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THE UNFOLDED SYNTAX OF THE INTELLECT:
PROCLUS ON THE METAPHYSICAL FOUNDATIONS
OF ELEMENTARY ARRANGEMENTS

διὰ γὰρ τὴν ἀλλήλων τῶν εἰδῶν
συμπλοκὴν ὁ λόγος γέγονεν ἡμῖν
(Pl. *Sph.* 259e5-6).

One of the most remarkable aspects of Proclean epistemology is its account of scientific knowledge. Although human beings are not granted immediate access to eternal realities through their natural faculties alone, Proclus maintains that human sciences, such as theology and mathematics, nonetheless concern such beings.

As is widely known, Proclus explains human temporal engagement with eternal beings by posing that expressions of noetic realities are found within the substance of the soul. The soul accesses these expressions of noetic beings by turning toward its eternal substance, thereby unfolding discursive images of its innate imprints of the Intellect, or «reason-principles», through its temporal powers¹. This dianoetic activity prepares the soul for the higher cognitive act of intellection (νόησις) – or even for the intellection which lies beyond the soul². Proclus' epistemology of

1. Procl. *El. Theol.* 191: «Every participated soul has an eternal substance (οὐσίαν αἰώνιον) but a temporal activity (ἐνέργειαν κατὰ χρόνον)». See also Helmig 2016, 183–296; Lernould 2021; Chlup 2012, chapter 4; Steel 1997; O'Meara 2016; O'Meara 1989, chapter 8, 9, and 10; MacIsaac 2001; MacIsaac 2002.

2. On Proclus' views on higher modes of intellection, see Vargas 2025. See also Lernould 2016; MacIsaac 2014; MacIsaac 2010.

discursive sciences and their apodeictic capacities raises several puzzles. This chapter explores these issues, focusing particularly on Proclus' views concerning the logical structures of dialectic and mathematics, their origins, relationship, and their metaphysical grounding. Its primary aim is to elucidate the metaphysical foundations underlying the logical structure exhibited by dialectic and mathematics and, on this basis, to trace Proclus' conception of the logical and doctrinal continuity among the contemplative sciences through the case study of «common mathematics».

Section 1 examines some aspects of Proclus' philosophy of language, focusing on the idea that the demonstrative structures of dialectic and mathematics are an «icon» or «intellectual paradigm» expressing the architecture of Intellect. Section 2 dwells on the justifications and consequences of this view, scrutinising the relationships between Intellect, the logical methods of dialectic and mathematics, and their elementary syntaxes. Section 3 applies the insights developed in the preceding sections to explore how, according to Proclus, a seamless logical transition between mathematics and theology can be achieved through the intermediary discipline of common mathematics. In this context, a formalisation of propositions 1-4 of the *Elementatio theologica* is proposed, showing them to be deducible from Euclid's arithmetic definitions (Book VII, def. 1 and 2). Finally, Section 4 offers concluding remarks, highlighting how Proclus' foundations of logical functions reveal that the composition of *elementationes* was more than a matter of literary form for the Diadochus: it embodied deep philosophical commitments.

Proclus' Philosophy of Language and the Intellectual Paradigm

In the *Platonic Theology*, Proclus provides some details regarding the science of dialectic, conceived as a demonstrative discourse about divine being (*Theol. Plat.* I 10.45.20-46.9):

The initial conclusions are immediately evident through very simple and familiar common notions, those that follow are demonstrated through more steps and variations, and the final ones are the most complex. He [Plato] always uses the initial conclusions for demonstrations in his arguments and presents the mutual connection amongst conclusions

in the order of geometry or other mathematical disciplines as an intellectual paradigm. If, therefore, definitions bear the image of the things of which they are the explanations, and if, as is the unfolding of demonstrations, such must the order necessarily be of the things demonstrated, then it is necessary, I think, that the things beginning from the most simple principles should be absolutely the most primary and arranged in connection with the One, whereas those that are more numerous and dependent on various demonstrations have come after a considerable interval from the One, if it is permissible to speak in this way³.

Proclus begins by reflecting on the inferential structures produced by the chains of demonstrations that are found in dialectic. He expresses his views that (i) Plato conducts his arguments according to the method that geometers *also* follow, namely by producing concatenations of proofs that depend on previously demonstrated theorems (this «also» will prove of the utmost importance). As Martijn has shown, *In Timaeum* and *In Parmenidem* provide clear examples of Proclus applying this conviction in his hermeneutics and seeking geometrical-like arrangements within the prose of the divine Plato⁴. To this, Proclus adds an important observation: (ii) that the order and intertwining of propositions constitutes an «intellectual paradigm» (παράδειγμα νοερόν). In other words, the inferential arrangement of theorems in dialectic (much as in mathematics) is itself an image of certain intellectual traits.

This implies that the different modes of theology described by Proclus (*scil.* entheastic, symbolic, mythical, and dialectical) over-

3. τὰ μὲν πρῶτα τῶν συμπερασμάτων δι' ἐλαχίστων ὡς οἶόν τε καὶ ἀπλουστάτων καὶ γνωριμωτάτων καὶ οἷον κοινῶν ἐννοιῶν εὐθὺς κατὰδηλα γίνεται, τὰ δὲ τούτων ἐφεξῆς διὰ πλείονων καὶ ποικιλωτέρων, τὰ δὲ ἔσχατα παντελῶς ἐστὶ συνθετώτατα. χρῆται γὰρ [ὁ Πλάτων] ἀεὶ τοῖς πρώτοις συμπεράσμασιν εἰς τῶν ἐχομένων ἀποδείξεις καὶ τῆς ἐν γεωμετρίας τάξεως ἢ τοῖς ἄλλοις μαθήμασι παράδειγμα προτείνει νοερόν τὴν τῶν συμπερασμάτων τούτων πρὸς ἄλληλα συνάρτησιν. εἰ τοίνυν οἱ λόγοι τῶν πραγμάτων εἰκόνα φέρουσιν ὧν εἰσὶν ἐξηγηταὶ καὶ ὡς ἔχουσιν αἱ [ἀπὸ] τῶν ἀποδείξεων ἀνελίσξεις, οὕτως ἀνάγκη καὶ τὰ δεικνύμενα τάξεως ἔχειν, ἀναγκαῖον οἶμαι τὰ μὲν δι' ἀπλουστάτων ἀρχόμενα πάντως ἀρχοειδέστατα καὶ τῷ ἐνὶ συνενωμένα τετάχθαι, τὰ δὲ ἀεὶ πληθυόμενα καὶ ποικίλων ἀποδείξεων ἡρηγμένα πορρώτερον προηληλυθέναι τῆς τοῦ ἐνὸς ἀποστάσεως, εἰ θέμις εἰπεῖν. See Solère 2003; Opsomer 2021; Opsomer 2020; Opsomer 2012; Kiosoglou 2022; Nikulin 2003. Unless otherwise indicated, the English translations are mine.

