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THE PLACE OF MUSIC IN PROCLUS' CONCEPTION OF MATHEMATICAL EXEGESIS

The Platonic Background

In book VII of the *Republic*, in the account of what is sometimes called the 'higher education' of the Guardians, Plato treats music as a branch of mathematics. Mathematics plays a key role as an essential preliminary to the highest kind of study, that of dialectic. Socrates describes five branches of mathematics: arithmetic, geometry, solid geometry or stereometry, astronomy and finally, at 530d-531c, harmonics, the mathematics of musical intervals. At 530d Socrates calls harmonics and astronomy «in some sense sister sciences [...] as the Pythagoreans say»¹. He claims that both sciences suffer from the same problem, namely that their practitioners are too focused on the physical world – the visible movements of the heavenly bodies in the case of astronomy, audible musical intervals in the case of harmonics – and the proper study of both requires abstraction from physical phenomena so as to think about numbers in themselves. As often, Plato here appears to be picking up on an idea which he found in Pythagorean sources and using it for purposes of his own.

The link between harmonics and astronomy reappears in *Republic* X where the myth of Er contains a description of the

I would like to thank the participants at the conference on «Proclus: Science and Theology» for comments on the earlier version of this essay which was delivered at the conference, especially Dominic O'Meara, Gerd Van Riel, Carlos Steel and Antonio Vargas.

1. Pl. *Resp.* VII 530d6-8: κινδυνεύει [...] καὶ αὐταὶ ἀλλήλων ἀδελφαὶ τινες αἱ ἐπιστῆμαι εἶναι, ὥς οἱ τε Πυθαγόρειοί φασι.

rotation of the heavenly bodies around «the spindle of Necessity» (see 616b–617d). Famously, this description includes a reference at 617b to the music of the spheres, another Pythagorean idea. Harmonics also plays an important role in the *Timaeus*, especially in the account of the composition of the world soul at 35b–36b. That is described in terms of a structure which is both mathematical and musical, according to the divisions of a musical scale. The result is the incorporeal circles of the Same and the Different, of which the latter is subdivided into seven concentric circles. Timaeus goes on to describe how the demiurge, having put the world soul together, forms the body of the world in accordance with the same structure, so that seven inner circles carry the visible heavenly bodies of the sun, the moon and five planets (Venus, Mercury, Mars, Jupiter and Saturn) while an undivided outer circle carries the fixed stars (see 38b–39e). So it turns out that once again mathematics, music and astronomy all go closely together².

Republic VII 530d–531c was picked up and developed by later writers on Platonic/Pythagorean harmonic theory such as Nicomachus of Gerasa and Iamblichus, often in connection with the music of the spheres alluded to in *Republic* X³. My concern here is with Proclus and with some of the ways in which he draws on the passage on harmonics in *Republic* VII, combining it not only with *Republic* X on the music of the spheres but also with exegesis of the *Timaeus*. Proclus was of course very familiar with the long tradition of Platonic interpretation that he inherited but he also looks directly to Plato. I shall briefly discuss Proclus' mention of music as one of the mathematical sciences in his *Commentary on Euclid* and his use of *Republic* VII in his *Commentary on the Republic*, especially, but not only, in the fifth essay of that commentary, before turning to his notion of mathematical exegesis in his *Commentary on the Timaeus*. I will examine the role of music in two passages of that *Commentary* in which Proclus contrasts mathematical exegesis with exegesis of other kinds, namely his comments on *Ti.* 31c–32a and 40c–d. In the last part

2. On the use of this harmonic structure in later passages of the *Timaeus* see Barker 2007, 323–26.

3. See O'Meara 2007, 147–61.

of the chapter I will address some aspects of Proclus' discussion of the musical scale of *Ti.* 35b and will attempt to clarify how he understands the distinctions between mathematical, physical and philosophical exegesis in that part of his *Commentary*.

Music as a Mathematical Science in the Commentary on Euclid and the Commentary on the Republic

As part of the prologue to his *Commentary on Euclid* Proclus discusses different views about the classification of the mathematical sciences, contrasting the Pythagorean classification with that of Geminus. When he presents the Pythagorean view, he offers a division of mathematics into four branches which may well be drawn from Nicomachus (*Ar.* 1.3.1) but also echoes the five-fold division of *Republic* VII: «Arithmetic [...] studies quantity as such, music the relations between quantities, geometry magnitude at rest, spherics [i.e. astronomy] magnitude inherently moving (35.28–36.3)»⁴. Solid geometry and astronomy are rolled up together here, but harmonics is picked out as a distinct branch and labelled not ἀρμονία but μουσική. A few lines further on Proclus develops the contrast between arithmetic and music in a way explicitly derived from the *Timaeus* (*In Eucl.* 36.17–38.1):

As the *Timaeus* has taught us, it is by virtue of her difference [...] that the understanding [...] projects numbers and the knowledge of numbers which is arithmetic; and by virtue of the unity of plurality in her and the community of bond that binds her together, she projects music. Hence arithmetic is elder than music, since the soul was first divided by the Demiurge and then bound together by ratios in the fashion explained by Plato⁵.

