I.

MATHEMATICS: BRANCHES, ONTOLOGICAL STATUS AND METHOD

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SYRIANUS AND PROCLUS ON WHY TRUE MATHEMATICS CANNOT BE EMPIRICALLY BASED

Introduction

The ontological status of the objects studied in the mathematical sciences, as conceived by the Platonist philosophers of Late Antiquity, has been examined by modern scholars in a number of publications and is familiar to specialists in the field[1]. In this chapter I would like to examine in more detail one aspect of the subject which, I believe, deserves further attention, namely the arguments given by these Platonists, in particular by Syrianus and by Proclus in Athens in the 5th century AD, in support of their ontology of mathematical objects, arguments designed to refute the position that such objects can be derived empirically, from sense perception. These arguments seek to show that 'true mathematics'[2] cannot be empirically based and lead to an important and novel conception of what mathematics is as a science and how it relates to other sciences, in particular to the science of transcendent divine being, i.e., metaphysics or "theology" (as they called this science).

Before looking at the arguments presented by Syrianus and Proclus, it may be of use to recall briefly the ontological land-

Earlier versions of this essay, presented in Amsterdam and Oslo, brought comments and questions, as did this version, presented in Naples, for which I am grateful.

1. See for example Mueller's Foreword to Morrow 1970; Cleary 2000; O'Meara 2016.

2. I indicate what is meant by this expression below, in section 4.

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scape in which these philosophers situated mathematical objects[3]. This landscape, if we take it in its simplest form as we find it in Plotinus, has at its summit the ultimate source of reality, the One, transcending all beings and thought. From the One derives a succession of lower levels of existence: a divine Intellect, which is one with transcendent Forms, and, deriving from Intellect, Soul, which produces the material world.

In Syrianus and Proclus this landscape acquires many more differentiating features, many more intermediate levels. Following Aristotle's reports on Plato, the Platonists of Late Antiquity held that mathematicals occupy an intermediary place in this ontological landscape, between Intellect/the Forms, on the one hand, and the material world, on the other. But in Plato, soul also seems to occupy this intermediate position. What then is the relation between mathematicals and soul? It is this question which will be pursued in the pages which follow.

In Syrianus and Proclus, and before them in Iamblichus, the theory of the ontological status of mathematicals replaces a position which derives these objects from material bodies, the objects of sense perception. The claim that mathematicals are taken from sensible objects goes back in particular to Aristotle, in his criticism of Platonism in *Metaphysics* Books M and N, more precisely in Book M, chapters 2 and 3. The great commentator on Aristotle of the second century AD, Alexander of Aphrodisias, commented on these books of the *Metaphysics*, a commentary which is now lost but which was probably used by Syrianus in his commentary on *Metaphysics* MN[4], where he sought to refute Aristotle's claim in arguments which we can also find, along with other such arguments, in the work of Syrianus' student and successor, Proclus, in particular in Proclus' *Commentary on Euclid's* Elements[5]. In the following pages I propose examining the argu-

3. I will refer to these objects in the following pages as 'mathematicals', term that can include both mathematical objects such as numbers or geometrical figures and the relations which obtain between them.

4. See below, note 37.

5. Longo 2010 has shown how Proclus, in his commentary on Euclid, takes up arguments given by Syrianus in his commentary on the *Metaphysics*. Helmig 2010 discusses these arguments and especially similar arguments given by Proclus in his commentary on Plato's *Parmenides*.

ments presented by Syrianus and Proclus refuting the derivation of mathematicals from the objects of sense perception, arguments supporting thereby their Platonist theory of the ontological status of mathematicals and their relation to soul – a theory which I will describe in the fifth section of this chapter.

Mathematical Objects Are Not to Be Found in the Sensible World

Syrianus points out that many of the objects studied in the mathematical sciences are not to be found among the objects of sense perception (*In Metaph.* 91.25-29):

> We have never seen any polygons of such size and quality as geometry concerns itself with by way of plane figures, nor such a variety of many-sided figures as are examined in stereometry, or divisions of angles or sides and surfaces[6].