4. Martijn 2014.

lap in certain cases⁵. Such an observation accords with Proclus' broader doctrine that (some) linguistic items provide insight into the realities they signify not through direct reference, but through representation – for instance, through etymologies or analogical naming. Proclus maintains that this also applies to complex linguistic items. For example, he holds that the dramatic structure of Plato's dialogues conveys metaphysical teachings. Similarly, in his commentary *In Timaeum*, Proclus accepts Socrates' claim that Timaeus' (demonstrative) cosmogony is, at the same time, a hymn (νόμος) to the Demiurge⁶. Once again tracing overlapping theological insights within a linguistic composition through the iconic function of language, Proclus suggests that the arrangement of demonstrations in dialectic and mathematical speculation reflects certain noetic relations.

How is this supposed to work?

Proclus provides some insight into this issue by introducing a distinctive theory of language, which he derives by fusing two Platonic passages. The first comes from the *Timaeus* (29b3–5), where Plato points to a kinship between explanatory λόγοι and the things of which they are expositors (ἐξηγηταί, συγγενεῖς) – the latter being both Forms and their cosmic images. The second passage is from the *Cratylus* (439a1–3), where Plato suggests that names are images of things (εἰκόνας τῶν πραγμάτων). Combining lexical and philosophical themes from both passages, Proclus formulates a theory of iconic language that unfolds on three levels of complexity: definitions, demonstrations, and elementary arrangements.

First (1), Proclus assumes that explanatory λόγοι bear a likeness to the things they describe. I take λόγος here to mean «definition», although apophantic propositions, such as those employed in demonstrations, could also fit. Proclus' doctrine appears to be that, insofar as a real definition or a true proposition conveys the nature of an entity, its subject-predicate relationship is an image which, in some respects, discursively expresses the defined entity.

5. Procl. *Theol. Plat.* I 4.17.9–23.10; See Gersh 2005. I wish to thank Prof. Gersh for bringing this point to my attention.

6. Pl. *Ti.* 27b9; Procl. *In Ti.* II 218.16 ff. Van Riel (= I 355.4 ff. Diehl); Martijn 2010, 11 ff. See *Theol. Plat.* V 20.65.10 ff.

Second (2), as a consequence of this view, Proclus claims that the arrangement of demonstrations is likewise an image of the ontological order (τάξις) among the referenced entities. He seems to think that if the elements of demonstrations (say, definitions) are images of the defined beings, then the resulting composition must also express the relationships among those beings. Arguably, this means that the way terms are arranged and connected within a syllogism reflects some structural features of the extra-logical being(s) the demonstration deals with.

Third (3), Proclus extends the notion of likeness between logical structures and the metaphysics of the entities they concern to the entire (actual or possible) dialectical system of theorems. He articulates this in a way reminiscent of both his understanding of the series of deductions from the second hypothesis in the *Parmenides*, as the unfolding of the noetic orders, and the layout of his *Elementatio theologica*, which proceeds deductively from the One-itself to the descended soul. Proclus notes that demonstrations concerning prior beings are more directly connected to (propositions about) the One, whereas derivative entities will be logically proven in advanced demonstrations that rely on the previously established theorems. Thus, the demonstrative system of dialectic works as an informative truth-bearer in two senses: (a) in a purely denotative sense, according to the content expressed in its conclusions; and (b) iconically, insofar as the entities involved in the demonstration are intertwined in a way whose relations of priority and posteriority are discursively represented through the logical structure of dialectic.

In the following sections, we shall explore in greater detail the philosophical justifications and implications of this view, and consider how this aspect of Proclus' philosophy of language – when applied to elementary arrangements – may deepen our understanding of the *Elementatio theologica* and illuminate the connections between dialectic and mathematics.

Unwrapping Intellectual Syntax: The Logical Methods

What are the intellectual traits of the geometrical-like inferential order of dialectic that make it an intellectual paradigm?

Whence do such traits come, and how are they philosophically justified? Part of the answer lies in a passage from Proclus' *In Cratylum* (III 5.1–8):

For Intellect is the projector of dialectic. From itself, as a whole, it generates the entirety of dialectic; and through the progression of all things from the One, it establishes division; through the gathering of each thing into the comprehension of a single property, it establishes definition; through the mutual presence of Forms, whereby each thing both is what it is and participates in the other Forms, it establishes demonstration; and through the return of all things to the One and to their proper principles, it generates analysis⁷.

In this passage, Proclus refers to Intellect as the «projector» (προβολεύς) of dialectic. This designation carries epistemological and metaphysical implications, as the verb προβάλλειν and its derived noun προβολή are employed by Proclus to denote the act of «putting forward» or «projecting» a further reality from a higher principle⁸. As the former passage of the *In Cratylum* concerns the science of dialectic, it is sensible to interpret the term προβολεύς there in a similar sense: it designates the process by which a superior reality is expressed through discursive powers – metaphysically translated into the temporal activity of the soul. Therefore, Proclus maintains that Intellect is projected as the logical powers that enable the soul to engage in demonstrative theology: human science can be understood as the soul's manifestation of its inner reasons according to the distinctive mode of being proper to each kind of knowledge (i.e. metaphysical in dialectic, quantitative in mathematics).

However, it is remarkable that, once again, instead of merely highlighting the doctrinal contents of dialectic as displayed ensouled reasons, Proclus also declares that its logical operations – division, definition, demonstration, and analysis – are likeness-

7. ὅτι νοῦς ἐστὶν ὁ τῆς διαλεκτικῆς προβολεύς. ἀφ' ἑαυτοῦ ὅλου ὅλην αὐτὴν ἀπογεννῶν· καὶ κατὰ μὲν τὴν ἀφ' ἐνὸς πάντων πρόδοον τὴν διαιρετικὴν ὑφίστησι, κατὰ δὲ τὴν συναγωγὴν ἐκάστου πρὸς μίαν ιδιότητος περίληψιν τὴν ὀριστικὴν ὑφίστησι· κατὰ δὲ τὴν ἐπ' ἄλληλα παρουσίαν τῶν εἰδῶν, δι' ἣν καὶ ἔστιν ὁ ἐστὶν ἕκαστον καὶ μετέχει τῶν λοιπῶν εἰδῶν, τὴν ἀποδεικτικὴν· κατὰ δὲ τὴν ἐπιστροφὴν πάντων εἰς τὸ ἐν καὶ τὰς οἰκείας ἀρχὰς τὴν ἀναλυτικὴν γεννᾷ. See also Procl. *Theol. Plat.* I 9.40.5–10.

