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INCOMMENSURABILITY IN GEOMETRICAL ENTITIES IN PROCLUS' *IN EUCL.* AND *IN PRM.*

Introduction

The central question I aim to explore in this chapter is the origin of incommensurability within Proclus' ontology. This issue is complex, and it has been observed that Proclus might not have regarded incommensurability as a subject of scientific inquiry at all. Yet, how does it emerge at a specific ontological level, and how does it relate to a particular mode of cognition? In this chapter, I will first identify the ontological status of geometrical entities. I will then show why neither sensible matter as such nor intelligible matter can unqualifiedly serve as the cause of incommensurability. I will demonstrate that incommensurability must rather be a property of certain specific geometrical items, belonging to them just as their other identifiable parts do. Finally, I will conclude by advancing what is surely a controversial position: that there might be a form of incommensurability.

Incommensurability in Proclus' Philosophy of Geometry

In his *Commentary on the First Book of Euclid's* Elements, Proclus situates the geometry presented by Euclid within a Neoplatonic framework. According to Proclus, Euclid's arguments in the *Elements* aim at the construction of the five regular bodies known from the *Timaeus*[1], progressively deriving them from the

1. «Solids» would imply their material solidity, which is not a given for geometric entities.

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highest principles[2]. The overarching argument begins with a first prologue, where Proclus locates mathematical entities within his ontological hierarchy, demonstrating how the common principles of general mathematics are responsible for the creation of geometrical entities[3]. In a second prologue, he elaborates on the goal of the first book of the *Elements* and the methods of geometry, including a general discussion of its postulates and axioms. After these two prologues, Proclus examines Euclid's definitions, followed by a detailed discussion of specific postulates and axioms, before finally addressing Euclid's propositions. The discussion Proclus presents thus becomes progressively more complex and increasingly removed from the higher and simpler causes.

Mathematical entities, generally speaking, occupy an intermediate level between the simple immaterial realities of the highest realm and the extended, composite, and confusedly complex objects of the sensible world[4]. They exist in the vestibule of the forms, being more exact than physical objects yet more complex than the forms themselves[5]. Proclus occasionally emphasises the complexity of mathematical entities by referring to them as *logoi*[6]. A closer look at mathematical entities reveals that they are further divided into a higher «arithmetical» part and a lower «geometrical» part. The well-known distinction between the two lies in their differing expressions of the relation between the Limit and the Unlimited. In the arithmetical part, the Limit ensures that each number remains finite, while the Unlimited accounts for the infinite additivity of numbers. In the geometrical part, the Limit accounts for the boundaries of entities, whereas the Unlimited accounts for their infinite divisibility[7]. As Proclus notes, «If there were no infinity, all magnitudes would be commensurable and there would be nothing inexpressible or irrational»[8]. Magnitudes

2. See Procl. *In Eucl.* 68.21-23; 70.19-71.5; 82.25-83.2.

3. See Procl. *In Eucl.* 5.14-18: τὰς δὲ ἀρχὰς τῆς μαθηματικῆς ὅλης οὐσίας ἐπισκοποῦντες ἐπ' αὐτὰς ἄνιμεν τὰς διὰ πάντων τῶν ὄντων διηκούσας ἀρχὰς καὶ πάντα ἀφ' ἑαυτῶν ἀπογεννώσας, λέγω δὲ τὸ πέρας καὶ τὸ ἄπειρον.

4. See Morrow 1970, XXXIII.

5. See Nikulin 2008, 154, referring to Procl. *In Eucl.* 5.2-3; 55.2.

6. See Nikulin 2008, 157.

7. See Nikulin 2008, 158, referring to Procl. *In Eucl.* 6.21

8. See Procl. *In Eucl.* 6.19-21: καὶ τῆς μὲν ἀπειρίας οὐκ οὔσης τά τε μεγέθη πάντα σύμμετρα ἂν ἦν καὶ οὐδὲν ἄρρητον οὐδὲ ἄλογον.

do not only exist on this intermediate ontological level. They also manifest within the lower realm of the sensible world and are present as *logoi* – though in a distinct, non-extended mode[9]. The *logos* of a geometrical object is the cause of both the geometrical and the sensible entity. The difference between geometrical and sensible objects is that the former are projected onto the soul's *phantasia* – i.e., intelligible matter[10] or *pathetikos nous*[11], a faculty of the soul – whereas the latter are projected onto the lower kind of matter that accounts for sensible objects. One key difference between the geometrical and the sensible projection is the degree of precision each allows, a point which will prove important for our understanding of incommensurability. Clearly, the *phantasia*, unlike the sensible world, can represent a geometrical figure such as a circle with precision. It is this projection onto the soul's *phantasia* that gives geometrical entities their extended nature, just as projection onto lower matter results in the extension of sensible objects[12]. Once the notions of matter, magnitude, and, with them, divisibility are introduced, we might infer that this is also the direct cause of incommensurability[13]. Yet, the problem is more complex than that.

The extended yet precise geometrical objects within the *phantasia* represent the mode by which the soul seeks to comprehend the unextended paradigms. Proclus describes this process as follows (*In Eucl.* 54.27-55.4):

For the understanding contains the ideas but, being unable to see them when they are wrapped up, unfolds and exposes them and pre-

9. Procl. *In Eucl.* 54.5-13.

10. As Proclus notes, the soul is not a passive receptacle but actively participates in the process of 'writing' these geometrical forms. See also Opsomer 2009, 215.

11. See Nikulin 2008, 163.

12. See Procl. *In Eucl.* 52.26-53.1: «(The soul) thinks the circle as extended, and although this circle is free of external matter, it possesses an intelligible matter provided by the imagination itself» (καὶ τόν τε κύκλον διαστατῶς νοεῖ τῆς μὲν ἐκτὸς ὕλης καθαρεύοντα νοητὴν δὲ ὕλην ἔχοντα τὴν ἐν αὐτῇ).

13. If *phantasia* is furthermore considered an irrational faculty of the soul, incommensurability, being *alogos*, seems to have found its proper place. See also Opsomer 2009, 220.

sents them to the imagination sitting in the vestibule; and in imagination, or with its aid, it explicates its knowledge of them [...][14].