4. καὶ τὴν μὲν ἀριθμητικὴν τὸ καθ' αὐτὸ τὸ ποσὸν θεωρεῖν, τὴν δὲ μουσικὴν τὸ πρὸς ἄλλο, γεωμετρίαν δὲ τὸ πηλίκον ἀκίνητον ὑπάρχον καὶ τὴν σφαιρικὴν τὸ καθ' αὐτὸ κινούμενον. Translations from the *Euclid Commentary* are taken from Morrow 1970. The echo of Nicomachus is pointed out in Baltzly 2009, 179 note 329.

5. ὥς ὁ Τίμαιος ἡμᾶς ἀνεδίδαξεν, λεκτέον ὅτι κατὰ μὲν τὴν ἐτερότητα τὴν αὐτῆς καὶ τὴν διαίρεσιν τῶν λόγων καὶ τὸ πλῆθος ἢ διάνοια στᾶσα καὶ νοήσασα ἑαυτὴν ἐν καὶ πολλὰ οὖσαν τοὺς τε ἀριθμοὺς προβάλλει καὶ τὴν τούτων γνῶσιν, τὴν ἀριθμητικὴν, κατὰ δὲ τὴν ἑνωσιν τοῦ πλῆθους καὶ τὴν πρὸς ἑαυτὸ κοινωνίαν καὶ τὸν σύνδεσμον τὴν μουσικὴν. δι' ὃ καὶ ἡ ἀριθμητικὴ πρεσβύτερα τῆς μουσικῆς, ἐπεὶ καὶ ἡ

Harmonics and music are not mentioned in this *Commentary* otherwise.

The *Commentary on the Republic* would seem to be the obvious place to look to find out more about Proclus' understanding of the treatment of music in *Republic VII*. None of the essays which make up this *Commentary* focus on the 'higher education' of the Guardians. However, the approach to music that we find in *Republic VII* is picked up when Proclus discusses Plato's views on music in the fifth essay of the *Commentary*. That essay discusses ten problems in the interpretation of Plato's views of poetry and music. The fifth of these problems addresses the inconsistencies between Plato's various remarks on μουσική both in different dialogues and within the *Republic* and tries to solve the problem in a way characteristic of later Neoplatonism, by separating out four different types of μουσική. The first type is philosophy, as in *Phaedo* 61a; the second is the inspired poetry described at *Phaedrus* 245a; the third is the one which «leads up from imperceptible harmonies to the perceptible beauty of the divine harmony (I 58.28–59.1)»⁶; and the fourth is the one which educates the character and the passions.

I have discussed Proclus' fourth type of μουσική elsewhere; it clearly derives from the discussion of music in the earlier passage on education in the *Republic*, in Book 3, where the focus is on the moral education of the young Guardians⁷. My interest here is in the third type of μουσική. This one is surely the μουσική of *Republic VII*, in which the proper mathematical study of harmonies can lead to an understanding of numbers in themselves, as a stepping stone to the supreme study of dialectic. Proclus stresses that someone who is μουσικός in this sense is reminded of the beautiful through hearing, rather than through sight, describing such a person at I 59.8–11 as someone whose «activities concern the beauty that is in harmonies and rhythms. From

ψυχή διαιρείται πρῶτον δημιουργικῶς, εἴθ' οὕτως συνδέεται τοῖς λόγοις, ὡς ὁ Πλάτων ὑφηγεῖται.

6. ἀναγωγὸν δὲ ὁμῶς ἀπὸ τῶν φαινομένων ἁρμονιῶν εἰς τὸ ἀφανὲς τῆς θείας ἁρμονίας κάλλος. Translations from the fifth essay of the *Republic Commentary* are taken from Baltzly *et al.* 2018, with some modifications.

7. See Sheppard 2005.

these, he ascends to the imperceptible harmonies and the rhythms that are never known through hearing but are instead apparent to the reasoning of discursive thought»⁸. Proclus' emphasis here is on the anagogic power of this type of μουσική, rather than on its mathematical basis.