8. Helmig 2012, 291–99.

bearers of the Intellect. This detail captures a well-known aspect of his philosophy that is crucial for grasping his notion of the elementary arrangement as an intellectual paradigm. The ontological relations of priority and dependence among intelligible beings translate into determined logical relationships; conversely, the logical connections of dialectic unfold and represent the metaphysical relationships that exist in an undifferentiated manner within the Intellect⁹. Therefore, as the dianoetic expressions of intellection, the methods of dialectic disclose a discursive image of the compressed architecture of Intellect in the form of a web of theorems.

We can illustrate this through Proclus' examples. When the dialectician engages in analysis and determines that a prior proposition must be postulated for the analysed proposition to hold true, his inferential movement mirrors the activity of (self-) reversion (ἐπιστροφή) within the noetic being under investigation. In this way, the dialectician, through his inferential process, retraces the dynamic ascent toward ontologically and logically prior principles, thereby uncovering a higher noetic being as a necessary, both logical and ontological, precondition of the one under study. Conversely, when the dialectician uses division, he is logically retracing the outpouring of power (πρόοδος) from a given Form, following its descent from its source to its effects. This movement proceeds from unity and simplicity toward multiplicity and complexity, as the further determinations of a genus are identified and progressively articulated.

In sum, the logical functions that weave together the theoretical corpus of dialectic present an unfolded image of the unitary interconnection of intelligible beings. Moreover, every discourse that employs these logical methods (see below) tends to converge toward an elementary arrangement – the very paradigm of discursive intelligibility.

What, then, of the axiomatic systems of mathematics? In what sense can they be regarded as images of the relationships that obtain among intellectual beings? To answer this, we must first understand how Proclus conceives the relationship between

9. Gersh 1973, 74 ff.

Intellect, dialectic, and mathematics. He addresses this issue in his commentary *In Euclidem* (*In Eucl.* 44.9–23):

But even higher than it [*scil.* common mathematics, see below], dialectic could be said to be the bond of union among the mathematical sciences or, to repeat Plato's designation in the *Respublica*, their capstone. For this perfects common mathematics and sends it up towards Intellect by means of its peculiar powers, showing that it is truly a science and rendering it steadfast and irrefutable. Yet highest in rank amongst the unifying bonds is that very Intellect which contains in itself all dialectical powers in undifferentiated fashion, combining their variety in simplicity, their partiality in completeness of insight, their plurality in unity. Intellect, then, wraps up the developments of the dialectical methods, binds together from above all the discursiveness of mathematical reasoning, and is the perfect terminus of the upward journey and of the activity of knowing¹⁰.

This section reveals some important details concerning Proclus' theory of discursive science.

First, it shows how the cascade of powers flowing from the noetic realm is mirrored in the soul as a progression of cognitive capacities or sciences. These powers reside initially within Intellect, where the Forms coexist in a unified manner (*μονοειδῶς*), beyond all discursivity. From there, they are transmitted to the activity of the soul and become differentiated as the logical capacities of dialectic. In turn, these dialectical powers generate (or can be employed as) lower cognitive powers, giving rise to diluted images of themselves. The offspring of dialectic stands as an intermediary science bridging noetic and sensible beings: mathematics. Just as dialectic originates from and reverts to Intellect, so mathematics proceeds from dialectic and attains its epistemic perfection by returning to it.

10. ἀνωτέρω δ' ἔτι ταύτης ἡ διαλεκτικὴ τῶν μαθημάτων ἂν εἴη σύνδεσμος, ἥ καὶ θριγγὸν αὐτῶν, ὥς ἔφην, ἐν πολιτείᾳ προσείρηκεν. αὕτη γὰρ καὶ τὴν ὅλην μαθηματικὴν τελειοῖ καὶ εἰς νοῦν ἀναπέμπει ταῖς ἑαυτῆς δυνάμεσι, καὶ ἐπιστήμην ὄντως ἀποφαίνει καὶ μόνιμον καὶ ἀνέλεγκτον ἀπεργάζεται. τρίτην δ' ἂν ἔχοι τάξιν ἐν τοῖς συνδέσμοις ὁ νοῦς αὐτὸς ὁ πάσας τὰς διαλεκτικὰς δυνάμεις ἐν ἑαυτῷ μονοειδῶς περιέχων καὶ τὴν ποικιλίαν αὐτῶν διὰ τῆς ἀπλότητος καὶ τὸν μερισμὸν διὰ τῆς ἡμεροῦς γνώσεως καὶ τὸ πλῆθος διὰ τῆς ἐνώσεως συνάγων. αὐτὸς δὴ οὖν συμπτύσσει μὲν τὰς ἀνελίζεις τῶν διαλεκτικῶν μεθόδων, συνδεῖ δὲ ἄνωθεν πᾶσαν τὴν διέξοδον τῶν μαθηματικῶν λόγων, τέλος δ' ἐστὶ τὸ ἄριστον τῆς ἀναγώγου πορείας καὶ τῆς γνωστικῆς ἐνεργείας; tr. Morrow 1970, slightly modified.

Second, just as dialectic is a discursive, demonstrative reflection of Intellect, so mathematics constitutes a quantitative reflection of dialectic. In this capacity, mathematics receives the four logical methods in a manner analogous to the way dialectic derives its powers from Intellect (*In Eucl.* 42.12–43.1):

Our answer is that, as Intellect is set over discursive thinking and dispenses principles to it from above, perfecting it out of its own riches, so in the same way dialectic, the purest part of philosophy, hovers attentively over mathematics, encompasses its whole development, and of itself contributes to the special sciences their various perfecting, critical, and intellectual powers: the procedures, I mean, of analysis, division, definition, and demonstration¹¹.

Thus, according to Proclus, the four methods by which the axiomatic structure of mathematics is constituted are, in reality, the envoys of dialectic within mathematics – restricted in their intellectual potency by the inferior nature of the mathematical beings with which they operate at this ontological level (see below). Hence, the elementary structures produced in the exercise of dialectic are not imitations of mathematical arrangements. On the contrary, the truth is precisely the reverse: mathematicians appropriate the methods and formal structure that properly belong to dialectic and manifest them in their own science. This ultimately means that if the mathematical sciences can be elementarily arranged at all, it is because mathematics inherits the footprints of the Intellect in the form of its logical movements, which allow (and require) a sort of elementary structure – a twice-removed imitation of the noetic syntax. Consequently, when Proclus composed the *Elementatio theologica* or sought deductive, elementary arrangements in Plato's dialogues, he could not have regarded himself as imposing a geometrical-like logical structure upon theology. Rather, from the standpoint of his philosophy,

11. καὶ λέγομεν, ὅτι καθάπερ ὁ νοῦς ἐπερίδρυνται τῆς διανοίας καὶ χορηγεῖ τὰς ἀρχὰς ἄνωθεν αὐτῇ καὶ τελειοῖ τὴν διάνοιαν ἀφ' ἑαυτοῦ, κατὰ τὰ αὐτὰ δὴ καὶ ἡ διαλεκτική, φιλοσοφίας οὕσα τὸ καθαρώτατον μέρος, προσεχῶς ὑπερήπλωται τῶν μαθημάτων καὶ περιέχει τὴν ὅλην αὐτῶν ἀνέλιξιν, καὶ δίδωσι δυνάμεις ἀφ' ἑαυτῆς ταῖς ἐπιστήμαις αὐτῶν παντοίας, τελεσιουργοὺς καὶ κριτικὰς καὶ νοεράς, τὴν ἀναλυτικὴν λέγω καὶ τὴν διαιρετικὴν καὶ τὴν ὀριστικὴν καὶ τὴν ἀποδεικτικὴν; tr. Morrow 1970, slightly modified.