Phantasia functions as a tool that enables the soul to grasp items that would otherwise be inaccessible to it[15]. Through the process of unfolding and dianoetic reasoning, the soul can reveal aspects of the *logoi* by systematically working through them. The unfolding, on the one hand, allows us to discover the properties of these entities through discursive reasoning – properties that cannot be apprehended merely by contemplating the *logoi* themselves[16]. Discursive reasoning, on the other hand, expands these properties through construction, employing techniques such as analysis, division, definition, and demonstration[17]. Proclus explicitly states that the relations and properties of geometrical entities are not derived from the lower level of the soul but are instead present in the higher realm, though in a non-extended form. He writes (*In Eucl.* 55.23-56.8):

Every true geometer should cultivate such efforts and make it his goal to arouse himself to move from imagination to pure and unalloyed

14. ἔχουσα γὰρ ἡ διάνοια τοὺς λόγους, ἀσθενοῦσα δὲ συνεπτυγμένως ἰδεῖν ἀναπλοῖ τε αὐτοὺς καὶ ὑπεκτίθεται καὶ εἰς τὴν φαντασίαν ἐν προθύροις κειμένην προάγει καὶ ἐν ἐκείνῃ ἢ καὶ μετ' ἐκείνης ἀνελίττει τὴν γνῶσιν αὐτῶν [...]. See also Procl. *In Eucl.* 56.13-23: ἀλλὰ καὶ ὅσα κρυφίως ἐστὶν ἐν ἐκείνῳ, διαστατῶς καὶ μεριστῶς εἰς φαντασίαν προάγεται καὶ τὸ μὲν προβάλλον ἡ διάνοια, τὸ δὲ ἀφ' οὗ προβάλλεται τὸ διανοητὸν εἶδος, τὸ δὲ ἐν ᾧ τὸ προβαλλόμενον παθητικὸς οὗτος καλούμενος νοῦς, ἐξελίττων ἑαυτὸν περὶ τὴν ἀμέρειαν τοῦ ἀληθοῦς νοῦ καὶ διϊστὰς ἑαυτοῦ τὸ ἀδιάστατον τῆς ἀκραιφνοῦς νοήσεως καὶ μορφῶν ἑαυτὸν κατὰ πάντα τὰ ἀμόρφωτα εἴδη καὶ πάντα γιγνόμενος, ἅ ἐστιν ἡ διάνοια καὶ ὁ ἀμερὴς ἐν ἡμῖν λόγος.

15. See Bechtle 2006, 337: «Wenn die Geometrie also - Proklos' Argumentation gemäß - über den Kreis, seinen Durchmesser, seine Tangenten o.ä. Aussagen macht, dann erfahren wir nichts über Kreise (allgemein *logoi*) in sinnlich wahrnehmbaren Dingen [...] – sagt Proklos - und auch nichts über die *logoi* in der *dianoia*. Denn der Kreis in der Geometrie ist auf der einen Seite immateriell, nämlich im Gegensatz zum Kreis in sinnlich wahrnehm-baren Dingen, auf der anderen Seite multipel, teilbar etc., nämlich im Gegensatz zu den *logoi* in der *dianoia*. Da die Geometrie sich also nur mit den projizierten *logoi* in der Vorstellung (beispielsweise also vorgestellten Kreisen) beschäftigt, sieht der Geometer die eingeborene Form oder Idee (den *logos*) auch nur indirekt als Spiegelbild, obwohl der *logos* selbst (der in der *dianoia*) das eigentliche Ziel seiner Beschäftigung ist». See O'Meara 2005, 138.

16. See Nikulin 2008, 164.

17. See Nikulin 2008, 165, referring to Procl. *In Eucl.* 42.9-43.21.

understanding, thus rescuing himself from extension and «passive nous» for the dianoetic activity that will enable him to see all things without parts or intervals – the circle, the diameter, the polygons in the circle, all in all and each separately. This is why even in our imagination we show circles as inscribed in polygons and polygons as inscribed in circles, in imitation of the proof that the part-less ideas exist in and through one another[18].

Thus, an unextended entity, such as the circle in the higher realm, possesses a diameter and relations to other geometrical entities, albeit in an unextended manner. Now, incommensurability might either result from the weakening power of matter, or it might be a property caused by the higher realm, on a par with the diameter or the angle, which become extended only within the *phantasia*.

Incommensurability and Matter?

Incommensurability and Imprecision

Having set the stage, we may now ask where incommensurability actually originates. The first answer that comes to mind is matter. The unextended *logoi* of geometrical objects are projected onto two distinct recipients: the soul's *phantasia* and the matter of the sensible world[19]. Both of these recipients weaken and extend the *logoi*. However, there is a clear distinction between the two projections. Sensible objects, by their very nature, are imprecise[20]. Everything in the sensible world is extended in three dimen-

18. καὶ ταύτην δεῖ τὴν μελέτην μελετᾶν τὸν ὡς ἀληθῶς γεωμετρικόν, καὶ πρὸς τὴν ἔγερσιν καὶ τὴν ἀπὸ τῆς φαντασίας μετάστασιν εἰς μόνην τὴν διάνοιαν αὐτὴν καθ' αὑτὴν ποιεῖσθαι τέλος, ἁρπάζοντα ἑαυτὸν ἀπὸ τῶν διαστάσεων καὶ τοῦ παθητικοῦ νοῦ πρὸς τὴν διανοητικὴν ἐνέργειαν, καθ' ἣν πάντα ἀδιαστάτως ὄψεται καὶ ἐν ἀμερεῖ τὸν κύκλον, τὴν διάμετρον, τὰ ἐν τῷ κύκλῳ πολύγωνα, καὶ πάντα ἐν πᾶσιν καὶ ἕκαστον χωρίς. διὰ γὰρ τοῦτο καὶ ἐν φαντασίᾳ δείκνυμεν ἔν τε τοῖς πολυγώνοις τοὺς κύκλους ἐγγραφομένους καὶ ἐν τοῖς κύκλοις τὰ πολύγωνα, μιμούμενοι τὴν τῶν ἀμερῶν λόγων δι' ἀλλήλων δεῖξιν.

19. See Procl. *In Eucl.* 51.13-15: καὶ τὰ μετέχοντα αὐτὰ διττὰ θέμενοι, τὰ μὲν αἰσθητὰ τὰ δὲ ἐν φαντασίᾳ τὴν ὑπόστασιν ἔχοντα.

20. See Nikulin 2019, 137. Procl. *In Eucl.* 12.23-26: οὐχ ὁρῶμεν, ὡς ἐν ἀλλήλοις πάντα τὰ αἰσθητὰ συμμέμικται καὶ ὡς οὐδὲν ἐν τούτοις εἰλικρινὲς οὐδὲ τοῦ ἐναντίου καθαρεῦον, ἀλλὰ μεριστὰ πάντα καὶ διαστατὰ καὶ κινούμενα.

sions. For instance, we will never encounter a one-dimensional line or an unextended point in the sensible world[21]. Moreover, in the sensible realm, magnitudes are conflated with one another: what we might identify as a circle displays features of straightness, and what strives to be straight exhibits traits of roundness. No comparison between two entities in the sensible world can ever be exact, owing both to their three-dimensional extension and to the confusion of their contrary properties. The absence of any precise interval in the sensible world makes it impossible to speak meaningfully of commensurability. This does not mean that everything in the sensible world is incommensurable. Rather, the characteristic feature of the sensible world is imprecision. Thus, there are no geometrical objects in the strict and precise sense in the sensible world[22] – only imprecise magnitudes.

Incommensurability only comes into play when the relationship between two intervals cannot be expressed in terms of a common unit. However – and this point is crucial – it must occur within a realm where *commensurability* is at least possible. Imprecision in the sensible world is not the same as incommensurability. Imprecision precludes any determination of whether something is commensurable or not. The sensible world, which is incapable of providing any precise unit owing to its inherent imprecision, instability, and motion, is therefore characterised neither by commensurability nor by incommensurability, but simply by imprecision. The sensible realm does not even permit the use of the anthyphairetic method to detect incommensurability, since each step of this method requires at least partial commensurability, precision, and a unity, followed by an interval that is not commensurable with the former, and so on. Incommensurability, then, is not a feature of the sensible world but of the *phantasia*. And if this is the case, the direct deduction of incommensurability from lower matter becomes implausible.