The thirteenth essay of the *Commentary on the Republic*, a lengthy treatment of the speech of the Muses in *Republic* VIII 546a–547a, includes a couple of passages which present the same conception of μουσική as a mathematical science with anagogic power. At II 36.3–46.17 Proclus interprets the mysterious nuptial number, described by the Muses in *Resp.* VIII 546c7 as a «geometric number» (ἀριθμὸς γεωμετρικός), first arithmetically, then geometrically, then «musically» (μουσικῶς), i.e. in terms of harmonics, and finally in terms of mathematical astronomy, using the division of the mathematical sciences from *Republic* VII. In a later passage of the same essay, II 68.3–16, Proclus contrasts the harmony (ἁρμονία) of the Muses with the harmony of the Sirens: the former saves souls and situates them among the gods while the latter is associated with the realm of becoming (γένεσις)⁹.

Proclus' conception of music as one of the mathematical sciences is further developed in his *Commentary on the Timaeus*, to which I now turn.

Mathematical Exegesis in Proclus' Commentary on the Timaeus I

Proclus' fullest presentation of a mathematical approach to music can be found in his exegesis of the musical scale of *Ti.* 35b. Before addressing this, it may be helpful to consider two other passages of the *Timaeus Commentary* where Proclus contrasts mathematical exegesis with exegesis of other kinds and to examine the role of music in these passages. The first one is *In Ti.* III 28.13–31.26 Van Riel (= II 20.19–23.8 Diehl). Here Pro-

8. τὸν μὲν περὶ τὶ καλὸν ἐνεργοῦντα τὸ ἐν ἁρμονίαις καὶ ῥυθμοῖς καὶ ἀπὸ τούτου πρὸς τὰς ἀφανεῖς ἁρμονίας ἀνιόντα καὶ ῥυθμοὺς ἐκείνους οὐκέτι δι' ἀκοῆς ὄντας γνωστούς, ἀλλὰ τῷ τῆς διανοίας λογισμῷ καταφανεῖς.

9. On the interpretation of the Sirens in this passage and its connection with the four types of music distinguished in the fifth essay, see Moro Torinese 2013.

clus is discussing *Ti.* 31c5–32a7. At this point in the dialogue Timaeus is describing how the body of the world is put together from the four elements; 31c5–32a7 sets out the formula for the geometric proportion of this composition. The passage of Plato is difficult and its interpretation is disputed¹⁰. My interest here is not in how we should understand Plato or even in exactly how Proclus understands him but in the way in which Proclus characterises the exegetical method he is using. At III 28.13–14 Van Riel (= II 20.19–20 Diehl) Proclus says «It is first necessary to speak about what is said in a mathematical manner, and then in a physical manner, since it is the latter in particular that is under discussion»¹¹. His mathematical interpretation runs through until III 31.26 Van Riel (= II 23.8 Diehl) and is then followed by his physical interpretation, from III 32.1–38.17 Van Riel (= II 23.9–28.7 Diehl). The physical interpretation is, as one might expect, concerned with the four elements as they are found in the physical world, although it is worth noting that Proclus emphasises the divine origin of the elements and concludes with a reference to the Neoplatonic allegorical interpretation of the binding of Ares and Aphrodite by Hephaestus in *Odyssey* 8 (see III 37.16–38.4 Van Riel = II 27.17–27 Diehl). The mathematical interpretation begins with a discussion of mathematical proportion but then moves into an examination of how these proportions apply in music: see III 30.5–6 Van Riel (= II 21.31–2 Diehl): «And of course one could do the same thing with musical values»¹². Proclus sets out how, in his view, first arithmetic and then geometric proportion are instantiated in music. As Baltzly points out, «The tradition that the arithmetic, geometric and harmonic means are instantiated in music goes back to Archytas». Our text of Proclus says nothing about the harmonic mean, perhaps because there is a lacuna or perhaps, as Baltzly suggests, following Winnington-

10. For a summary of the modern interpretations of this difficult passage of Plato and of Proclus' view see Baltzly 2007, 11–18.

11. πρῶτον εἰπεῖν χρὴ περὶ τούτων μαθηματικῶς, ἔπειτα, ὅπερ μάλιστα πρόκειται, φυσικῶς. All translations of passages from the *Timaeus* Commentary are taken from Baltzly 2007, 2009, 2013, with some modifications.

