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RESTORING HARMONY THROUGH THE PROPAEDEUTIC SCIENCE OF MUSIC: A RECONSIDERATION OF THE BRETHREN OF PURITY'S EPISTLE ON MUSIC AND ITS RELATION WITH THE EPISTLE ON PROPORTIONS

At the turn of the seventh to eighth/thirteenth to fourteenth centuries, the Persian compendium *Mujmal al-ḥikma*[1] describes the work of the Ikhwān al-Ṣafā' (Brethren of Purity) as "obscure, mysterious, redundant". Although more than eight centuries have passed since the *Mujmal*'s probable date of writing, we can still gain insight into its perception by examining "music" and "harmony"[2], subjects that scholars have traditionally considered to be systematized and extensively discussed.

This article reconsiders topics from the commonly known "Epistle on Music" (*Epistle* 5). It proposes that the treatise, unlike any previous work in the field of propaedeutic sciences, makes a meaningful distinction between "music" and "harmony" as well as their divine and human aspects. In light of this, it can be argued that when reading *Epistle* 5, the art of harmony should be viewed as the human counterpart of divine Harmony and the art of music as the human counterpart of divine Music.

1. Anonymous, *Mujmal al-ḥikma, tarjume-yi guneh-i kehan az Rasā'il Ikhwān al-Ṣafā'*, ed. M. Dāneshpazūh and I. Afshār (Tehrān 1996), 29.

2. As discussed later in this article, the Greek tradition frequently used the terms «music» and «harmony» interchangeably, without distinguishing between their divine and human aspects. In this article, we employ quotes to refer to these concepts in a general sense as «music» and «harmony». However, when they denote a divine dimension, we capitalize them as «Music» and «Harmony». This convention is particularly relevant when referring to the Brethren's texts.

«Micrologus. Nature, Sciences and Medieval Societies» XXXIII (2025), pp. 335-367
ISSN 1123-2560 • e-ISSN 2975-1535 • ISBN 978-88-9290-371-5 • e-ISBN (PDF) 978-88-9290-372-2
DOI 10.36167/M33PDF •

Considering that the relationship between "music" and "harmony" was never formalized in the supposedly well-structured Greek tradition transmitted to the Arabic scientific literature, this distinction seems justified and interesting.

In addition, this article aims to stimulate a rethinking of the original purpose of *Epistle* 5, which, according to our reading, explains how divine Harmony can be traced and the art of harmony achieved through the exemplary study of the art of music. Since it is essentially composed of harmonic proportions, the art of music is the noblest art and the ideal model (lit., *mithāl*) for exploring, understanding, and reproducing divine Harmony over other human arts. Thus, music is not the only human art discussed in *Epistle* 5 and in the "Epistle on Proportions" (*Epistle* 6), since the art of harmony and proportions can be grasped and achieved through all the arts.

This article also seeks to show how *Epistle* 5 and *Epistle* 6, generally considered two independent treatises[3], may have been conceived, originally, as two related parts of the same subject, namely "harmony". Indeed, *Epistle* 5 provides a detailed description of how to discover and achieve "harmony" by recognizing affinities (*munāsabāt*) and applying proportions. Complementarily, *Epistle* 6 explains the mathematical relations on which the same affinities and proportions are based. By sharing the same subject matter and reorganizing earlier Greek material on this subject, the epistles demonstrate an intimate connection.

In this article, I have made a few assumptions about how the Brethren approached a purportedly, but inorganic, Greek tradition on the propaedeutic science of music, a subject that deserves greater attention.

In addition, I have revised earlier readings of *Epistle* 5, recognizing that even the most accurate interpretation of past sources is not without inconsistencies. This new perspective on the treatise is not an exception.

3. Previous studies indicated that *Epistle* 6 belongs to the propaedeutic sciences, but do not investigate the reasons for this.

What the Epistle On Music *Is Not About*

ليس غرضنا في هذه الرسالة تعليم الغناء وصنعة الملاهي ... بل غرضنا معرفة النسب وكيفية التأليف...

Our aim in this treatise is not to teach the practice of music (*al-ghinā'*) nor the construction (*ṣan'a*) of instruments… Our purpose is the knowledge of proportions and the modality by which Harmony manifests [itself][4]…

Music should not be studied in its instrumental or practical aspect since it corresponds to the mathematical fundamentals of divine Harmony.