21. See Procl. *In Eucl.* 49.12-17: ποῦ δὲ καὶ τεθεάμεθα ἐν τοῖς αἰσθητοῖς τὸ ἀμερὲς σημεῖον ἢ τὴν ἀπλατῆ γραμμὴν ἢ τὴν ἀβαθῆ ἐπιφάνειαν ἢ τὴν ἰσότητα τῶν ἐκ τοῦ κέντρου γραμμῶν ἢ ὅλως τὰ πολύγωνα καὶ πολύεδρα σχήματα πάντα, περὶ ὧν ἡ γεωμετρία διδάσκει.

22. See Nikulin 2019, 137.

Incommensurability and Intelligible Matter

Given this, it makes sense to take not lower matter but intelligible matter as the cause of incommensurability. Incommensurability, then, would not result from the weakening force of lower matter. Proclus, after all, viewed incommensurability primarily as a feature of the geometrical realm of the *phantasia*[23]. However, to take intelligible matter as the cause of incommensurability is also problematic, since only certain items projected upon it exhibit incommensurability, while others do not[24]. Con-

23. See Procl. *In Eucl.* 6.19-22: καὶ τῆς μὲν ἀπειρίας οὐκ οὔσης τά τε μεγέθη πάντα σύμμετρα ἂν ἦν καὶ οὐδὲν ἄρρητον οὐδὲ ἄλογον, οἷς δὴ δοκεῖ διαφέρειν τὰ ἐν γεωμετρίᾳ τῶν ἐν ἀριθμητικῇ […]. See Procl. *In Eucl.* 278.19-24: ὅταν γὰρ δεικνύωσιν ὅτι ἔστιν τὸ ἀσύμμετρον ἐν τοῖς μεγέθεσι καὶ οὐ πάντα σύμμετρα ἀλλήλοις, τί ἄλλο δεικνύναι φήσει τις αὐτοὺς ἢ ὅτι πᾶν μέγεθος εἰς ἀεὶ διαιρεῖται καὶ οὐδέποτε ἥξομεν εἰς τὸ ἀμερές, ὅ ἐστι κοινὸν μέτρον τῶν μεγεθῶν ἐλάχιστον.

24. This distinction was already clearly recognised by Aristotle. See Karasmanis 2011, 389: «Does he think that incommensurability is a result of infinite divisibility? We do not have evidence that Aristotle thought so. On the contrary, from a passage in the *Prior Analytics* (65b16-20) we see that Aristotle thinks that infinite divisibility and incommensurability are two rather independent properties». See Karasmanis 2011, 393: «Now, if we agree that Plato says (25a6-7 [in the *Philebus*, DW]) that apeiron admits opposite characteristics to those of *peras*, then we have to conclude that incommensurability is a further and very important characteristic of *apeiron*. If we define continuity in terms of Zenonian infinite divisibility, as Aristotle does, then all the pairs of the infinite cuts of a continuous thing (e.g., a line) have a specific ratio (a):(b) (where (a) and (b) are numbers). Therefore, according to Plato's conception of *peras*, the Aristotelian continuum belongs to peras and not to apeiron. Hence, we cannot interpret Plato's apeiron just in terms of continuity or infinite divisibility». Commensurability depends on the 'cuts' we make in the substrate. If those cuts are in a 'Zenonian' ratio, there is no incommensurability. According to Karasmanis, Proclus did not work this argument into his *Commentary on Euclid*. Nevertheless, it is significant for our investigation, as it highlights yet another reason to distinguish incommensurability from unlimitedness. See *ibid.*: «Let us look at some propositions from Book X of Euclid's *Elements*: Proposition 5: «Commensurable magnitudes have to one another the ratio which a number has to a number». Proposition 6: «If two magnitudes have to one another the ratio which a number has to a number, the magnitudes will be commensurable». Proposition 7: «Incommensurable magnitudes do not have to one another the ratio which a number has to a number». Proposition 8: «If two magnitudes do not have to one another the ratio which a number has to a number, the magnitudes will be incommensurable».

sider the circle, which according to Proclus does not exhibit incommensurability; the same applies to some triangles. These figures can be incommensurable with other entities, but not in themselves. Although all such entities are projected onto intelligible matter, only some manifest incommensurability, while others do not. The difference between the "effect" of lower matter, which causes *everything* to become imprecise and three-dimensionally extended, and that of intelligible matter, where only certain items display incommensurability, requires explanation. For now, it is clear that we must determine why incommensurability arises in some cases but not in others.

Commensurability and Incommensurability in the Cosmos

So far, we have established that incommensurability is a feature of the *phantasia*, yet there must be a reason why it arises in some extended geometrical entities but not in others. If intelligible matter were the sole cause, all such entities would exhibit incommensurability in themselves.

In Ti. II 261.20–262.2 Van Riel (= I 384.29-385.3 Diehl) and *In Eucl.* 6.19-22 suggest a link between matter and incommensurability. However, it is in *In Prm.* VII 1205.9-1206.28 that Proclus develops the concepts of commensurability and incommensurability in greater detail[25]. Proclus introduces first equality and

25. I quote the entire passage in the footnote and comment on specific lines in the text. The full passage from *In Prm.* VII 1205.1-1206.20 reads as follows: ἐπεὶ γάρ εἰσι καὶ ἐν ταύτῃ πολλαὶ δυνάμεις, αἱ μὲν ἀλλήλαις σύστοιχοι καὶ πρὸς τὸ αὐτὸ τέλος καὶ ἀγαθὸν ἀνατεινόμεναι, αἱ δὲ διαφέρουσαι κατά τε ὑπεροχὴν καὶ ὕφεσιν, ἐκείνας μὲν κατὰ τὴν ἰσότητα χαρακτηρίζεσθαι ῥητέον – τὸ γὰρ αὐτὸ ἀγαθὸν μέτρον ἐστὶν ἑκάστων· τὰ οὖν τῷ αὐτῷ ἀγαθῷ συνηνωμένα τῷ αὐτῷ μέτρῳ μεμέτρηται καὶ ἔστιν ἴσα ἀλλήλοις –, τὰς δέ γε μὴ συστοίχους ἀλλήλαις κατὰ τὸ ἄνισον πεποιῆσθαι τὴν πρόοδον θετέον, ἐπειδὴ τὰς μὲν ὑπερέχειν, τὰς δὲ ὑφεῖσθαι λέγομεν. ἀλλ' ἐπεὶ καὶ τῶν ἀνίσων τὰ μὲν σύμμετρα, <τὰ δὲ ἀσύμμετρα>, δῆλον ὅτι καὶ ταῦτα τοῖς θείοις ἐφαρμόσομεν, τὴν μὲν συμμετρίαν ἐφ' ὧν (10) τὰ δεύτερα συνανακεράννυται τοῖς πρὸ αὐτῶν καὶ ὅλων μετέχει τῶν κρειττόνων – οὕτω γὰρ καὶ ἐν τοῖς συμμέτροις τὸ ἔλαττον ἐθέλει μέτρον κοινὸν ἔχειν πρὸς τὸ μεῖζον, ὅλον ἑκάτερον τοῦ αὐτοῦ μετροῦντος –, τὴν δὲ ἀσυμμετρίαν ἐφ' ὧν τὰ καταδεέστερα διὰ τὴν τῶν κρειττόνων ἐξῃρημένην ὑπεροχὴν μετέχει (15) μέν πως αὐτῶν, ὅλοις δὲ αὐτοῖς συνάπτεσθαι διὰ τὴν ἑαυτῶν ὕφεσιν ἀδυνάτως ἔχει· ταῖς γὰρ μερικαῖς αἰτίαις καὶ πολλοσταῖς ἀπὸ τῶν πρώτων ἀσύμμετρός ἐστιν ἡ πρὸς ἐκεῖνα κοινωνία καὶ ἡ πρὸς

inequality as broad principles before coming to our notions (*In Prm.* VII 1204.24-28):