12. καὶ δὴ καὶ ἐν δυνάμεσι τὸν αὐτὸν τρόπον. It is clear from what follows that δυνάμεις here means 'musical values'.

Ingram, because he «set this difficult point to one side for the moment»¹³.

The way in which Proclus concludes his discussion of the mathematical interpretation makes it very clear that he considers music, understood here as harmonics, as an integral part of mathematics. At III 31.4–8 Van Riel (= II 22.16–20 Diehl) he says:

All these [ratios] may be seen in numbers as well as in volumes and in musical values, but number has been particularly adapted to the arithmetic [proportion], volume to the geometric and musical values to the harmonic. It is for these reasons that Plato has made use of these three things: numbers, volumes and musical values¹⁴.

He continues at III 31.20–22 Van Riel (= II 23.2–5 Diehl):

One *might* say that, strictly speaking, sameness is set over the geometric proportion – for the ratio [between the terms] is that of sameness or identity – but equality is set over the arithmetic proportion, while similarity is set over the harmonic¹⁵.

Proclus also discusses the distinction between arithmetic, geometric and harmonic ratios in the immediately preceding passage of his commentary, III 26.1–28.3 Van Riel (= II 18.21–20.9 Diehl), where the statement in *Ti.* 31c4 that the bonding between the elements is best brought about by proportion (ἀναλογία) leads him to expound these three types of proportion, explicitly focusing on Plato and setting aside the complex accounts of different types of proportion found in philosophers such as Nicomachus and Moderatus (see III 26.15–16 Van Riel = II 19.4–5 Diehl). The three types of proportion are derived from what follows in the *Timaeus* at 34a–36a, as Baltzly explains in the introduction to his translation of Proclus¹⁶. Overall, then, music,

13. See Baltzly 2007, 66 note 85 and 67 note 92. Cf. also Van Riel 2022, note on III 30.15 Van Riel (= II 22.8–9 Diehl).

14. πᾶσαι ἄρα καὶ ἐν ἀριθμοῖς ὁρῶνται καὶ ἐν ὄγκοις καὶ ἐν δυνάμεσι, μᾶλλον μὴν ὥκειώται ὁ μὲν ἀριθμὸς τῇ ἀριθμητικῇ, ὁ δὲ ὄγκος τῇ γεωμετρικῇ, ἡ δὲ δύναμις τῇ ἀρμονικῇ. καὶ διὰ ταῦτα μὲν οὖν τοῖς τρισὶν ἐχρήσατο τοῦτοις ὁ Πλάτων, ἀριθμοῖς, ὄγκοις, δυνάμεσιν.

15. Λέγοιτο δ' ἂν κυρίως ἐπὶ μὲν τῆς γεωμετρικῆς ἡ ταυτότης (ὁ γὰρ αὐτός ἐστι λόγος), ἐπὶ δὲ τῆς ἀριθμητικῆς ἡ ἰσότης, ἐπὶ δὲ τῆς ἀρμονικῆς ἡ ὁμοιότης.

16. See Baltzly 2007, 8–11.

in the sense of harmonics, forms an integral part of Proclus' mathematical interpretation of this part of the *Timaeus*. The mathematical interpretation of the text, in terms of different types of proportion, appears to be at a higher level than the physical interpretation. This is in accordance both with the treatment of mathematics in *Republic* VII and with the part played by mathematics in structuring the world soul in the *Timaeus*. In both dialogues mathematics, including harmonics, operates at a higher, more abstract level than perception of the physical world.

Proclus again contrasts mathematical exegesis with exegesis of another kind at *In Ti.* IV 188.17–193.21 Van Riel (= III 145.32–149.28 Diehl). This time the contrast is not between mathematical and physical exegesis but between mathematical and philosophical exegesis. The passage under examination here is *Ti.* 40c–d where the heavenly bodies are treated as visible gods whose regular movements form a cosmic dance. Proclus offers first a mathematical interpretation of the passage, according to which the movements of the stars are to be understood in astronomical terms, and secondly a philosophical one, according to which the passage is concerned not so much with the heavenly bodies as with the divine souls which animate them. He begins his lengthy exposition of the mathematical interpretation by saying (IV 188.17–19 Van Riel = III 145.32–146.2 Diehl): «If first you wish to proceed in the mathematical [mode of interpretation], let us interpret their dances as their well-ordered and harmonised revolutions»¹⁷. The word «harmonised» (ἐναρμονίους) might lead us to expect that the mathematical interpretation will also involve something about music (harmonics). That is not the case: what we get is a very detailed discussion of astronomy which does include references to *Resp.* X 616–17 as well as to *Ti.* 36–39 (see IV 191.20–191.4 Van Riel = III 147.21–31 Diehl). But we should remember that for a Platonist astronomy and music are «sister sciences» and also that the stars are dancing. Dancing is usually