Proclus was attempting to present dialectical theology in its most proper and authentic form: in a logical configuration of which mathematicians are mere vicars. In short, for Proclus, axiomatised mathematical systems are but attenuated versions of the exemplary dialectical syntax.

But then, in which sense is the elementary structure of mathematics weaker than that of dialectic?

This follows from the metaphysical principle that the transmission of power from cause to effect entails a diminution of perfection in the recipient's activity. Thus, according to Proclus, the fullest epistemic realisation of the logical methods – and therefore the highest possible form of discursive science – can be attained only within dialectic¹². Lernould has convincingly argued that such an epistemic superiority lies in the capacity of dialectic to achieve an unhypothetical status¹³. Both dialectic and mathematics employ hypotheses in their analytic phase, the stage of inquiry aimed at uncovering higher principles. This process eventually leads to the first principles specific to each discipline: definitions, postulates, and common notions in mathematics; and the One in dialectic. However, whereas the hypotheses of dialectic culminate in this unhypothetical principle, mathematics, according to Platonic epistemology, can never be freed from hypotheses. The first principles of mathematics remain suppositions of existence¹⁴ and can be justified only when demonstrated by the superior, unhypothetical science (see next section).

Lastly, Proclus holds that mathematics as a whole is endowed with the dialectical methods. This means that all its branches are epistemologically dependent upon dialectic. Because of this, he introduces an overarching, general mathematical discipline that mediates between dialectic and the particular mathematical

12. A further question arises as to whether Proclus truly attained the archetypal scientific rigour he envisioned for dialectic in his *Elementatio theologica*. By his own standards, a genuine *elementatio* of the supreme discursive science would have to surpass Euclid's *Elementa* in precision, irrefutability, and freedom from hypotheses. Yet, whether Proclus actually achieved this remains open to legitimate doubt.

13. Lernould 1987.

14. For Proclus' understanding of definitions as suppositions of existence, see Martijn 2010, 84 ff. See also MacIsaac 2014.

sciences, which he calls «common mathematics» (κοινή μαθηματική). Succinctly, Proclus characterises this discipline through two main traits: (i) common mathematics comprises the most general theorems and methods shared by geometry and arithmetic and thus stands as the general science upon which all mathematical branches depend; (ii) common mathematics functions as the hinge between dialectic and mathematics: it serves as the bridge leading the learner from mathematical inquiry to dialectical speculation. Proclus illustrates this by noting that through the study of fundamental mathematical concepts, the practitioner is gradually drawn toward reflection on the fundamental principles underlying mathematical truth, thereby advancing toward the science of being *qua* being (τοῦ ὄντος, ἧ ὄν ἐστίν) – the Aristotelian expression Proclus employs to denote dialectic in this section¹⁵.

The Interweaving of Discursive Sciences: Common Mathematics and Dialectic

So far, we may conclude that, according to Proclus, (a) the underlying logical structures of dialectic and mathematics stand in an exemplar-imitation relationship; (b) this structure is what may be termed ‘geometric-like’ or, more properly, ‘elementary’; (c) the syntax of Intellect necessitates this structure within dialectic; and (d) the same structure is transmitted from dialectic to mathematics, which constitutes the quantitative image of dialectic.

From this, it follows that (1) the syntax of Intellect is manifested at each ontological level in a manner appropriate to that

15. Procl. *In Eucl.* 7.13–22: ἀλλ’ ὅτι μὲν ἀρχαὶ καὶ τῶν μαθημάτων αὗται προεστήκασιν, αἱ καὶ τῶν ὄντων ἀπάντων, φανερόν. ὥσπερ δὲ τὰς κοινὰς ἀρχὰς αὐτῶν τεθεωρήκαμεν καὶ διὰ πάντων διηκούσας τῶν μαθηματικῶν γενῶν, οὕτω δὴ καὶ τὰ κοινὰ αὐτῶν θεωρήματα καὶ ἀπλᾶ καὶ τῆς μιᾶς ἐπιστήμης ἔγγονα τῆς πάσας ὁμοῦ τὰς μαθηματικὰς γνώσεις ἐν ἐνὶ ἐπεχούσης ἀναλογισώμεθα, καὶ ὅπως ἐφαρμόττει πάσαις καὶ δύνатаι καὶ ἐν ἀριθμοῖς καὶ ἐν μεγέθεσι καὶ ἐν κινήσει θεωρεῖσθαι σκοπήσωμεν. *In Eucl.* 9.11–19: οὐ γὰρ δήπου τῶν μὲν μεριστῶν εἰσὶν ἐπιστήμαι καὶ γνώσεις, τῶν δὲ αὐλῶν καὶ τῆς νοερᾶς θεωρίας ἐγγυτέρω τεταγμένων οὐδὲ μίαν ἔχομεν ἐπιστήμην, ἀλλὰ πολλῷ πρότερον ἢ ἐκείνων γνώσις ἐστίν ἐπιστήμη καὶ ἀπ’ ἐκείνης αἱ πολλὰ τοὺς κοινούς ὑποδέχονται λόγους καὶ μέχρι τοσούτου γνώσεων ἄνοδος ἀπὸ τῶν μερικωτέρων ἐπὶ τὰς ὀλιγωτέρας, ἕως ἂν ἐπ’ αὐτὴν ἀναδράμωμεν τὴν τοῦ ὄντος, ἧ ὄν ἐστίν, ἐπιστήμην. See also *In Eucl.* 7.26–27; 8.4–6. See Steel 2005.

level. As noted above, one of the main differences between dialectic and mathematics lies in the hypothetical status of the latter. Another distinction may be found in the fact that whereas dialectic consists entirely of theorems, mathematics also includes problematic propositions – i.e. steps that require the mathematician to construct the objects about which theorems will be demonstrated. These constructive procedures reveal that mathematics stands closer to the realm of generation and change.

Second, it also follows that (2) if mathematics both ontologically and epistemically depends on dialectic and is perfected by it, then a logical bridge must exist between the logical syntaxes of the two sciences. Proclus locates this connection in what he calls common mathematics. We will now focus on the mediatory role of this science. In doing so, we will take propositions 1–4 of the *Elementatio theologica* as a token of dialectic and, as indicated by Proclus, attempt to reach them through an analysis of mathematical concepts.