> Now, as we have said, equality and inequality are to be taken as penetrating throughout the whole of divinity in the cosmos, even if the proofs also suit physical equals and mathematic equals and those in reason-principles of soul and those in the intellectual forms[26].

As subclasses of the equal and the unequal, commensurability and incommensurability are also introduced. Unlike in the *Commentary on Euclid's* Elements, Proclus here first discusses what we might call vertical commensurability and incommensurability. He says (*In Prm.* VII 1205.9-28):

τὸ αὐτὸ νοητὸν ἀγαθὸν ἐκείνοις ἀνάτασις. καὶ ἔοικεν, εἰ τῶν ἐγκοσμίων ἐστὶ θεῶν ταῦτα συνθήματα, τὸ ἴσον (20) λέγω καὶ τὸ ἄνισον, εἰκότως ἐνταῦθα καὶ τὸ σύμμετρον ἀναφανῆναι τοῦτο καὶ τὸ ἀσύμμετρον· ἐν μὲν γὰρ τοῖς ἀσωμάτοις καὶ ἀΰλοις χώραν ἡ τούτων ἀντίθεσις οὐκ ἔχει, πάντων ἐκεῖ ῥητῶν ὄντων καὶ ἐν εἴδεσι καθαροῖς ὑφεστώτων, ὅπου δέ ἐστι καὶ ὑλικὸν ὑποκείμενον καὶ μίξις εἴδους καὶ ἀνειδέου τινός, εἰκότως καὶ συμμετρίας ἐστὶν ἐνταῦθα ⌞καὶ ἀσυμμετρίας⌟ ἀντίθεσις, ὧν οἱ ἐγκόσμιοι προσεχῶς εἰσι συνεκτικοί, ψυχῶν καὶ σωμάτων, εἴδους καὶ ὕλης. εἰκότως ἄρα πέφυκεν ἐν αὐτοῖς κατὰ τὴν <τοῦ> ἀνίσου διαίρεσιν τὸ σύμμετρον καὶ ἀσύμμετρον, διὸ καὶ τούτων ὁ παρὼν ἐμνημόνευσε (30) λόγος. (1206) Ταῦτα μὲν οὖν καὶ εἰσαῦθις ἐπὶ πλέον ἀνασκεψόμεθα. νῦν δὲ ὅτι γεωμετρικῶς πρὸ τῶν ἀποδείξεων ὅρους παραλαμβάνει τοῦ ἴσου, τοῦ συμμέτρου, τοῦ ἀσυμμέτρου, λεκτέον· ἴσον μὲν λέγων τὸ τοῖς αὐτοῖς μέτροις μετρούμενον, σύμμετρον δὲ τὸ τῷ αὐτῷ μέτρῳ μετρούμενον, ἐὰν μὲν μεῖζον ᾖ, (5) πλεονάκις ἢ τὸ ἔλαττον, ἐὰν δὲ ἔλαττον, ἐλαττονάκις ἢ τὸ μεῖζον, ἀσύμμετρον δὲ τὸ διαιρούμενον εἰς ἴσα μὲν κατ' ἀριθμόν, ἄνισα δὲ κατὰ μέγεθος, εἰ μὲν ἔλαττον εἴη, δῆλον ὡς εἰς ἐλάττονα, εἰ δὲ μεῖζον, δῆλον ὡς εἰς μείζονα· τὸ μὲν ⌞γὰρ⌟ ἔλαττον σμικροτέρων ἔσται μέτρων ἴσων (10) κατ' ἀριθμόν, τὸ δὲ μεῖζον μειζόνων· ἐὰν γὰρ λάβῃς τὴν πλευρὰν καὶ τὴν διάμετρον καὶ διέλῃς ἑκατέραν δίχα, δῆλον ὡς ἡ μὲν μεῖζον ἕξει τῶν μερῶν ἑκάτερον, ἡ δὲ ἔλαττον. ἐπ' ἀμφοτέρων δὲ τῶν συμμέτρων καὶ τῶν ἀσυμμέτρων ὁμοίως πρὸς μείζονα καὶ ἐλάττονα τῆς παραβολῆς οὔσης, ἐφ' ὧν μὲν τῶν (15) μέτρων τῶν αὐτῶν ὄντων, ἀλλ' ἢ πλεονάκις ἢ ἐλαττονάκις, ἐφ' ὧν δὲ εἰς ἴσα μὲν τῆς διαιρέσεως γιγνομένης, ἀλλ' ἢ εἰς μείζονα κατὰ τὸ μέγεθος ἢ εἰς ἐλάττονα, ὅπου μὲν κατὰ τὸ πλῆθος τῆς διαφορᾶς οὔσης, ὅπου δὲ κατὰ τὸ μέγεθος ⌞αὐτῶν. καὶ γὰρ ὅπου μὲν κοινὸν μέτρον πάντα μετρεῖ, ὅπου δὲ οὐ δυνατὸν εἶναι πάντων ταὐτὸ μέγεθος⌟ ἀσυμμέτρων ὑπαρχόντων. τούτων δὴ τῶν ὅρων προειλημμένων ἑξῆς ἐπάγει τὰς ἀποδείξεις, δι' ὧν ἀποδείκνυται πρῶτον μὲν ὅτι οὐκ ἔστιν ἴσον τὸ ἓν οὔτε ἑαυτῷ οὔτε ἄλλῳ, ἔπειτα ὅτι οὔτε ἄνισον πάλιν οὔτε ἑαυτῷ οὔτε ἄλλῳ. λέγει δὲ οὕτως·

26. ἀλλ' ὥσπερ εἴπομεν, ἑνὸς ἰσότητα καὶ ἀνισότητα ληπτέον τὴν διὰ πάσης φοιτῶσαν τῆς ἐγκοσμίου θεότητος, εἰ καὶ ἐφαρμόζουσιν αἱ ἀποδείξεις καὶ τοῖς φυσικοῖς ἴσοις καὶ τοῖς μαθηματικοῖς καὶ τοῖς ἐν τοῖς λόγοις τοῖς ψυχικοῖς καὶ τοῖς ἐν τοῖς εἴδεσι τοῖς νοεροῖς.