17. καὶ πρότερον, εἰ βούλει, μαθηματικῶς {τε καὶ φιλοσόφως} χορείας μὲν ἀκούσωμεν τὰς εὐτάκτους αὐτῶν καὶ ἐναρμονίους περιφοράς. The words τε καὶ φιλοσόφως («and also in the philosophical [mode of interpretation]») seem out of place here and are treated by both Van Riel and Diehl as an interpolation that should be deleted. I follow Festugière and Baltzly in not including them in my translation of the text.

accompanied by music; the music to which the stars are dancing is surely not music audible to human ears but the music of the spheres. The philosophical interpretation to which Proclus turns at IV 193.17 Van Riel (= III 149.23 Diehl) is introduced by a reference to another dialogue which is important for the tradition of the cosmic dance, namely the *Phaedrus*: «Surely then the dances of the souls are the Bacchic ones that they make around the intelligible, and the intellectual periods and complete cycles for they [*scil.* souls] also dance when they are following the more divine among their leaders, as Socrates says in the *Phaedrus*»¹⁸. The interpretation which follows offers a philosophical allegory of the cosmic dance.

If we put these two passages of the *Timaeus Commentary* together, we can see that for Proclus a mathematical interpretation is superior to a physical interpretation but inferior to a philosophical one. We can also see that a mathematical interpretation can include either harmonics or astronomy or both. None of this is very surprising, but it is deeply Platonist and reflects both the *Republic* and the *Timaeus*.

Mathematical Exegesis in Proclus' Commentary on the Timaeus II

Finally, I come to Proclus' interpretation of the musical scale of *Ti.* 35b. In this context too Proclus distinguishes between mathematical, physical and philosophical interpretations but, as we shall see, he also distinguishes more than one level of both mathematical and physical interpretation, partly in order to structure his presentation of the interpretations of earlier commentators. At *In Ti.* III 236.20–237.12 Van Riel (= II 174.15–28 Diehl), Proclus introduces what he describes as the mathematical interpretation of 35b4–6 with these words:

Mathematical theory ought neither to be entirely scorned, nor sought after for its own sake. The latter option means that Plato will not be able to indicate to us, as he wishes to, the things in their images,

18. οὐκοῦν χορεῖται μὲν εἰσι τῶν ψυχῶν αἱ περὶ τὸ νοητὸν βακχεῖται καὶ αἱ περίοδοι καὶ ἀποκαταστάσεις αἱ νοεραὶ· συνεπόμεναι γὰρ τῷ θειοτέρῳ τῶν ἡγεμόνων καὶ αὐταὶ χορεύουσιν, ὥς φησιν ὁ ἐν Φαίδρῳ Σωκράτης. Cf. *Phdr.* 246e4–247a7, 250b6 and 252d1.

while the former option makes the whole exegesis of the text as unstable as a ship without ballast, for it is necessary to be firmly anchored, as it were, when one is heading off after the essence of those realities with which the dialogue is concerned. We, however, will steer a middle course, as we said before, having first set out the text mathematically, then, after this we will provide an exegesis of the proposed division [of the psychic essence] that is appropriate to the subject. Now the Pythagoreans were doubtless very wise in as much as they discovered the division of the monochord, but it was *Plato* who imparted the division of the soul in these terms – disclosing the substantial causes and the generative *logoi* of mathematical theorems¹⁹.

The mathematical interpretation of this difficult passage of Plato in which mathematics and music are combined takes Proclus a long time. He starts with an extended account of the numbers and musical intervals involved, including discussion of the views of some earlier commentators. At III 260.25 Van Riel (= II 193.9 Diehl) he turns to what is described as *πραγματειώδης ἐξήγησις*, which I suggest should be translated as «the exegesis that correlates the text with reality». The word *πραγματειώδης*, taken from Plato's *Parmenides* (137b2), is used a number of times in Proclus' *Commentary on the Parmenides* to contrast a purely logical interpretation of that dialogue with an interpretation in terms of metaphysical reality. In this passage of the *Timaean Commentary* there is a parallel contrast between two levels of mathematical interpretation, as described in the text quoted above: one is the purely mathematical interpretation, which would be unstable on its own, while the other is the interpretation that «discloses the substantial causes and the generative *logoi* of mathematical theorems»²⁰. The distinction between two levels of mathematical