Some details about the doctrinal transition from mathematics to dialectic are revealed in Proclus' *In Rem publicam* (I 291.28–292.15):

[...] Intellect contemplates that which lies beyond dianoetic beings, not by proceeding toward a result, as in reasoning from hypotheses to derived conclusions while leaving the hypotheses unproven, but instead by ascending from principles to ever higher principles, until it reaches the indemonstrable and unhypothetical first principle that does not exist hypothetically but as the true principle of all things, beyond which nothing can be thought. It is not subordinate to anything else, but everything else is subordinated to it. Thus, while the point is the principle of all things in geometry, it derives from the common principle of all things and is subordinate to it. The unit (μονάς) is also the principle of certain things and not of all in the same way; it is the principle of numbers and of everything in arithmetic, but it too is subordinated to the principle of all things. Such steps may be used by those ascending toward that principle of everything¹⁶.

16. [...] ὁ δὲ τῶν διανοητῶν ὑπέρτερον ὄν νοῦς ἐπισκοπεῖ καὶ οὗτος οὐκ ἐπὶ τελευτῇ πορευόμενος, οἷον ἐξ ὑποθέσεων τινῶν ἐπὶ συμπεράσματα, τῶν ὑποθέσεων ἀναποδείκτων κειμένων, ἀλλ' ἀπὸ τῶν ἀρχῶν ἐπ' ἄλλας ἀνωτέρω κειμένας ἀρχάς, ἕως ἂν ἐπὶ τὴν ἀναποδείκτον ἀναδράμῃ καὶ ἀνυπόθετον ἀρχήν, οὐ καθ' ὑπόθεσιν ταύτην οὖσαν, ἀλλὰ κατ' ἀλήθειαν τῶν πάντων ἀρχήν, ἧς οὐδὲν ἐπέκεινα νοεῖν θέμις, μηδενί

Proclus here illustrates the analytical ascent of the Intellect toward the supreme principle. He contrasts this movement with the synthetic progression from the hypothetical principles of mathematics to the proofs of propositions. Moreover, he declares that the first principles of arithmetic and geometry, as images of the One, can each be traced back to their metaphysical origin in dialectic, and that this is the path to be followed by the learner.

Similarly, in his *In Parmenidem*, Proclus observes that the beginner should be led to understand the divine principles of dialectic by reflecting upon arithmetical and geometrical units as images of divine undifferentiation (*In Prm.* IV 926.16–23):

For whatever our discursive thinking is unable to contemplate in the gods, it can see it in these [*scil.* mathematics] as in images (ἐν εἰκόσι); and having seen them, it will rest in the contemplation of those [*scil.* the gods] and believe also the things said about them; I mean, for example, that if it wonders how the many are in one and how everything is in the indivisible, it will reflect upon unity (τὴν μονάδα), and how within it all things are shown to be both even and odd, circle and the sphere, and the other Forms of numbers¹⁷.

From this it becomes clear that Proclus regards the notion of unity in arithmetic and geometry as the starting points of the soul's ascent toward dialectic. It is therefore reasonable to take as our own starting points the two authoritative Euclidean definitions (well-known to Proclus) of such principles: Book I, def. 1 «A point is that which has no part(s)» (σημεῖόν ἐστιν, οὗ μέρος οὐθέν) and Book VII, def. 1 «A unit is that through which each of the beings is said one» (μονάς ἐστιν καθ' ἣν ἕκαστον τῶν ὄντων ἐν λέγεται)¹⁸.

μὲν ὑποκειμένης, αὐτῇ δὲ τῶν ἄλλων ὑποκειμένων. τὸ μὲν γὰρ σημεῖον, εἰ καὶ ἀρχὴ τῶν ἐν γεωμετρίᾳ πάντων ἐστίν, ἀλλὰ καὶ ἐκ τῆς κοινῆς ἀνήπται πάντων ἀρχῆς καὶ ὑπόκειται ἐκείνῃ. τινῶν οὖν ἀρχὴ καὶ μονάς καὶ οὐ πάντων ὡσαύτως, ἀριθμῶν μὲν ἀρχὴ καὶ τῶν ἐν ἀριθμητικῇ πάντων, ὑπόκειται δὲ τῇ πάντων ἀρχῇ. καὶ ἐπαναβασμοῖς τοῦτοις χρῆσθαι δυνατόν τοις ἐπ' ἐκείνην ἀνιοῦσιν τὴν τοῦ παντὸς ἀρχήν.

17. ὅσα γὰρ ἡμῶν ἡ διάνοια θεωρεῖν ἐν τοῖς θεοῖς ἀδυνατεῖ, ταῦτα ἐν τοῦτοις ὡς ἐν εἰκόσι κατιδοί, καὶ κατιδῶν ἐπαναπαύσεται τῇ τούτων θεωρίᾳ, καὶ πιστεύσει καὶ τοῖς περὶ ἐκείνων λεγομένοις· λέγω δὲ οἶον, εἰ θαναμάζοι πῶς ἐν ἐνὶ τὰ πολλὰ καὶ ἐν ἀμερεῖ πάντα, τὴν μονάδα ἐννοήσει, καὶ ὅπως ἐν αὐτῇ δέκνυνται ὄντα πάντα καὶ ἄρτια καὶ περιττά, κύκλος τε καὶ σφαῖρα, καὶ τὰλλα εἶδη τῶν ἀριθμῶν. See O'Meara 1989 chapter 10.

18. Accessing theology through meta-mathematical speculation does not

To attempt to grasp how the ascent to theology through common mathematics works according to Proclus, we must first clarify some aspects of his philosophy of mathematics. We shall focus on the notion of number, since Euclid's definition of unity seems philosophically promising. Indeed, this definition – already in use and metaphysically interpreted in the pre-Euclidean age – extends beyond the disciplinary boundaries of mathematics, confronting the reader of the *Elementa* with a metaphysical proposition: unity, or *a* unity (the statement is ambiguous), is that by virtue of which each *being* is said to be «one». This definition is meant to pave the way for the following one, namely that of number (Euclid's VII, def. 2): «A number is a multitude composed of units» (ἀριθμὸς δὲ τὸ ἐκ μονάδων συγκεῖμενον πλῆθος). From these definitions, it follows that (i) the unit is not itself a number; (ii) numbers are not units (but collections of units); and (iii) each number is *one* number – that is, each multitude (πλῆθος) of units is, *qua* number, *one* multitude of units.