But since even of those that are unequal some are commensurate and others incommensurate, it is plain that we must fit these concepts into the divine realm, (A) commensurability being taken as referring to those cases in which the secondary entities are mixed with those prior to them and participate in the whole of what is superior to them (for in this way also, in entities that are commensurate, the lesser desires to have a common measure with the greater, the same unit of measurement being applied to either of them as a whole); whereas (B) incommensurability applies in those cases where the lesser entities, by reason of the transcendent superiority of the greater, participate in them up to a point, but are unable to be joined to them as a whole because of their own inadequacy. For in the case of partial causes which are at many removes from the primal, communion with them is incommensurate and so is the striving upwards for them towards the same intelligible good, and it would seem, if these things are symbols of the encosmic gods, I mean the equal and the unequal, that it is reasonable that this "commensurate" and "incommensurate" should appear at this point. For among incorporeal and immaterial entities such an antithesis can have no place, since everything there is rationally related and has its basis in pure forms; but where there is also a material substratum and a mixture of form and a formless element, then it is reasonable that there should also be an antithesis of commensurability and incommensurability between those things which the encosmic gods maintain directly – souls and bodies, Form and Matter.

This passage is highly complex, yet Proclus appears to maintain that commensurability and incommensurability coexist[27] when matter is involved. In the context of his argument, he must mean that these are manifested in the relation between the higher and lower realms within a given object. He does not seem to be referring here to the horizontal commensurability and incommensurability discussed in the commentary on Euclid. By contrast, incorporeal and immaterial entities possess only rational relations to one another, and incommensurability thus has no place among them[28]. From this passage, we might infer that the

27. [...] εἰκότως καὶ συμμετρίας ἐστὶν ἐνταῦθα ⌊καὶ ἀσυμμετρίας⌋ ἀντίθεσις [...].

28. See Procl. *In Prm.* VII 1205.22-24: «For among incorporeal and immaterial entities such an antithesis can have no place, since everything there is rationally related and has its basis in pure forms» (ἐν μὲν γὰρ τοῖς

sensible-material realm is, as such, incommensurable. This would contradict the argument we tried to make about the difference between incommensurability and imprecision. However, the discussion of incommensurability in the *Commentary on the* Parmenides focusses on the relation between the higher and the lower realms. The lower realm is incommensurable insofar as it cannot reach the higher realm. This vertical incommensurability concerns not the entities of the lower realm themselves, but the relation those entities bear to the higher standard.

Incommensurability in Specific Entities

Simple Entities

Given the foregoing, neither lower matter nor intelligible matter can be the sole cause of incommensurability, as shown by the fact some geometrical entities do not exhibit it. Thus, we need to examine these entities more closely. Two straight lines of equal length are clearly commensurable with one another, even though they possess magnitude and extension and are, consequently, infinitely divisible. Thus, while infinitely divisible items exist within the geometrical realm, their ratios may or may not conform to numerical relations. The mere presence of infinity and magnitude does not, by itself, cause incommensurability, but it can create the conditions for incommensurability to arise[29]. At this point, the central question becomes what exactly is being related to what in this specific case. We could cut off a portion of a line such that it becomes incommensurable with another chosen line. But this would be arbitrary. Instead, we must shift the focus to a single geometrical entity and its internal proportions. When we speak of incommensurability, we are speaking of a property that an item possesses within itself, not of a comparison between two randomly chosen magnitudes.

Let us examine more closely the causes and properties of geometrical entities. Commensurability and incommensurability

ἀσωμάτοις καὶ ἀΰλοις χώραν ἡ τούτων ἀντίθεσις οὐκ ἔχει, πάντων ἐκεῖ ῥητῶν ὄντων καὶ ἐν εἴδεσι καθαροῖς ὑφεστώτων).

29. See Procl. *In Eucl.* 60.26-61.7.

seem to arise concretely from the interplay of the two principles: Limit and Unlimited. Proclus clarifies this when he explains that material extension is governed by the Unlimited, whereas the higher realm, which is free of extension, is governed by the Limit[30]. The interplay between Limit and Unlimited recurs at every level of being. Proclus states, in *In Eucl.* 151.13-152.5[31], that the *circular line* belongs to the realm and principle of the Limit[32]. Let us compare this with his account of the *straight line*, where he notes that the straight line belongs to the Unlimited (ἡ δὲ εὐθεῖα πρὸς τῆς ἀπειρίας)[33]. This prioritisation of the circular over the straight line has significant implications for understanding incommensurability. The *circular line*, as the first among extended geometrical entities, functions as the formative and limiting cause of the *straight line*, such as the diameter. It also holds precedence over the semicircle, which it governs[34]. The circle's priority is further evidenced by its direct dependence on the point rather than on the straight line. When Proclus explains

30. See Procl. *In Eucl.* 85.14-86.4.

31. See Procl. *In Eucl.* 151.13-152.5: ἀλλὰ ταῦτα μὲν μέχρι τούτων διαπεπεράνθω, τὴν δὲ μαθηματικὴν ἀπόδοσιν τοῦ κύκλου θεωρήσομεν εἰς πέρας ἀκριβείας ἥκουσαν. σχῆμα μὲν οὖν αὐτὸν ἔθετο διότι δὴ πεπέρασται καὶ περιέχεται πανταχόθεν ὑφ' ἑνὸς ὅρου καὶ οὐκ ἔστι τῆς ἀπείρου φύσεως, ἀλλὰ τῷ πέρατι σύστοιχος, καὶ ἐπίπεδον δὲ αὖ, καθόσον τῶν σχημάτων ἢ ἐν ἐπιφανείαις ὁρωμένων ἢ ἐν στερεοῖς, ὁ κύκλος τῶν ἐπιπέδων ἐστὶ τὸ πρώτιστον, ἁπλότητι μὲν τῶν στερεῶν ὑπερφέρων, μονάδος δὲ πρὸς τὰ ἐπίπεδα λόγον ἔχων, ὑπὸ μιᾶς δὲ γραμμῆς περιεχόμε νον, ὡς τῷ ἑνὶ προσήκοντα καὶ κατὰ τὸ ἓν ἀφοριζόμενον, τὴν δὲ ποικιλίαν τῶν περικειμένων ἔξωθεν ὅρων οὐ παραδεχόμενον, πρὸς δὲ ταύτην τὴν γραμμὴν ἴσας ἔχοντα πάσας τὰς ἀφ' ἑνὸς σημείου τῶν ἐντὸς αὐτοῦ κειμένων, διότι καὶ τῶν ὑπὸ μιᾶς ὁριζομένων γραμμῆς τὰ μὲν ἐκ μέσου πάσας ἴσας ἔχει, τὰ δὲ οὐ πάσας.

32. See also its priority for motion in Procl. *In Eucl.* 17.21-22: πάντων γὰρ τῶν κινουμένων ὁ κύκλος ἀρχὴ καὶ ἡ κύκλῳ κίνησις.