Aristotle would agree with this statement, as indeed would Alexander[7]. However, both would maintain that mathematicals correspond to aspects to be found in the objects of sense perception, that are derived from sense perception by certain procedures in the thinking subject. Alexander speaks of the «help» provided by the thinking subject in deriving mathematicals from sense perception[8]. But what is this «help», and what are these procedures? In his *Commentary on Euclid*, Proclus distinguishes between two procedures, «abstraction» (ἀφαίρεσις) and (what I will translate as) «accumulation» (ἄθροισις)[9]. I will take each of these procedures and the refutation of them by Syrianus and Proclus insofar as they might constitute possible means for the derivation of mathematicals from sense perception.

6. See Syr. *In Metaph.* 91.25-29, commenting on Arist. *Metaph.* M 2: μηδὲ ἑωρακέναι ἡμᾶς ποτε μήτε πολύγωνα τοσαῦτα καὶ τοιαῦτα, οἷα ἐπίπεδα ἡ γεωμετρία θεωρεῖ, μήτε πολύπλευρα οὕτω ποικίλα <οἷα> ἡ στερεομετρία ἐπισκέπτεται, ἢ γωνιῶν διαιρέσεις ἢ πλευρῶν ἢ ἐμβαδῶν; tr. Dillon and O'Meara 2006. See also Syr. *In Metaph.* 176.26-31; Procl. *In Eucl.* 12.19-23; 49.12-17; 139.26-140.2.

7. Arist. *Metaph.* M 2.1077b15; Alex. Aphr. *In APr.* 4.7-9.

8. See Alex. Aphr. *De an.* 87.28.

9. Procl. *In Eucl.* 12.4-7.

Abstraction

According to Aristotle and his commentator Alexander, abstraction, as a procedure, allows for the derivation of mathematicals from the objects of sense perception[10]. Against this possibility Proclus gives two arguments[11], of which this is the first (*In Eucl.* 12.26-13.3):

How then will we give stable being to unchanging [mathematical] *logoi*, if they are derived from things that are ever changing from one state to another? For they agree [axiom 1] that all that which results from changing beings receives from them a changeable being[12].

If mathematicals (*logoi*) are unchanging, and the objects of sense perception, i.e., bodies, are in constant flux, then the former cannot derive from the latter, on the principle (which I label as «axiom 1»[13]) that all that comes from changing beings is itself changing. The principle is one to which «they» – the Aristotelians, I take it – subscribe[14], who consequently cannot derive mathematicals from the objects of sense perception without violating a principle to which they themselves adhere.

Proclus then offers a second argument intended to refute Aristotelian abstractionism (*In Eucl.* 13.3-5):

10. Arist. *APo.* 74a33-b1; *Metaph.* Z 11.1036a34-b3; b22-23; on Alexander see below, note 37 and Mueller 2016 who presents Alexander as championing 'abstractionism'. On Alexander's abstractionism see also De Libera 1999 ch. 1.

11. On these issues see Helmig 2012, 212; Iambl. *Comm. Math.* 34.9-10 (a cue inspiring Proclus' approach?).

12. Tr. Morrow 1970, modified as in the following quotations. πῶς οὖν τοῖς ἀκινήτοις λόγοις ἐκ τῶν κινουμένων καὶ ἄλλοτε ἄλλως ἐχόντων αὐτὴν τὴν μόνιμον οὐσίαν δώσομεν; πᾶν γὰρ τὸ ἀπὸ κινουμένων οὐσιῶν ὑφιστάμενον καὶ ὕπαρξιν μεταβλητὴν ἔχειν ὁμολογεῖται παρ' αὐτῶν. See 49.17-24.

13. The principle is formulated as a universal affirmative proposition: «All that which results [...]», reminiscent of the universal affirmative propositions of Proclus' *El. Theol.*

14. See Procl. *El. Theol.* 76 (part of which corresponds word-for-word to axiom 1). In his edition Dodds 1963, 345 refers us to Arist. *Ph.* Θ 6.259b32-260a10. See Proclus' commentary on Plato's *Parmenides* III 786.17-18.

And how can we give precision to forms which are precise and irrefutable from things which are not precise? For [axiom 2] all that is cause of unadulterated knowledge must itself be such to a greater degree[15].