Here a philosophical puzzle arises: how can each number be one number if it is a multitude of units, yet not itself a unit? Proclus' philosophy of arithmetic, which he arguably inherited from Syrianus¹⁹, resolves this difficulty by applying to number two general features of his metaphysics: (a) that all begins except the first principle are not one by themselves, and (b) that all beings except the first principle are unified (or said to be «one») by virtue of their subordination to some unifying principle. Accordingly, Proclus endorses the long-held distinction between monadic numbers (numbers composed of units) and eidetic or divine numbers (indivisible and unique numbers). In his view, the oneness of a monadic number derives from its participation in an eidetic number: five units are said to be *one* multitude – and thus a single monadic number – because of the Form of five, or five-ness itself. Hence, the eidetic number is conceived as a monad that bestows the character of «being one-five» upon a

exclude the simultaneous and necessary pursuit of other paths, such as theurgic practice and moral refinement.

19. See Syr. *In Metaph.* 133.8–14. See also Mueller 2002; Mouzala 2015; Opsomer 2014.

series of fitting, unified manifolds²⁰. Ultimately, the source of the oneness imparted by the eidetic number is the One-itself, in which both eidetic and monadic numbers participate.

Considering all this, we now return to Euclid's arithmetical definitions. If a monadic number, *qua* that number, is said to be «one», then, according to Euclid's definition of the unit, a collection of units (say, five units) can only be said to be one number (one token of five-ness) through (a) unit. What does this mean precisely? If we interpret this statement through the lens of Proclus' philosophy of arithmetic, the function «Y is said to be X through a unit» must be reformulated as «Y participates in some way in X-ness». Consequently, a number is said to be one X by participating in and being unified through X-ness. If this principle applies not only to numbers but to all unified beings, as Proclus upholds and Euclid's VII def. 1 declares (ἕκαστον τῶν ὄντων), then a metaphysical claim follows: every composite being is said to be «one» by participating in oneness (or *a* oneness). From this, propositions 1-4 of the *Elementatio theologica* can be logically derived. Seeking to remain faithful to Proclus' own manner of reasoning, we may now propose the following tentative formalisation:

Analysis from arithmetic

(o) Every (arithmetic) number is a (one) being (i.e., arithmetic numbers exist).

(1) Euclidean definitions:

(1.a, VII, def. 2) A number is a multitude composed of units.

(1.b, VII, def. 1) A unit is that by virtue of which each of the beings is said one.

20. See, for instance, Procl. *Theol. Plat.* IV 34.101.8-11: ἐπεὶ κὰν τοῖσδε τοῖς μοναδικοῖς ἀριθμοῖς ἄλλα μὲν τὰ εἶδη τῶν ἀριθμῶν, οἷον ἡ τριάς, ἡ πεντάς, ἡ ἐπτάς, ἄλλαι δὲ αἱ τῶν εἰδῶν ἐνώσεις· καὶ γὰρ τούτων ἕκαστον τῶν εἰδῶν ἓν ἐστὶ καὶ πλῆθος. Note that, in this passage, Proclus emphasises that every monadic number is simultaneously one and a multitude – a detail which will be crucial below. For Proclus' views on eidetic numbers, see *Theol. Plat.* IV 29.86.1-10; *In Ti.* II 173.13-174.3 Van Riel (= I 324.4-14 Diehl); IV 132.14-19 Van Riel (= III 102.17-23 Diehl). On monadic number, see *In Prm.* VII 1175.5-14; *In Eucl.* 95.26-96.3.

Analysis within common mathematics (from arithmetic to dialectic)

(1.c, Proclean postulate) No number is one by itself.

(2) If every number (*qua* that number) is *one* multitude composed of units (from 0 and 1.a), then every number is said to be one (from 1.a and 1.b).

(3) Every multitude composed of units is a number (identity inversion of 1.a).

(4) Every being is said to be one either by itself or through another (from 1.b).

(4.a) Every being said to be one through another is not one by itself (from 4).

(5) If every multitude composed of units is a number (3), and every number is said one (2), and no number is one by itself (1.c), and every being is said to be one either by itself or through another (4), then every multitude composed of units is said to be one through another.

Analysis within dialectic

(6) If every multitude composed of units is said to be one through another (5), and a unit is that through which each of the beings is said to be one (1.b), then there is some unit through which every multitude composed of units is said to be one.

(7, re-definition as participation) If a being Y is said to be X through another (from 4), this means that Y participates in some way in X-ness, and X-ness is the unit through which any being is said to be X.

(8) If every multitude composed of units is said to be one through another (5), and every being is one either by itself or through another (2), and every being said to be X participates in some way in X-ness (from 7), then *every multitude* (composed of units) *participates in some way in one(ness)* (Proclus, prop.1).

(9) If every multitude composed of units is said to be one (5) and every being is said to be one either by itself or through another (2), and a being said to be X through another participates in some way in X-ness (from 7), and every being which is said to be one through another is not one by itself (4.a), then *every being that participates in one(ness) is both one* (through another) *and not one* (by itself) (Proclus, prop. 2).

(10) If there is some unit through which every multitude composed of units is said to be one (6), and a being said to be X through another participates in some way in X-ness, and X-ness is the unit through which any being is said X (7), then *every being that becomes one* (= it is said to be one through another) *does so by participation in one(-ness)* (Proclus, prop. 3).

(11) If *every being that participates in one(ness)* is both one (through another) and not one (by itself) (9, Proclus, prop. 2) and *every being that becomes one* (= it is said to be one through another) *does so by participation in one(-ness)* (10, Proclus, prop. 3), then *every being that is unified* (= it is said to be one through another) *is other than the One-itself* (*sive* oneness, *sive* one by itself) (Proclus, prop. 4).

Although this logical transition could be structured in alternative ways that may be more direct or more comprehensive, its overall features provide valuable insight into the logical continuity between mathematics and dialectic that Proclus postulates. Four aspects are particularly noteworthy:

First, it helps us to understand more clearly the preliminary role of common mathematics in relation to dialectic. As Proclus explains, it is through reflection on the first principles of mathematics that the learner is ultimately drawn into metaphysical speculation. Questions such as «What is a number?» lead the learner to consider the higher concepts upon which mathematical definitions depend. This constitutes one of the defining traits of common mathematics: it begins with propositions about mathematical concepts and concludes with metaphysical ones (see steps 1.c and 2 to 5).

Second, as is well known, Proclus' *Elementatio theologica* rests on certain implicit assumptions. Opsomer has convincingly argued that Proclus most likely avoided definitions and postulates in order to preserve the unhypothetical status of dialectic²¹. However, we should bear in mind that the Platonic hierarchy of the sciences is not merely a classification of disciplines according to their proximity to the Forms; it also outlines a structured cur-

21. Opsomer 2022, part 3.

riculum for students progressing toward dialectic. Following Plato, Proclus regarded mathematics as preparatory to dialectic, situating common mathematics as the final pre-dialectical stage. Consequently, it is reasonable to assume that Proclus' *Elementatio theologica* was intended for readers who, following the Platonic progression, were already familiar with the preliminary concepts of geometry and arithmetic. Thus, some of the notions developed within common mathematics would form part of the conceptual toolkit required for dialectical speculation, *while remaining part of common mathematics as a distinct discipline*. In this way, theology could be kept free from all hypotheses while nonetheless drawing its necessary background notions from common mathematics. Further evidence for this view lies in the fact that the key terms employed by Proclus in the opening propositions of the *Elementatio theologica* partially overlap with the lexicon of mathematics: unit (μονάς), multitude (πλῆθος), one (ἓν), as well as unlimited (ἄπειρον), and limit (πέρας). These are all terms found in Euclid – a Platonic philosopher himself, according to Proclus²².