33. See Procl. *In Eucl.* 107.15.

34. In contrast, the modern approach often seeks to measure the circle by means of an infinite number of triangles, gradually approximating the length of its circumference. Proclus, however, would likely reject this procedure, as it attempts to measure something that belongs to the domain of the Limit by means of something inferior, rooted in the Unlimited. Arguably, he would maintain that incommensurability arises from the inferiority of the straight line, whereas the circular line itself does not exhibit incommensurability. One can, of course, select two segments of a circle that are incommensurable with one another, but this is a matter of chance and has no bearing on the nature of the circle.

how both circular and straight lines are constructed, he traces each back to the movement of the point[35]. Moreover, he explicitly rejects the claim that the straight line is necessary for the construction of the circular line. I will quote the entire passage (*In Eucl.* 106.20-107.11) below, as it is important for the continuation of the argument:

> One might think that, although both the straight line and the circle are simple lines, the straight line is the simpler. For it contains not even any dissimilarity in thought, whereas concavity and convexity in the circle indicate difference; and the straight line does not suggest the circle, whereas the circular line does bring to mind the idea of the straight line, if not through its mode of generation, at least by its relation to a center. What, then, if someone should say that the circle needs the straight line for its existence? For if one end of a finite line remains stationary and the other moves, it will describe a circle whose center is the stationary extremity of the straight line. Should we not reply that what describes the circle is not the line, but the point that moves about the stationary point? The line only defines its distance from the center, whereas what produces the circle is the point in circular movement[36].

This passage brings us closer to understanding why some geometrical entities exhibit incommensurability while others do not, as will become clear in what follows.

Complex Geometrical Entities

An intricate interplay across different ontological levels is emphasised, for instance, in *In Eucl.* 108-109. There, Proclus

35. The straight line in Procl. *In Eucl.* 97.6-7: ἀφορίζονται δὲ αὐτὴν καὶ κατ’ ἄλλας μεθόδους, οἱ μὲν ῥύσιν σημείου λέγοντες, and Proclus’ approval in 13-14: ἡ δὲ ῥύσις τὴν πρόοδον ἐνδείκνυται καὶ τὴν γόνιμον δύναμιν [...].

36. δόξειε δ’ ἂν ἀμφοτέρων οὐσῶν ἁπλῶν τῶν γραμμῶν, τῆς εὐθείας καὶ τῆς περιφεροῦς, ἁπλουστέρα μᾶλλον ἡ εὐθεῖα εἶναι. ἐν ταύτῃ μὲν γὰρ οὐδὲ κατ’ ἐπίνοιάν ἐστιν ἀνομοιότης, ἐπὶ δὲ τῆς περιφεροῦς τὸ κοῖλον καὶ κυρτὸν ἑτεροίωσιν ἐμφαίνει. καὶ ἡ μὲν εὐθεῖα τὴν περιφέρειαν οὐ συνεισάγει κατὰ τὴν ἐπίνοιαν, ἡ δὲ περιφέρεια τὴν εὐθεῖαν, εἰ καὶ μὴ κατὰ τὴν γένεσιν, ἀλλὰ κατὰ τὴν πρὸς τὸ κέντρον σχέσιν. τί οὖν, εἰ λέγοι τις καὶ τὴν περιφέρειαν δεῖσθαι τῆς εὐθείας εἰς τὴν ὑπόστασιν; εἰ γὰρ εὐθείας πεπερασμένης θάτερον μὲν τῶν περάτων μένοι, θάτερον δὲ κινοῖτο, γράψει κύκλον, κέντρον δὲ αὐτοῦ τὸ μένον τῆς εὐθείας πέρας. ἢ τὸ γράφον τὸν κύκλον τὸ σημεῖόν ἐστι περὶ τὸ μένον φερόμενον, οὐχ ἡ εὐθεῖα; τὴν γὰρ ἀπόστασιν αὕτη μόνον ἀφορίζει, τὴν δὲ κυκλικὴν γραμμὴν τὸ σημεῖον ὑφίστησι κινούμενον κυκλικῶς.

explains that the straight represents procession, while the round represents return. These principles govern not only spatial but also spiritual motion. Essentially, the circular line is associated with the Limit, whereas the straight line is associated with the Unlimited[37]. Therefore, the circle defines or limits, whereas the straight line, taken on its own, is indefinitely extensible. However, these principles interact in complex ways. As a diameter within a circle, the straight line depends on the limit imposed by the circle. Its unlimited extension is thus limited and made contingent upon it. With these two basic elements in view, we can now investigate how their interplay informs the construction of geometrical entities in the *phantasia*.

The circular line is not identical with the circular surface: while the circular line represents the Limit in one-dimensional space, the straight line represents the Unlimited in the same space[38]. Beginning with *In Eucl.* 115, we learn that in two-dimensional space another opposition arises – one crucial to our argument – namely, that between round and rectilinear figures (*In Eucl.* 115.3-8):

> For the circle, which is the principle of all curvilinear figures, carries a hidden trinity in its center, diameter, and circumference; and the triangle is the premier of all rectilinear figures, as everyone can see, because it is determined by the number three and formed by it[39].

The circle's surface is, furthermore, the most perfect of all surfaces[40]. This suggests, by analogy with the circular line, its affinity

37. See Procl. *In Eucl.* 107.11-16: ἔοικεν δὲ ἡ μὲν περιφέρεια πρὸς τοῦ πέρατος εἶναι καὶ τοῦτον ἔχειν τὸν λόγον πρὸς τὰς ἄλλας γραμμάς, ὃν πρὸς πάντα τὰ ὄντα τὸ πέρας – καὶ γὰρ ὥρισται καὶ σχῆμα ἀποτελεῖ μόνη τῶν ἁπλῶν – ἡ δὲ εὐθεῖα πρὸς τῆς ἀπειρίας· ἐπ' ἄπειρον οὖν ἐκβαλλομένη οὐδὲ παύεται. See for the priority of circular lines O'Meara 2005, 140.

38. Proclus does not appear to address the fact that the circular line, though itself one-dimensional, necessitates two-dimensional space.

39. ὁ μὲν γὰρ κύκλος, ὅς ἐστιν ἀρχὴ τῶν περιφερομένων, ἐν κρυφίῳ ἔχει τὸ τριαδικὸν τῷ κέντρῳ τῇ διαστάσει τῇ περιφερείᾳ· τὸ δὲ τρίγωνον ἁπάντων ἡγεμονοῦν τῶν εὐθυγράμμων παντί που δῆλον, ὅτι τῇ τριάδι κατέχεται καὶ κατ' ἐκείνην μεμόρφωται.

40. See Procl. *In Eucl.* 146.24-147.3: «The first and simplest and most perfect of the figures is the circle. It is superior to all solid figures because its being is of a simpler order, and it surpasses other plane figures by reason

with the Limit in two-dimensional space. The triangle, by contrast, as the principle of the other class of figures, must bear a closer resemblance to the Unlimited in two-dimensional space. This is hinted at, e.g., in *In Eucl.* 214, where Proclus presents the circle as the cause of the construction of a triangle – analogous to the way the circular line gives rise to the straight line, expressed through its diameter[41].

Even though the triangle appears to represent the Unlimited, it remains unclear why some triangles do not exhibit incommensurability within themselves while others do[42]. For instance, the

of its homogeneity and self-identity. It corresponds to the Limit, the number one, and all the things in the column of the better» (τὸ πρώτιστον καὶ ἁπλούστατον τῶν σχημάτων καὶ τελειότατον ὁ κύκλος ἐστί. τῶν μὲν γὰρ στερεῶν πάντων ὑπερφέρει τῷ ἐν ἁπλουστέρᾳ τάξει τὴν ὕπαρξιν ἔχειν, τῶν δὲ ἐν τοῖς ἐπιπέδοις ὑφισταμένων τῇ ὁμοιότητι καὶ ταυτότητι τὴν ὑπεροχὴν ἔλαχεν).