The second argument has the same structure as the first. A principle is stated, which I label as "axiom 2"[16], which the Aristotelians are taken to uphold[17]. Mathematicals are precise and certain, but deriving their precision and certainty from things which are imprecise, impure, and unstable is to violate this principle.

In reading Proclus' arguments, we might make the following observations. Proclus assumes that mathematicals are of an unchanging, eternal nature: they are precise and clear in their nature, whereas bodies are forever changing, confused, imprecise. He also seems to consider *solely* the derivation of mathematicals from the objects of sense perception, without taking into account the role which the thinking subject, i.e., the mathematician, might have in generating mathematicals from sense perception, even if the concept of abstraction would seem to include such a role. However, by ruling out derivation solely from sensible objects, the arguments turn the focus on the role played by the thinking subject, the rational soul, as we see in the conclusion Proclus gives to his two arguments against abstractionism (*In Eucl.* 13.6-8):

Soul must therefore be what generates mathematical forms and *logoi*[18].

The role played by soul in generating mathematical science will therefore require examination: can it be reduced to a procedure of abstraction? Or can soul be the only source of mathematicals, as Proclus appears to claim? I will come back to these questions below.

15. πῶς δὲ τοῖς ἀκριβέσικαὶ ἀνελέγκτοις εἴδεσιν ἀπὸ τῶν μὴ ἀκριβῶν τὴν ἀκρίβειαν προσθήσομεν; πᾶν γὰρ τὸ τῆς ἀκιβδήλου γνώσεως αἴτιον μειζόνως ἐστὶν αὐτὸ τοιοῦτον. See also 139.26-140.4. See Syr. *In Metaph.* 91.25; 95.29-32; 96.31-33; see Pl. *Phd.* 75a-b.

16. Again, this principle is formulated as a universal affirmative proposition: «All that is cause [...]». See Procl. *El. Theol.* 7.

17. See Alex. Aphr. *De an.* 88.17-89.8.

18. ψυχὴν ἄρα τὴν γεννητικὴν ὑποθετέον τῶν μαθηματικῶν εἰδῶν τε καὶ λόγων.

Accumulation

A second procedure is mentioned by Proclus as put forward by those who seek to derive mathematicals from sense perception: «accumulation». Who might be the proponents of «accumulation»? What precisely is this procedure and how does it relate to the procedure of abstraction? Is it an alternative procedure, as Proclus' argumentative strategy seems to imply, or is it in some way complementary to abstraction? I would like to take up these questions before coming to the analysis of Proclus' arguments against the procedure of accumulation.

It might be thought that «accumulation» refers to a Stoic theory[19], which would mean that in attacking abstraction Proclus criticises Aristotelianism, and in refuting accumulation he is rejecting a Stoic theory of mathematicals. However, I will argue here that accumulation is a procedure which can also be found in Aristotelianism[20], in particular in Alexander of Aphrodisias, and that Proclus' arguments against the procedure of accumulation, like his arguments against abstraction, are addressed to an Aristotelian opponent, most likely Alexander.

According to a report in Simplicius, the verb «to accumulate» was used by Alexander (*In Ph.* 1075.7-8):

> This, then, is the way Alexander explained [it], wishing the universal and the cognition of the universal to be assembled from the particulars, and he said that it was stated that 'it somehow knows the particulars through the universal' [Arist. *Ph.* H 3.247b5-7] as a sign that the cognition of the universal is accumulated (ἀθροίζεσθαι) by means of the particulars, since the universal knowledge is of each of the things [falling] under the universal and it has been accumulated (ἠθροίσθη) from these[21].

If this is good evidence that the term «accumulation» would have been used by Alexander in a relevant context, we can ask if the procedure which the term designates was used by him to

19. See *SVF* II 841 (Chrysippus, as reported in Galen), Sext. Emp. *Pyr.* III 188; see Helmig 2012, 31 (a very useful collection of references to the use of the term «accumulation») and 280.