19. τὴν μαθηματικὴν θεωρίαν οὕτε παντάπασιν ἀτιμαστέον οὕτε μόνην αὐτὴν καθ' ἑαυτὴν ζηλωτέον· τὸ μὲν γὰρ οὐ δείκνυσιν ἡμῖν, ὥσπερ ὁ Πλάτων βούλεται, τὰ πράγματα ἐν ταῖς εἰκόσι, τὸ δὲ ἀνεργάτιστον ποιεῖ τὴν ὅλην ἐξήγησιν· δεῖ γὰρ ὡς ἐπ' ἀσφαλούς πείσματος ὁρμεῖν τῆς τῶν πραγμάτων περὶ ὧν ὁ λόγος ἐστὶν οὐσίας· ἐν μέσῳ τοῖσιν, ὥσπερ εἵπομεν καὶ πρότερον, καταστήσαντες τὸν λόγον μαθηματικῶς μὲν πρῶτον, οἰκειῶς δὲ τοῖς ὑποκειμένοις μετὰ ταῦτα τὴν προκειμένην ἐξηγησώμεθα κατατομὴν. οἱ μὲν οὖν Πυθαγόρειοι μέγα φρονοῦσιν, ὡς τὴν τοῦ κανόνος κατατομὴν ἀνηρνηκότες· ὁ δὲ γε Πλάτων ψυχῆς κατατομὴν ἐν τοῦτοις παραδίδωσι τὰς αἰτίας τὰς οὐσιώδεις καὶ τοὺς λόγους τοὺς γεννητικούς τῶν μαθηματικῶν θεωρημάτων ἀναπτύσσων.

20. τὰς αἰτίας τὰς οὐσιώδεις καὶ τοὺς λόγους τοὺς γεννητικούς τῶν μαθημα-

interpretation is expressed in musical terms at III 263.11-19 Van Riel (= II 195.11-24 Diehl):

Let the manner of exegesis of the soul's essence be naturally consonant with the essence of the soul itself. You must free yourself from the [mere] appearance of harmony, and lift yourself up to the substantive and immaterial harmony, and be led back from images to the paradigms [of these images]. For the concordances that flow through our ears and which consist in soundings and striking differ entirely from the concordance of what is life-giving and intellectual. Therefore, let no one stop at the point of the mathematical consideration of the subject at hand. Instead he should urge himself on to examine this subject in a manner fitting to the essence of the soul²¹.

After expounding both levels of mathematical interpretation, Proclus finally moves at III 285 Van Riel (= II 212 Diehl) to other types of interpretation, introducing them as follows (III 285.23-286.5 Van Riel = II 212.3-9 Diehl):

As we said earlier, it is necessary to interpret what has been said by Plato not only mathematically, but also physically or philosophically. For the soul's essence has not been composed from mathematical numbers and ratios, but instead all these ratios and numbers are images of the soul's real essence and the demiurgic and life-generating divisions within it²².

τικὼν θεωρημάτων ἀναπτύσσων. On the meaning of *πραγματειώδης* and the content of this part of the commentary, cf. Baltzly 2009, 23-4 and 168 note 302. Baltzly translates *πραγματειώδης ἐξήγησις* as «the exegesis that correlates the text with things». I am doubtful about his suggestion that *πραγματειώδης* refers specifically to 'correspondences between levels'. Morrow-Dillon 1987 sometimes translate the word by «substantive» or «important», as he says, but at other times use phrases such as «concerned with the reality behind the text», «concerned with the underlying realities» and «from the perspective of metaphysical reality», all of which effectively capture the contrast with a purely logical (or purely mathematical) interpretation. Cf. also Procl. *In Eucl.* 82.13-20.

21. ὁ δὲ τρόπος τῆς περὶ αὐτὴν ἐξηγήσεως ἔστω τῇ οὐσίᾳ συμφυῆς, ἀπὸ μὲν τῆς φαινομένης ἁρμονίας ἀπολυόμενος, ἀνάγων δὲ ἑαυτὸν ἐπὶ τὴν οὐσιώδη καὶ ἄυλον ἁρμονίαν καὶ ἀπὸ τῶν εἰκόνων ἐπὶ τὰ παραδείγματα ἀναπεμπόμενος· ἡ γὰρ δι' ὧτων εἰσρέουσα συμφωνία καὶ ἐν ἡχοῖς καὶ πληγαῖς ὑφισταμένη τῆς ζωτικῆς καὶ νοερᾶς ἐξήλλακται. μὴ τοίνυν τις ἄχρι τῆς μαθηματικῆς τῶν προκειμένων ἰστάσθω θεωρίας, ἀλλ' ἐπὶ τὸν πρέποντα τρόπον τῇ οὐσίᾳ τῆς ψυχῆς ἑαυτὸν ἐγειρέτω.