41. Procl. *In Eucl.* 214.4-8: «If, then, the circle is the likeness of intelligible being, and the triangle the likeness of the first soul because of the similarity and equality of its angles and its sides, it would seem reasonable to demonstrate it by means of circles as an equilateral middle area cut off in them» (εἰ τοίνυν ὁ μὲν κύκλος εἰκών ἐστι τῆς νοερᾶς οὐσίας, τὸ δὲ τρίγωνον τῆς πρωτίστης ψυχῆς διά τε τὴν ἰσότητα καὶ τὴν ὁμοιότητα τῶν γωνιῶν καὶ πλευρῶν, εἰκότως ἂν καὶ τοῦτο διὰ τῶν κύκλων ἐν αὐτοῖς μέσον ἀπολαμβανόμενον ἰσόπλευρον ἀποδεικνύοιτο).

42. Clearly, even though the triangle projected onto the soul represents the Unlimited, its distance from the first cause renders it a mixed entity. By this, I mean that the triangle, possessing a three-sided boundary, also exhibits features of the Limit. See Procl. *In Eucl.* 136; 143. See Procl. *In Eucl.* 142.24-143.5: «Since figure too, like the idea of the angle, exhibits in its own subdivisions the twofold progression of the Limit and the Unlimited, it applies the single boundary and the simple form to the things it bounds when it acts in accordance with the Limit and the many boundaries by virtue of the Unlimited. This is why everything figured has either one or more than one boundary» (ἐπεὶ γὰρ καὶ αὐτὸ τὴν τοῦ πέρατος καὶ ἀπείρου δυοειδῆ πρόοδον ἐν τοῖς οἰκείοις εἴδεσι προτείνει, καθάπερ δὴ καὶ ὁ τῆς γωνίας λόγος, τὸν μὲν ἕνα ὅρον καὶ τὸ ἁπλοῦν εἶδος ἐπάγει τοῖς ὑφ' ἑαυτοῦ περιεχομένοις κατὰ τὸ πέρας, τοὺς δὲ πολλοὺς κατὰ τὴν ἀπειρίαν. καὶ διὰ τοῦτο πᾶν τὸ ἐσχηματισμένον ἢ ἑνὸς ὅρου μετείληφεν ἢ πλειόνων). See also Procl. *in Eucl.* 144.6-18: «But whence comes the idea of figure and from what sort of principles is it perfected? I answer, first, that it owes its being to the Limit and the Unlimited and the Mixture of the two. This is why it generates some kinds by virtue of the Limit, others by reason of the Unlimited, and others according to the Mixed. For circular figures it invokes the idea of the Limit, for rectilinear that of the Unlimited, and for figures derived from both the idea of the Mixed» (ἀλλὰ πόθεν πρόεισιν ὁ τοῦ σχήματος λόγος καὶ

equilateral triangle does not exhibit incommensurability among its sides. Proclus distinguishes three types of geometrical triangles, each reflecting a different degree of proximity to the circle, and thus to the Limit. In *In Eucl.* 213 ff., we learn that the equilateral triangle is the most beautiful and the most akin to the circle[43]. The isosceles triangle stands somewhat further removed from it, while the scalene triangle occupies the opposite end of this spectrum, being furthest from the circle. It is in these latter two cases that incommensurability between sides may arise[44]. To be sure, incommensurability may also appear when we arbitrarily compare one geometrical entity with another. However, there are only a limited number of geometrical entities which display incommensurability in themselves and by themselves.

The Form of Incommensurability or the Display of Infinity?

So far, it remains unclear why only some triangles display incommensurability within themselves while others do not. We learned from Proclus that the task of the geometer's *phantasia* is to make visible what the understanding cannot perceive[45]. The entities that exist in an unextended manner, without shape or magnitude in the higher realm, nonetheless have their inherent proper parts. Though unextended and shapeless, the diameter and other elements are present within the form of the circle. Analogously, the angle and the limits of the triangle must likewise be present in an unextended manner within the form of the triangle. But what of incommensurability itself? Could it also be a property like these others? If incommensurability is coextensive

ἀπὸ ποίων αἰτίων τελειοῦται; λέγω δή, ὅτι πρῶτον μὲν ἐκ τοῦ πέρατος ὑφίσταται καὶ τοῦ ἀπείρου καὶ τοῦ ἐκ τούτων μεμιγμένου, διὸ καὶ αὐτὸς τὰ μὲν ἀπογεννᾷ κατὰ τὸ πέρας τῶν εἰδῶν, τὰ δὲ κατὰ τὸ ἄπειρον, τὰ δὲ κατὰ τὸ μικτόν, τοῖς μὲν περιφερέσι τὴν τοῦ πέρατος ἰδέαν ἐπάγων, τοῖς δὲ εὐθυγράμμοις τὴν τοῦ ἀπείρου, τοῖς δὲ ἐκ τούτων τὴν τοῦ μικτοῦ).

43. See O'Meara 2005, 140.

44. The elementary triangles of the *Timaeus* which make up the regular bodies are of the latter two classes, as Proclus acknowledges. See Procl. *In Eucl.* 383.17-384.4: «[...] the half-square; and that the scalene half-triangle [...] I do not mention these matters without a purpose, but because they prepare us for the teaching of the *Timaeus*».

45. See Procl. *In Eucl.* 55.23-56.8.

with infinity, then clearly infinity is beyond the grasp of the imagination. It suspends any thought about what is immeasurable and thus incomprehensible[46]. It cannot traverse the infinite[47]. Knowing the infinite is akin to seeing darkness in the dark[48]. However, it seems that incommensurability should not be considered coextensive with infinity. In *In Eucl.* 278.19-24, we learn that:

> For when geometers demonstrate that there is incommensurability among magnitudes and that not all magnitudes are commensurable with one another, what else could we say they are demonstrating than that every magnitude is divisible indefinitely and that we can never reach an indivisible part which is the least common measure of magnitudes[49]?