20. See, for example, Sext. Emp. *Math.* VII 224.

21. Tr. Hagen 1994, slightly modified.

refer to the generation of mathematicals. This procedure, as Proclus describes it, seems to involve the accumulation from sensible particulars of what is common to them, which includes mathematicals[22]. A comparable procedure can be found in the extant works of Alexander, when he speaks of intellect putting together (σύνθεσις) similar particulars, grasping the universal through the similarity in sensible particulars. Intellect sees (συνιδών) what is common in particulars, grasping this common form, separating form from matter[23]. If this synthetic (or synoptic) procedure corresponds to what Proclus refers to as «accumulation», then I think that we can say that the procedures of abstraction and of accumulation, as Alexander seems to have seen them, were not alternatives, as Proclus' critique might lead one to think, but were linked: to accumulate perceptions of sensible particulars, seeing what is similar in them, what is common to them, allows abstraction of the form common to them, i.e., the universal.

To confirm that Proclus is indeed attacking an *Aristotelian* procedure of accumulation, and not, for example, a Stoic theory, we can observe that this procedure is mentioned by Late Antique commentators on Aristotle's *Posterior Analytics* when they deal with the famous chapter on induction (II 19). Both Themistius, in his paraphrase of this chapter, and the author of a commentary on it attributed to Philoponus[24], discuss the topic of the «principles» (ἀρχαί) of demonstration. Themistius speaks in this connection of how experience (ἐμπειρία) is «accumulated» (ἀθροίζεται) from many perceptions and from memories by the soul's «collecting» (συλλέγουσαν) what is similar in sensible particulars, the universal[25]. 'Philoponus' also speaks of the accumulation (he uses the verbs συναθροίζειν and ἐπισυναθροίζειν) of per-

22. *In Eucl.* 12.6-7; 14.1; 15.17-18. The relation between universals and mathematicals cannot be discussed here. In speaking of what is common to sensible particulars, Proclus seems to associate them, as had Alexander of Aphrodisias (*De an.* 90.2-11). Proclus also speaks of the "accumulation", i.e., the combination of theorems as premises in order to constitute demonstrations (71.13-17); see Alex. Aphr. *In APr.* 17.24-25; Ammon. *In APr.* 26.4-6.

23. See *De an.* 83.11-15; 85.18-20; *Quaest.* 79.16-18; Helmig 2012, 161.

24. On the question of the attribution to Philoponus, see most recently Lagnerini 2023.

25. Them. *In APo.* 63.19-64.2; see *In Ph.* 206.15.

ceptions and memories leading to knowledge of the universal[26]. Now it is precisely with regard to the topic of the principles of demonstration that Proclus' two arguments against the procedure of accumulation are formulated, seeking to show that to derive such principles through accumulation is to violate rules for demonstration laid down in the *Prior Analytics*[27]. Here then is the first of Proclus' two arguments (*In Eucl.* 13.27-14.15):

If we accumulate mathematical *logoi* from below, from sensibles, must we not say that demonstrations which are constituted from sensibles are superior to those constituted from ever more universal and simpler forms? For we say that everywhere the causes for demonstrations should be appropriate to the hunt for what is sought[28]. If therefore particulars are causes of the universal and sensibles of the objects of discursive thought, how can the term[29] in a demonstration refer to what is universal rather than to what is particular[30] and the being of objects of discursive thought be declared more related to demonstrations than the being of sensibles? For the man, they say[31], who demonstrates that the isosceles triangle has the sum of its angles equal to two right angles and that the same is true of the equilateral and the scalene triangles does not properly understand these propositions; rather it is he who demonstrates about any triangle, without qualification, that knows in the strict sense of the term[32].

26. *In APo.* 435.16-23. Philoponus' distinguishes between accumulating perceptions and (furthermore) collecting (ἐπισυνάγεται) the universal from these.

27. The connection of Proclus' argument with the *An. post.* is noted by Helmig 2010, 51-52.

28. See Arist. *APo.* 71b19-33.

29. ὅρος, the subject or predicate in a premise (Arist. *APr.* 24b16).

30. *APo.* 85b23-27.

31. *APo.* 73b28-74a3; 74a25-26.