In pedagogical terms, therefore, when Proclus employs, for example, the abstract noun πλῆθος in the opening propositions of the *Elementatio theologica*, he most likely assumes that the learner, as part of his philosophical training, has already advanced from the study of physical multitudes to the contemplation of incorporeal, countable multitudes (i.e., monadic numbers) by studying Euclid's *Elementa*. Such a learner would thereby have acquired the necessary familiarity with immaterial objects and the conceptual background required to embark upon dialectical speculation. Hence, the reader would know from his previous training in common mathematics that a πλῆθος is «a composite being that is said to be one, composed either of units or of other composite beings that are also said to be one» – a definition implied in Proclus' first proposition and easily inferred from Euclid's arithmetical definitions.

Third, once proposition 4 of the *Elementatio theologica* has been analytically deduced from arithmetical principles, the reverse operation should, in principle, also be logically possible. In such

22. See *In Eucl.* 68 ff.

a synthetic operation, arithmetical definitions – assumed as existential hypotheses within the domain of mathematics – are granted the status of demonstrable metaphysical theorems from the higher standpoint of dialectic. Thus, what Proclus' theory ultimately maintains is that arithmetical definitions, such as «A number is a multitude composed of units», should be read as existential claims when viewed from the perspective of dialectic: «There exists such a thing as a multitude composed of units, i.e., a number». This proposition would be established as a theorem derived from dialectical propositions: it follows from the One-itself that numbers – i.e., unitary multitudes – must factually exist. Therefore, every genuine mathematical principle ultimately stands as a metaphysical theorem synthetically demonstrated within the *interregnum* of common mathematics. This, in turn, shows how Proclus interpreted Plato's claim in the *Respublica* (510b) that mathematics becomes truly scientific through dialectic.

Lastly, we are now in a position to better understand the nature of the all-pervading syntax of Intellect. According to Proclus, the logical structure of discursive sciences, taken as a whole, mirrors the underlying relations of priority and dependence among beings. Hence, the theorematic arrangements of discursive sciences cannot be regarded as discrete systems: the seamless continuity of powers structuring Proclus' cosmos allows for no gaps; ontological continuity in the procession and reversion of beings entails logical – and therefore doctrinal – continuity. Inferential bridges must thus exist between sciences whose objects of study occupy adjacent positions within the metaphysical hierarchy. With the sole exception of dialectic, the principles of every subordinate science must be derivable from certain elements belonging to the science, or sciences, that stand above it.

This leads us to a further question – one that, for the sake of simplicity, I have set aside in this chapter – namely, the place of Proclus' *Elementatio physica* within this framework and its relation to prior mathematical sciences. As things currently stand, we can draw two preliminary conclusions. (1) First, according to Proclus, the physical world also bears the imprints of Intellect, as it receives it from the Soul. This transmission endows the sensible realm with some degree of intelligibility and, ultimately, makes it

possible to learn about it demonstratively. (2) Second, if Proclus is consistent in his thought, he must, *mutatis mutandis*, also assume the existence of a doctrinal transition between the principles of physics and those of mathematics.

Concluding Remarks: The Elementary Structure of Scientific Knowledge

In this chapter, we have explored how, for Proclus, the hallmark of scientific knowability lies in a being's capacity to be propositionally formalised and integrated into an inferential web governed by the operations of analysis, division, demonstration, and definition. When combined, these cognitive powers converge upon a precise structure in which propositions are deductively interwoven according to an order of explanatory priority and logical dependence: the elementary system. According to Proclus' philosophy of language, the ordering produced by these logical powers reflects the syntax of Intellect, which stands as the ultimate source of these powers and the compressed formal model of this logical arrangement.

Proclus also maintains that likenesses of the intellectual syntax cannot be achieved with equal efficacy across all sciences. The extent to which a science's logical structure can formally represent the noetic architecture depends on the ontological proximity of its subject matter to Intellect. Hence, the discursive science concerned with the loftiest object must be the one capable of representing the most accomplished logical expression of the undifferentiated noetic syntax. Therefore, whereas dialectical elementations can, at their finest, be arranged as self-sustained elementary systems (imitating the self-sufficiency of divine being), mathematical or physical systems (dealing with derivative entities) are structurally constrained to rely on hypothetical principles that they cannot justify. Consequently, as illustrated above through the case of Proclus' common mathematics, formal transitions between disciplines must exist, allowing for the ascent from lower to higher sciences and enabling the hypothetical principles of the former to be demonstrated from the perspective of the latter. In this resulting hierarchy of sciences, unhypothetical dialectic stands as the pinnacle.

It must also be emphasised that, according to Proclus, it would be utterly mistaken to claim that the elementary arrangements of theology attempt to emulate mathematical methodologies: the elementary syntax inherently belongs to theology; the so-called geometrical method is only an imperfect reflection, dependent upon the dialectical one.

All of this provides insight into Proclus' conception of the *elementatio* as a literary genre. According to his view, the elementary structure is not merely one mode of philosophical exposition among others. Rather, when composed correctly, the logical structures of the elementations are closer or more distant iconic instantiations of paradigmatic intelligibility²³. Therefore, for Proclus, composing *elementationes* was more than writing pedagogical textbooks aimed at clarity and thoroughness. Understood in his own terms, by producing these elementary works, Proclus also sought to retrace the syntax of Intellect itself, envisioning it as iconically expressed across the three theoretical sciences.

Bibliography

- R. Chlup, *Proclus: An Introduction*, Cambridge, Cambridge University Press, 2012.
- S. Gersh, *Κίνησις ἀκίνητος: A Study of Spiritual Motion in the Philosophy of Proclus*, Leiden – Boston, Brill, 1973.
- S. Gersh, *Proclus' Theological Methods: The Programme of Theol. Plat. I. 4*, in Id., *Reading Plato, Tracing Plato: From Ancient Commentary to Medieval Reception*, London – New York, Routledge, 2005, 15–27.
- C. Helmig, *Forms and Concepts: Concept Formation in the Platonic Tradition*, Berlin, De Gruyter, 2012.
- C. Helmig, *Proclus on Epistemology, Language, and Logic*, in P. d'Hoine – M. Martijn (eds), *All From One: A Guide to Proclus*, Oxford – New York, Oxford University Press, 2016, 183–206.