We learn that, although all geometrical magnitudes are infinitely divisible, some are nevertheless incommensurable[50]. Infinity and incommensurability are therefore not coextensive. This allows us to identify in incommensurability a different kind of explanation – one that points toward its being something definable. A passage from Proclus' *Commentary on the* Parmenides moves in this direction and offers the following definition of incommensurability. In *In Prm.* VII 1206.4-8, he states:

46. See Procl. *In Eucl.* 285.10-13.
47. See Procl. *In Eucl.* 285.18-19.
48. See Nikulin 2008, 170.
49. ὅταν γὰρ δεικνύωσιν ὅτι ἔστιν τὸ ἀσύμμετρον ἐν τοῖς μεγέθεσι καὶ οὐ πάντα σύμμετρα ἀλλήλοις, τί ἄλλο δεικνύναι φήσει τις αὐτοὺς ἢ ὅτι πᾶν μέγεθος εἰς ἀεὶ διαιρεῖται καὶ οὐδέποτε ἥξομεν εἰς τὸ ἀμερές, ὅ ἐστι κοινὸν μέτρον τῶν μεγεθῶν ἐλάχιστον;
50. The notion of infinity discussed a little later in Procl. *In Eucl.* 285-287 also appears to concern infinite extension. See Procl. *In Eucl.* 285.15-24: «It produces it (i.e. the infinite, DW) indeed, because it has an indivisible power of proceeding without end, and it knows that the infinite exists because it does not know it. For whatever it dismisses as something that cannot be gone through, this it calls infinite. So if we supposed the infinite line to be given in imagination, exactly like triangles, circles, angles, lines, and all the other geometrical figures, should we not ask in wonder how a line can actually be infinite and how, being indeterminate, it associates with determinate notions?» (γεννᾷ μὲν οὖν αὐτὸ τῷ δύναμιν ἔχειν ἀμερῆ προϊέναι δυναμένην ἀκαταλήκτως, νοεῖ δὲ ὡς ὑποστὰν ὅτι μὴ νοεῖ τὸ ἄπειρον. ὃ γὰρ ἀφῆκεν ὡς ἀδιεξίτητον, τοῦτο ἄπειρον λέγει, ὥστε τὴν ἄπειρον γραμμὴν τὴν δοθεῖσαν ἐν τῇ φαντασίᾳ θέμενοι, καθάπερ δὴ καὶ τὰ ἄλλα πάντα γεωμετρίας εἴδη, τὰ τρίγωνα, τοὺς κύκλους, τὰς γωνίας, τὰς γραμμάς, οὐ θαυμασόμεθα, πῶς κατ' ἐνέργειαν ἄπειρός ἐστι γραμμὴ καὶ προστίθησιν ἀορισταίνουσα ταῖς ὡρισμέναις νοήσεσιν).

[...] commensurate is «that which is measured by the same measure - if it is greater, more times than less, if it is less, less times than the greater»; and the incommensurate is «that which is divided into parts which are equal numerically, but are unequal in size»[51].

It is not common to speak of a form of incommensurability, yet in the context of other forms that Proclus defines in his *Commentary on the* Parmenides, this may be a valid explanation. Likeness and unlikeness, for example, are defined in terms of identity and otherness[52] – and both are forms. Equality and inequality are likewise defined as deriving from identity and otherness[53]. As we have seen above, commensurability and incommensurability appear immediately after Proclus' and Plato's discussion of equality and inequality, and they too are directly defined in terms of identity, equality, and inequality. Thus, even though these notions apparently belong to a lower rank – insofar as they are dependent on these other forms – incommensurability, like unlikeness, seems to possess its own definable form. Proclus does not call it a form of incommensurability; therefore, this interpretation is only valid insofar as we accept the analogy with likeness and unlikeness. If likeness and unlikeness are definable in terms of identity and otherness and are themselves forms, then commensurability and incommensurability, too, would appear to be definable forms. As in the case of likeness and unlikeness, this does not imply that the form of incommensurability must itself be incommensurable. Rather, analogously to likeness and unlikeness, commensurability and incommensurability are forms of relatives that are themselves not relative. They are «non-relational, though productive of a relation» (ἄσχετος [...] ἐστιν ὑπόστασις

51. [...] σύμμετρον δὲ τὸ τῷ αὐτῷ μέτρῳ μετρούμενον, ἐὰν μὲν μεῖζον ᾖ, πλεονάκις ἢ τὸ ἔλαττον, ἐὰν δὲ ἔλαττον, ἐλαττονάκις ἢ τὸ μεῖζον, ἀσύμμετρον δὲ τὸ διαιρούμενον εἰς ἴσα μὲν κατ' ἀριθμόν, ἄνισα δὲ κατὰ μέγεθος.

52. See d'Hoine 2014, 1-37.

53. Morrow-Dillon 1987, 481: «Apart from showing that 'equal' and 'unequal' follow on logically from sameness and otherness, he is chiefly here concerned with the concept of 'measure' and commensurability which Plato introduces here. First, as he notes (1206.1 ff.), Plato produces definitions of 'equal', 'unequal', and 'incommensurable', in a proper geometrical manner, before going on to his proofs».

ἐκεῖ, σχέσεως ὑποστατική τινος, *In Prm.* IV 936.25-26)[54]. If incommensurability is analogous to unlikeness – a property that appears as extended in its proper place while depending on a form – and if it is neither merely an 'effect' of matter[55] nor identical with infinity, then it can be displayed in the geometer's *phantasia*. In the same way as the diameter in the circle, incommensurability appears as an 'element' – and it can be shown in an extended way in the *phantasia* by the triangles in which it is present.

What remains unclear is why there is only one form of the triangle, which nevertheless differentiates into geometrical triangles – some displaying incommensurability, others not. I have not found explicit textual evidence for the claim I wish to propose, but its systematic basis seems to be the following. As noted above, Proclus distinguishes three kinds of geometrical triangles, the equilateral, the isosceles, and the scalene. The last one of these is said to be the most remote from the circle, which I take to mean that it represents the triangle in its purest form. Now, every triangle – whether isosceles, equilateral, or scalene – can be divided by a line drawn from the apex to the base, into two right-angled triangles. Most right-angled triangles exhibit incommensurability, while its absence appears to be the exception (cf. Pythagorean triples). This comes close to the way Plato describes the two elementary triangles in the *Timaeus*.

Conclusion

The argument I have laid out shows that incommensurability in Proclus' philosophy is a complex issue. Scholarly literature rightly emphasises that intelligible matter plays a crucial role in its manifestation within geometrical entities, just as sensible matter accounts for depotentiation, magnitude, and imprecision in sensible entities. However, we have seen that intelligible matter cannot be the sole cause of the appearance of incommen-

54. See d'Hoine 2014, 22 in the context of a thorough discussion of Likeness and Unlikeness in Proclus.

55. See two forms of dissimilarity, d'Hoine 2014, 31-32, referring to *In Prm.* II 748.12: «The one derives its character from above, the other springs from the indeterminateness of matter [...]».

surability. Some objects display this feature within the geometrical 'realm' when projected onto the *phantasia* of intelligible matter, while others do not. The reason, it seems, is that Proclus acknowledges a form of incommensurability – one that is defined and constitutes an inherent of certain entities.

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Denis Walter, *Incommensurability in Geometrical Entities in Proclus'* In Eucl. *and* In Prm.

This chapter investigates the origin of incommensurability within Proclus' ontology through an analysis of his *Commentary on the First Book of Euclid's* Elements and his *Commentary on the* Parmenides. The central question concerns how incommensurability arises at a specific ontological level and how it corresponds to a distinct mode of cognition. The chapter first situates geometrical entities in an intermediate position between the intelligible and the sensible realms. It then demonstrates that sensible matter cannot serve as the unqualified cause of incommensurability, since it accounts only for imprecision rather than for incommensurability. Likewise, intelligible matter (the soul) alone cannot explain it, as not all objects within the soul exhibit incommensurability. Instead, the chapter argues that incommensurability is an intrinsic property of certain geometrical entities, comparable to their other identifiable features. Ultimately, the essay supports the interpretation that incommensurability is a definable and necessary feature of some geometrical objects.

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