32. εἰ κάτωθεν καὶ ἀπὸ τῶν αἰσθητῶν ἀθροίζομεν τοὺς τῶν μαθημάτων λόγους, πῶς οὐκ ἀνάγκη τὰς ἀποδείξεις ἀμείνους λέγειν, ὅσαι ἀπὸ τῶν αἰσθητῶν συνίστανται, καὶ οὐ τὰς ἀπὸ τῶν καθολικωτέρων ἀεὶ καὶ ἁπλουστέρων εἰδῶν; τὰ γὰρ αἴτια πανταχοῦ ταῖς ἀποδείξεσιν οἰκεῖα πρὸς τὴν τοῦ ζητομένου θήραν εἶναι φαμέν. εἰ οὖν τὰ μερικὰ τῶν καθόλου καὶ τὰ αἰσθητὰ τῶν διανοητῶν αἴτια, τίς μηχανὴ τὸν ὅρον τῆς ἀποδείξεως ἐπὶ τὰ καθόλου μᾶλλον ἀναφέρειν ἀντὶ τῶν μερικῶν καὶ τῶν διανοητῶν τὴν οὐσίαν πρὸ τῶν αἰσθητῶν ταῖς ἀποδείξεσιν συγγενεστέραν ἀποφαίνειν; οὐδὲ γὰρ εἴ τις, φασὶν, ἀποδείξειεν ὅτι τὸ ἰσοσκελὲς δυεῖν ὀρθαῖς ἴσας ἔχει τὰς γωνίας, καὶ ὅτι τὸ ἰσόπλευρον ὡδὶ καὶ τὸ σκαληνὸν ἐπίσταται κατὰ τρόπον, ἀλλ' ὁ πᾶν τρίγωνον καὶ ἁπλῶς ἀποδείξας ἔχει τὴν ἐπιστήμην καθ' αὐτό.

Sharing with Aristotelians fundamental premises concerning scientific demonstration as laid out in Aristotle's *Posterior Analytics*, Proclus («we») argues that this is incompatible with the procedure of generation of mathematicals by accumulation. For «they» (the Aristotelians) agree that the universal is prior and superior to particulars and is the principle of scientific demonstration, not the particulars.

The same point is made in Proclus' second argument against accumulation (*In Eucl.* 14.15-20):

> Again [they say] that the universal is better than the particular for demonstration and consequently that demonstrations from what is universal are more demonstrative. But those [things] from which demonstrations are constituted are prior and naturally superior to particulars and the causes of what is demonstrated[33].

Proclus' arguments against «accumulation» can already be found in large part in Syrianus' commentary on Aristotle's *Metaphysics* Book M[34]. Although Syrianus does not mention the term «accumulation» in this context, I think that we can take it that there is a very good chance that he is arguing against a position he found in Alexander's (now no longer extant) commentary on Aristotle's Book M[35], where Proclus would have read the term[36]. At any rate, we are now in a position to explain why in particular the topic of the principles of demonstration comes up in Proclus' argument against accumulation and to see that Aristotelianism is the target of the arguments against both abstraction and accumulation. Attention to Alexander's views also suggests that abstraction and accumulation are not alternative procedures

33. καὶ πάλιν ὅτι τὸ καθόλου βέλτιον τοῦ κατὰ μέρος πρὸςἀπόδειξιν, καὶ ἑξῆς ὅτι αἱ ἀποδείξεις ἐκ τῶν καθόλου μᾶλλον, ἐξ ὧν δὲ αἱ ἀποδείξεις, ταῦτα πρότερα καὶ τῇ φύσει προηγούμενα τῶν καθ' ἕκαστα καὶ αἴτια τῶν δεικνυμένων. See Arist. *APo.* 71b25-72a7; 86a5-30.

34. Syr. *In Metaph.* 90.17-91.2; 164.4-12.

35. Alexander certainly spoke there of abstraction, as we can see in Syrianus' report (*In Metaph.* 96.14-19), as quoted below, note 37.

36. Syrianus (*In Metaph.* 54.12-13) regarded Alexander's commentary on Aristotle's *Metaphysics* as adequate for use, except of course where Platonism and Pythagoreanism were the object of criticism, which Syrianus undertook to refute.

for generating mathematicals, contrary to what seems to be suggested by the structure of Proclus' critique.

As in the case of Proclus' arguments against abstraction, we note that no account is taken of the activity of the thinking subject, i.e., the rational soul, in generating mathematicals; sensible particulars alone are considered as the source of mathematicals. Yet, the rejection of sensible objects as sources of mathematicals allows Proclus to turn now to soul and to its role in constituting mathematicals. Before coming to this subject, I would like to introduce two descriptions of what an empirically based mathematical science might look like, in the eyes of Syrianus and Proclus.

