claims about mathematics as a bridge to theology. The theological speculations present in his work are best understood as digressions that illustrate how it is possible, from our discursive perspective (the proper domain of mathematics), to gain an intellectual insight into the divine causes. Specifically, the chapter examines the crucial difference between the technical 'textual' analysis and the speculative approach (*theoria*) to reality.

Carlos Steel
KU Leuven (Belgium)
carlos.steel@kuleuven.be