22. ὅτι μὲν οὐ μόνον μαθηματικῶς, ἀλλὰ καὶ φυσικῶς ἢ φιλοσόφως ἀκροῶσθαι δεῖ τῶν ὑπὸ τοῦ Πλάτωνος λεγομένων ἐν τούτοις, καὶ πρότερον εἶπομεν· οὐ γὰρ ἐκ

The second sentence here once again expounds the two levels of mathematical interpretation which Proclus has treated at length. In what follows he presents physical interpretations of the same lemma as before, *Ti.* 35b4-6, and then, at the end of his discussion, the philosophical interpretation.

Proclus organises the physical interpretations into two groups, at two different levels, just as he did with the mathematical interpretations, starting with a brief account of interpretations of the musical scale in terms of the seven heavenly spheres and the movement of the heavenly bodies. However, this kind of interpretation purely in terms of the physical world is swiftly rejected and he moves to discussion of the views of first Amelius and then Porphyry, Iamblichus and Theodore of Asine with another reference to *πραγματειώδης ἐξήγησις*, introducing their views by saying «After these people [i.e. those who offer a purely physical interpretation of the text] there is yet another crowd of interpreters who hold views that are more in keeping with reality (III 287.15-16 Van Riel = II 213.8-9 Diehl)»²³. At this second level of physical interpretation, physical phenomena are correlated in various ways with numbers and ratios. For Proclus none of the philosophers discussed under this heading offers a fully satisfactory account of the text, although he expresses approval for some aspects of the interpretations of Porphyry and Iamblichus²⁴. At III 295.11 Van Riel (= II 218.20 Diehl) he comes at last to his preferred interpretation, that of his teacher Syrianus, introducing it as «another mode of reasoning (*ἄλλον τρόπον λόγων*)». This

μαθηματικῶν ἀριθμῶν ἐστὶ καὶ λόγων ἢ οὐσίας τῆς ψυχῆς, ἀλλ' οὗτοι πάντες οἱ λόγοι καὶ οἱ ἀριθμοὶ τὴν ὄντως οὐσίαν αὐτῆς ἀπεικονίζονται καὶ τὰς ἐν αὐτῇ διαιρέσεις τὰς δημιουργικὰς τε καὶ ζωογονικὰς.

23. μετὰ δὲ τούτους ἄλλο πλῆθος ἐστὶν ἐξηγητῶν πραγματειωδεστέρων λόγων ἀντεχόμενον·

24. Theodore of Asine is criticised at III 294.20-295.3 Van Riel (= II 218.8-12 Diehl) for offering an interpretation which is limited to mathematics and fails to explain «how it compares with the reality which is the subject matter of our discourses or how one might interpret this in a Pythagorean manner» (ὅπως δὲ ἀπεικασταὶ τοῖς πράγμασιν ὑπὲρ ὧν οἱ λόγοι, καὶ ὅπως ἂν τις αὐτὰ Πυθαγορικῶς διερμηνεύσειεν). I take this to mean that Theodore's interpretation correlates physical phenomena with mathematical ones but fails to ascend to the higher level of mathematical exegesis, let alone to a fully philosophical interpretation.

interpretation sets the understanding of the soul within the big Neoplatonist metaphysical picture of procession from and reversion to first principles and offers a clear example of what Proclus means by a philosophical interpretation. An interpretation of this kind is like the philosophical interpretation of the cosmic dance, i.e. it interprets the text in a way that goes beyond correlations between physical phenomena and numbers and ratios and also beyond correlations between the properties of numbers and ratios and the properties of the soul. At III 299.17-19 Van Riel (= II 221.24-26 Diehl) Proclus argues that such an interpretation offers the best explanation of Plato's text and that by referring to it «we will resolve many of the puzzles (πολλὰ τῶν ἀπόρων διαλύσομεν)».