23. However, not every elementation is equally thorough, elegant, or clear. As an accomplished mathematician, Proclus was aware that alternative elementary models of a given science could be constructed, differing in their completeness, propositional arrangement, and inferential pathways. For Proclus' account of the literary genre of elementation and its composition, see *In Eucl.* 71 ff.

- S. A. Kiosoglou, *Proclus' Theory of Seemingly 'Useless' Theorems*, «Méthexis», 34 (2022), 148-65.
- A. Lernould, *La dialectique comme science première chez Proclus*, «Revue des Sciences Philosophiques et Théologiques», 71 (1987), 509-36.
- A. Lernould, *La διάνοια chez Proclus*, «Chôra», 14 (2016), 139-54.
- A. Lernould, *La ΣΤΟΙΧΕΙΩΣΙΣ ΘΕΟΛΟΓΙΚΗ de Proclus. Science première et dépassement de la méthode hypothético-déductive*, in G. Aubry - L. Brisson - P. Hoffmann - L. Lavaud (éd.), *Relire les Éléments de théologie de Proclus: Réceptions, interprétations antiques et modernes*, Paris, Hermann, 2021, 70-91.
- D. G. MacIsaac, *The Soul and Discursive Reason in the Philosophy of Proclus*, Doctoral Dissertation, University of Notre-Dame, 2001.
- D. G. MacIsaac, *Projection and Time in Proclus*, in J. Inglis (ed.), *Medieval Philosophy and the Classical Tradition in Islam, Judaism and Christianity*, London - New York, Routledge, 2002, 83-105.
- D. G. MacIsaac, *Νόησις, dialectique et mathématique chez Proclus*, in A. Lernould (éd.), *Études sur le Commentaire de Proclus au premier livre des Éléments d'Euclide*, Villeneuve d'Ascq, Presses Universitaires du Septentrion, 2010, 125-38.
- D. G. MacIsaac, *Geometrical First Principles in Proclus' Commentary on the First Book of Euclid's Elements*, «Phronesis», 59 (2014), 44-98.
- M. Martijn, *Proclus on Nature: Philosophy of Nature and Its Methods in Proclus' Commentary on Plato's Timaeus*, Leiden - Boston, Brill, 2010.
- M. Martijn, *Proclus' Geometrical Method*, in P. Remes and S. Slaveva-Griffin (eds), *The Routledge Handbook of Neoplatonism*, London - New York, Routledge, 2014, 145-59.
- G. R. Morrow (ed.), *Proclus. A Commentary on the First Book of Euclid's Elements*, Princeton, Princeton University Press, 1970.
- M. G. Mouzala, *Syrianus on the Platonic Tradition of the Separate Existence of Numbers*, «Peitho», 6 (2015), 167-94.
- I. Mueller, *Syrianus and the Concept of Mathematical Number*, in G. Bechtle - D. J. O'Meara (éd.), *La philosophie des mathématiques de l'Antiquité tardive*, Fribourg, Éditions Universitaires, 2000, 71-83.
- D. Nikulin, *Physica more geometrico demonstrata: Natural Philosophy in Proclus and Aristotle*, «Proceedings of the Boston Area Colloquium in Ancient Philosophy», 17 (2003), 183-209.
- D. J. O'Meara, *Pythagoras Revived: Mathematics and Philosophy in Late Antiquity*, Oxford - New York, Oxford University Press, 1989 (Oxford - New York, Oxford University Press, 2010).
- D. J. O'Meara, *Mathematics and the Sciences*, in P. d'Hoine - M. Martijn (eds), *All From One: A Guide to Proclus*, Oxford - New York, Oxford University Press, 2016, 167-82.

- J. Opsomer, *Mathematical Explanation and the Philosophy of Nature, Late Ancient Philosophy: Astronomy and the Theory of the Elements*”, «Documenti e studi sulla tradizione filosofica medievale», 23 (2012), 65-106.
- J. Opsomer, *Syrianus, Proclus, and Damascius*, in J. Warren – F. Sheffield (eds), *The Routledge Companion to Ancient Philosophy*, London-New York, Routledge, 2014, 626-42.
- J. Opsomer, *Proclus' Elements of Physics and the Axiomatization of Kinematics*, in F. Baghdassarian – I. Papachristou – S. Toulouse (éd.), *Relectures néoplatoniciennes de la théologie d'Aristote*, Baden-Baden, Academia, 2020, 83-102.
- J. Opsomer, *Organiser la philosophie selon ses éléments. Structures argumentatives dans les Éléments de théologie*, in G. Aubry – L. Brisson – P. Hoffmann – L. Lavaud (éd.), *Relire les Éléments de théologie de Proclus: Réceptions, interprétations antiques et modernes*, Paris, Hermann, 2021, 133-76.
- J. Opsomer, *Proclus' Elements of Theology and Platonic dialectic*, in D. Calma (ed.), *Reading Proclus and the Book of Causes*, Vol. 3: *On causes and the noetic triad*, Leiden – Boston, Brill, 2022, 17-36.
- J. L. Solère, *L'ordre axiomatique comme modèle d'écriture philosophique dans l'Antiquité et au Moyen Âge*, «Revue d'histoire des sciences», 56 (2003), 323-45.
- C. Steel, *Breathing Thought: Proclus on the Innate Knowledge of the Soul*, in J. J. Cleary (ed.), *The Perennial Tradition of Neoplatonism*, Leuven, Leuven University Press, 1997, 293-309.
- C. Steel, *Theology as First Philosophy. The Neoplatonic Concept of Metaphysics*, «Quaestio», 5 (2005), 2-21.
- A. Vargas, *Proclus on Direct Knowledge of the Forms*, «Methexis», 37 (2025), 73-89.

Álvaro José Campillo Bo, *The Unfolded Syntax of Intellect: Proclus on the Metaphysical Foundations of Elementary Arrangements*

This chapter examines Proclus' epistemology and ontology of the discursive sciences, focusing on the foundations that ground the logical structures of dialectic and mathematics. It explores Proclus' account of demonstrative reasoning and elementary arrangements as the iconic reflection of the architecture of Intellect. The chapter considers aspects of Proclus' philosophy of language to show how the structures unfolded by discourse reproduce the relational patterns of the Intellect. It analyses how the dynamics of Intellect manifest in the logical operations of dialectics and, in a more attenuated way, in mathematical demonstration, establishing a hierarchical relation between the two sciences. This hierarchy clarifies the dependence of mathematics on dialectic and their mutual intertwining through the discipline of "common mathematics". This dependence is examined by exploring the continuous logical syntax linking both sciences. As a case study, the chapter explores how the opening propositions of the *Elementatio theologica* (1 to 4) can be analytically derived from Euclidean arithmetic definitions. The chapter concludes that, for Proclus, the elementary logical syntaxes in contemplative sciences are not mere methodological tools but discursive enactments of the noetic order.

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