What Would Be an Empirically Based Mathematics?

Syrianus gives us the following description of such a mathematics (*In Metaph.* 96.14-19):

> But how could it [soul] achieve that, if it concerned itself with the products of abstraction? [...] if in the words of Alexander of Aphrodisias, it moulds for itself certain objects of intellection, which are not by their own nature intelligible, and then vainly plays shadow battles (ἐσκιαμάχει) with them[37].

Syrianus is quoting, it appears, Alexander's (lost) commentary on *Metaphysics* M 3, on the topic of abstraction. Syrianus regards a mathematics which produces its objects by abstraction from sensible particulars (which in themselves are not intelligible objects) as a sort of shadow-battle. The reference is to the image of the cave in Plato's *Republic*, according to which prisoners contend with each other in elaborating conceptions and theories on the basis of the shadows that they see moving on the wall of the cave[38]. To construct such a chimerical world from shadows is what it is to construct a mathematical science from the shadows of reality which are sensible particulars.

37. ὃ περὶ τὰ ἐξ ἀφαιρέσεως διατρίβουσα πότ' ἂν κατεπράξετο [...] εἰ καθάπερ φησὶν ὁ Ἀφροδισιεὺς Ἀλέξανδρος, νοητὰ ἄττα ἑαυτῇ ἀναπλάσασα, οὐκ ὄντα τῇ οἰκείᾳ φύσει νοητά, περὶ αὐτὰ ταῦτα μάτην ἐσκιαμάχει.

38. Pl. *Resp.* VII 515a-b, 516c-d, 520c-d (σκιαμαχούντων).

Such a mathematical science cannot be considered as true, as is suggested by Proclus in another description of an empirically based mathematics (*In Eucl.* 13.13-21):

But if it [soul] weaves this great immaterial structure and generates such a great knowledge without knowing or having previously known these *logoi*, how can it judge whether the offspring it bears are fertile or wind eggs [Pl. *Tht.* 151e6], whether they are phantoms instead of truth? What criteria could it use for measuring the truth in them? And how could it even produce so great a variety of *logoi*, if they did not have their being in it? For then we would be making their being come about at random, without reference to any standard[39].

If rational soul is entirely dependent on what it experiences of material things through sense perception, what independent criteria could it have for judging of the truth of its mathematical thoughts, what standard? Without such a standard, soul's mathematical constructions would be unverifiable and random, and the result of chance encounters in sense experience. Proclus also refers to the fertility of mathematical thought, namely the great variety of mathematical truths elaborated by the rational soul. Can this soul's creativity be simply reduced to the procedures of abstraction/accumulation? We note that, in Proclus' view, soul's contribution to the constitution of mathematical science is far greater and of another nature than what could be produced by the procedures of abstraction or accumulation.

This point is made clear in Proclus' final argument against those who have the soul derive mathematicals from sensibles[40]. To subordinate the soul to sensibles and to make it fabricate (διαπλάττει) images derived from them is to disvalue the soul and to make it inferior to matter which receives forms from nature, forms which the soul would then artificially separate from matter in making images of them.

39. εἰ δὲ μὴ ἔχουσα μηδὲ προειληφυῖα τοὺς λόγους τοσοῦτον ὑφαίνει διάκοσμον ἄϋλον καὶ τοσαύτην ἀπογεννᾷ θεωρίαν, πῶς τὰ γεννηθέντα δύναται διακρίνειν, εἴτε γόνιμα τυγχάνει ὄντα εἴτε ἀνεμιαῖα καὶ εἴδωλα ἀντ' ἀληθῶν, ποίοις δὲ κανόσι χρωμένη τὴν ἐν τούτοις ἀλήθειαν παραμετρεῖ; πῶς δὲ καὶ μὴ ἔχουσα τὴν οὐσίαν αὐτῶν ἀπογεννᾷ τοσαύτην ποικιλίαν λόγων; ηὐτοματισμένην γὰρ οὕτω τὴν ὑπόστασιν αὐτῶν ποιήσομεν καὶ πρὸς οὐδένα ὅρον ἀναφερομένην.