Conclusion

We have seen that, like Plato, Proclus regards music as one of the mathematical sciences. It follows that Proclus' conception of mathematical exegesis includes music, alongside astronomy. In this he clearly follows the lead given by the text of both the *Republic* and the *Timaeus*. In several places in the *Timaeus* commentary Proclus distinguishes between mathematical, physical and philosophical exegesis; when he comes to discuss the musical scale of 35b he subdivides both mathematical and physical exegesis into two levels to take account of the complexity of the subject matter and the diversity of earlier interpretations. Yet his overall conception of music (μουσική) is much wider, as we can see from the fifth and thirteenth essays of the *Republic* commentary. I argued earlier that the third type of music distinguished in the fifth essay is the mathematical type («harmonics»). This type of music reappears in the thirteenth essay, both as harmonics and as the anagogic harmony of the Muses, contrasted with the dangerously bewitching harmony of the Sirens. The first two types described in the fifth essay, which are equated with philosophy and with inspired poetry, might have a part to play in philosophical exegesis as we find it in the *Timaeus Commentary* (for example, in the philosophical exegesis of the cosmic dance or in Syrianus' exegesis of 35b4-6 in terms of overarching metaphysical princi-

ples). The fourth type, which educates the character and the passions, operates at the same level as physical exegesis, although I have not so far come across any texts which put these two things together. However that may be, Proclus follows Platonic and Pythagorean tradition in including at least one type of music within mathematics.

Bibliography

- D. Baltzly (ed.), *Proclus. Commentary on Plato's Timaeus*, Vol. 3, Book 3, Part 1: *Proclus on the World's Body*, Cambridge, Cambridge University Press, 2007.
- D. Baltzly (ed.), *Proclus. Commentary on Plato's Timaeus*, Vol. 4, Book 3, Part 2: *Proclus on the World Soul*, Cambridge, Cambridge University Press, 2009.
- D. Baltzly (ed.), *Proclus. Commentary on Plato's Timaeus*, Vol. 5, Book 4: *Proclus on Time and the Stars*, Cambridge, Cambridge University Press, 2013.
- D. Baltzly – J. F. Finamore – G. Miles (eds), *Proclus. Commentary on Plato's Republic*, Vol. 1, Cambridge, Cambridge University Press, 2018.
- A. Barker, *The Science of Harmonics in Classical Greece*, Cambridge, Cambridge University Press, 2007.
- A.-J. Festugière, *Proclus, Commentaire sur le Timée*, 5 vols., Paris, Vrin, 1966–1968.
- S. F. Moro Tornese, *Music and the Return of the Soul in Proclus' Commentaries on Plato's Timaeus and Republic*, in A. Sheppard (ed.), *Ancient Approaches to Plato's Republic*, London, Institute of Classical Studies, School of Advanced Study, University of London, pp. 117–28. 2013.
- G. R. Morrow (ed.), *Proclus. A Commentary on the First Book of Euclid's Elements*, Princeton, Princeton University Press, 1970.
- G. R. Morrow – J. M. Dillon (eds), *Proclus' Commentary on Plato's Parmenides*, Princeton, Princeton University Press, 1987 (1992²).
- D. J. O'Meara, *Hearing the Harmony of the Spheres in Late Antiquity*, in: M. Bonazzi – Carlos Lévy – C. Steel (eds), *A Platonic Pythagoras. Platonism and Pythagoreanism in the Imperial Age*, Turnhout, Brepols, 2007, 147–61.
- A. Sheppard, *Music Therapy in Neoplatonism*, in R. W. Sharples (ed.), *Philosophy and the Sciences in Antiquity*, Aldershot, Ashgate Publishing, 2005, 148–55.
- G. Van Riel (ed.), *Procli Diadochi in Platonis Timaeum Commentaria*, 5 vols., Oxford, Oxford University Press, 2022.

Anne Sheppard, *The Place of Music in Proclus' Conception of Mathematical Exegesis*

This chapter investigates the role of music (*mousike*) within Proclus' concept of mathematical exegesis, drawing upon the Platonic and Pythagorean traditions. In Plato's *Republic* VII and the *Timaeus*, harmonics is established as the «sister science» of astronomy and a necessary branch of mathematics for the Guardians' education toward dialectic. The author argues that this Platonic conception of the mathematical sciences informs Proclus' application of the notion of mathematical exegesis in his *Commentary on the Timaeus*. The chapter specifically examines passages in *In Tim.* and Proclus' distinction between mathematical, physical, and philosophical interpretation, including his exegesis of the musical scale from *Timaeus* 35b. The core argument is that Proclus' conception of mathematical exegesis intentionally includes music alongside astronomy, adhering to the ancient tradition that incorporates a specific type of music into the domain of mathematics, which is also evident in his discussions in the *Commentary on Euclid* and the *Commentary on the Republic*.

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