40. Procl. *In Eucl.* 14.24-15.15.

Soul, Intellect and the Generation of Mathematicals

From Proclus' arguments in his *Commentary on Euclid*, it follows that if mathematicals do not derive from sensible particulars, then their source must be sought elsewhere. In the metaphysical landscape of the Late Antique Platonist, this means, in the first place, in soul and/or in divine Intellect. As regards the soul, Proclus dismisses soul as the sole origin of mathematicals on the grounds that mathematicals are images of the paradigmatic Forms which inspire divine Intellect, as demiurge, as maker of the sensible world. Nor can this Intellect, however, be the sole source of mathematicals in Proclus' view, for this would reduce soul to a merely passive role, receiving mathematicals from Intellect as if it were matter, depriving it of the activity and the dynamic creativity it displays in developing mathematics[41]. From this argument it thus follows that mathematicals must be located in and derive from both soul and Intellect. But how?

The solution to this problem might be briefly summarised as follows[42]. According to the cosmological account presented in Plato's *Timaeus*, divine Intellect, as demiurge of the world, constituted the nature of soul out of mathematical structures (*Ti.* 35a-d). For both Syrianus and Proclus, these mathematical structures constituting the soul, which they call «substantial reasons» (*ousiodeis logoi*)[43], derive from and are images of the paradigmatic Forms which the demiurge uses in ordering the material world. Soul, in developing mathematical science, elaborates, i.e., unfolds («unrolls») in discursive thought, these «substantial reasons» so as to bring out their implications. Accordingly, for example, the geometer «projects» these reasons in extension, which is provided by the representative faculty (*phantasia*), so as to grasp them more easily and make manifest what they imply. Sensible particulars, imaging the paradigmatic Forms, may serve to remind soul[44] of the innate knowledge (the substantial reasons) which it already

41. Procl. *In Eucl.* 15.21-16.4.

42. See Procl. *In Eucl.* 16.4-18.4.

43. Procl. *In Eucl.* 17.23-26 (quoted below, note 47); 37.11-26; Syr. *In Metaph.* 91.29-92.5; 95.15-16; 179.8.

44. Procl. *In Eucl.* 18.17-18; 45.7-15; 140.15.

possesses as constitutive of its own nature[45]. In mathematical thought, these substantial reasons are explored by the rational soul by means of various methods proper to discursive thought. Geometry, for example, conceptually articulates the substantial reasons in the form of first principles or hypotheses: these are precisely the definitions, postulates, and axioms (common notions) of Euclid's *Elements*. By means of the methods of division, definition, demonstration, and analysis, the first principles are elaborated to form a logically rigorous deductive chain of truths and a system of seamless progression of conclusions based on demonstrations, whose conclusions, by the method of analysis, can also lead back in stages to the first principles of the science[46]. Proclus describes this dynamic progression and return of mathematical thought as follows (*In Eucl.* 17.22-18.4):

> So the mathematical *logoi* which fill souls are substantial and self-moving. By projecting them and unfolding them discursive reason brings into existence the mathematical sciences in all their variety; and it will never stop, forever generating and discovering more and more of them as it explicates the indivisible *logoi* within itself. For it contains in advance all of them as being their principle, and by virtue of its infinite power projects manifold theorems from the principles pre-contained in it[47].

Rational soul thus exhibits immense creativity in developing the mathematical sciences and an ability to generate an elaborate system of universal, necessary, and eternal truths which is anchored in the canons of truth represented by the substantial reasons innate in soul, which are themselves rooted in the truth of the transcendent Forms known by divine Intellect. This creativity and the system it generates stand in strong contrast to a

45. This theory of the ontological status of mathematicals appears to go back to Iamblichus (*Comm. Math.* 43.15-23; 44.4-9) and has its roots in Plotinus (*Enn.* IV 3 [27], 30; VI 6 [34], 16.38-39); see Sheppard 1997; Maggi 2010, 178-84; Kalligas 2015.

46. Procl. *In Eucl.* 18.10-19.20; 42.15-43.10; 69.13-19.

47. οὐσιώδεις ἄρα καὶ αὐτοκίνητοι τῶν μαθημάτων εἰσὶν οἱ λόγοι <οἱ> συμπληροῦντες τὰς ψυχάς, οὓς δὴ καὶ προβάλλουσα ἡ διάνοια καὶ ἐξελίττουσα πᾶσαν τὴν ποικιλίαν ὑφίστησι τῶν μαθηματικῶν ἐπιστημῶν, καὶ οὐ μή ποτε παύσηται, γεννῶσα μὲν ἀεὶ καὶ ἀνευρίσκουσα ἄλλα ἐπ' ἄλλοις, τοὺς δὲ ἀμερεῖς αὐτῆς λόγους ἐξαπλοῦσα. πάντα γὰρ προείληφεν ἀρχοειδῶς καὶ κατὰ τὴν ἄπειρον ἑαυτῆς δύναμιν ἐκ τῶν προειλημμένων ἀρχῶν παντοδαπῶν θεωρημάτων ποιεῖται προβολάς.

contribution of the rational soul which would consist in abstracting or accumulating generalities from sensible particulars[48]. Both the character of rational soul's methods and the objects which it elaborates are of a different order than that which the Aristotelians envisage as the abstracting/accumulating activity of soul and the conceptions which it distils from sensibles. Proclus is convinced that his view of mathematical science is what we find in the mathematicians, in particular in Euclid's *Elements*[49].

Mathematics and the Science of the Divine (Theology) and of Nature

The Late Antique Platonist theory of the nature of the mathematical sciences has implications concerning the relations between these sciences and other sciences, in particular metaphysics and physics. If in mathematics the first principles of science (the «substantial reasons») are images of the transcendent paradigmatic Forms thought by divine Intellect, then, by articulating and reaching a better grasp of these first principles, the mathematician comes nearer to reaching a knowledge of transcendent divine beings, that is the domain of metaphysics (theology). Mathematics thus prepares the way for the study of metaphysics, and the mathematical world which it constructs images the transcendent hierarchy of the divine. Similarly, since mathematics has innate knowledge imaging the paradigmatic Forms

48. These generalities are what Late Antique Platonists term «later-generated» (ὑστερογενῆ): as universals post rem they are seen as ontologically inferior to the sensible objects from which they are derived and epistemologically inferior to ante rem Forms such as are known by the substantial reasons innate in soul; see Syr. *In Metaph.* 171.16; Herm. *In Phdr.* 171.9-11 Couvreur = 178.31-179.3 Lucarini - Moreschini; Procl. *In Eucl.* 14.25-15.3. For Aristotelians, the form abstracted from matter is purely intelligible and identical (see Alex. Aphr. *De an.* 85.16-18), whereas for Platonists form is diminished by its presence in matter, and the conceptual form derived (as abstracted) from this enmattered form is even more degraded.

49. Proclus' views on mathematical science, which were to be of interest to Renaissance mathematicians and philosophers, correspond only in part to modern versions of Platonism in the philosophy of mathematics as described, for example, by Linnebo 2023.

which inspire the divine maker of the world, it provides access to the rational structures which underlie the organization of the material world while also supplying physics with scientific first principles and methods. These themes, however, are the subject of other investigations[50].

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Dominic J. O'Meara, *Syrianus and Proclus on Why True Mathematics Cannot Be Empirically Based*

This chapter presents arguments by Syrianus (in his *Commentary on Aristotle's* Metaphysics) and Proclus (in his *Commentary on Euclid's* Elements) to refute the Aristotelian explanation of the empirical origin of mathematical concepts, known as abstractionism. The authors seek to establish a higher ontological status for mathematical objects. According to Proclus and Syrianus, mathematical objects do not derive from sensible experience but are discursive projections of a priori concepts (*ousiodeis logoi*) innate in the human soul. These concepts, in turn, originate from a transcendent demiurgic intellect. This Neoplatonic ontology demonstrates how mathematical objects maintain qualities of precision, eternity, and universal validity, while simultaneously allowing for the creative activity of the mathematician. Crucially, this perspective positions mathematics as a fundamental science that serves as a paradigmatic model for physics and a crucial foreshadowing of metaphysics, highlighting its vital role in Proclus' overall philosophical system.

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