Despite its position among propaedeutic sciences, after arithmetic, geometry, and astronomy/geography, traditionally, the commonly known *Risāla fī l-mūsīqī* ("Epistle on music") has been identified as a treatise on the fundamentals of music, belonging to the same tradition as al-Kindī's *Risāla fī khubr ṣinā'at al-ta'līf* and al-Fārābī's *Kitāb al-mūsīqī l-kabīr*.

In spite of this identification, note that, with the exception of a few sections dedicated to music (the fundamentals of acoustics, lute tuning, rhythm, and prosody), the treatise does not include instruction about musical intervals[5], how to construct them, how to derive them through arithmetic operations, or how to play them; also it does not discuss genres, modes, tonalities, melodies, instrument construction, and instrument classification.

When viewed as a treatise for teaching music as a mathematical propaedeutic science, one will not find the section on harmonic proportions and rational numbers. Instead this topic is covered in *Risāla fī l-nisab al-'adadiyya wa-l-handasiyya wa-l-ta'līfiyya* ("Epistle on arithmetic, geometric, and harmonic proportions", corresponding with *Epistle* 6)[6]. In addition, note that after the explication of ratios and

4. [Ikhwān al-Ṣafā'], *On Music: An Arabic Critical Edition and English Translation of Epistle 5*, ed. O Wright (Oxford 2010); Ar., 6. The excerpts included in this article are translated by L. Tribuzio.

5. The only reference to a list of ratios that express intervals appears in chapter 8: «4:3, 3:2, 5:4, 9:8». [Ikhwān al-Ṣafā'], *On Music*, Eng. 116, Ar. 71.

6. [Ikhwān al-Ṣafā'], *On Composition and the Arts: Arabic Edition and English Translation of Epistles 6-8*, ed. N. El-Bizri and G. de Callataÿ (Oxford 2018).

proportions, the same *Epistle* 6 deals with the domains in which proportions can be used, and shares a good deal of content with *Epistle* 5[7].

What the Epistle On Music *Is About*

The Brethren provide insight into the content of the treatise at the beginning of the epistle when they describe it as "commonly known *On Music*" (*mulaqqaba bi-l-mūsīqī*). This description associates the treatise with the art of harmony and the ways in which divine Harmony and proportions manifest (*nisab wa-kayfiyya al-ta'līf alladhīna bi-himā*).

فنريد أن نذكر في هذه الرسالة الملقبة بالموسيقي الصناعة المركبة بين الجسمانية والروحانية التي هي صناعة التأليف ومعرفة النسب، ليس غرضنا في هذه الرسالة تعليم الغناء وصنعة الملاهي، وإن كان لا بد من ذكرها، بل غرضنا معرفة النسب وكيفية التأليف الذين بهما وبمعرفتهما يكون الحذق في الصنائع كلها.

In this epistle, which is commonly known as (*al-risāla al-mulaqqaba*) "Epistle on music," we want to mention the art, which is composed by bodily and spiritual [matter] which is the art of harmony (*ṣinā'at al-ta'līf*), and the knowledge of proportions (*nisab*). Our aim in this treatise is not to teach the practice of music (*ghinā'*) nor the construction (*ṣan'a*) of instruments, although such matters deserve to be addressed. Our purpose is [to establish] the knowledge of proportions and the modality by which Harmony manifests (*kayfiyya al-ta'līf*) [itself], which by both (*alladhīna bi-himā*) of them and their acquaintance the dexterity in all the arts is attained[8].

Thus, the epistle *On Music* relates to "music" in the sense that it expresses the art of harmony (*ṣinā'at al-ta'līf*), the human and crafted realization of divine Harmony[9]. Specifically, it aims to explain the

7. [Ikhwān al-Ṣafā'], *On Composition and the Arts*, chapter 5, *On the Merit of the Science of Arithmetic, Geometric, and Musical Proportions*, Ar. 27-44, Eng. 57-68.

8. [Ikhwān al-Ṣafā'], *On Music*, Ar. 6-7. A similar purpose is stated in *Epistle* 2 and *Epistle* 7. See *infra*, 357-59. «Harmony in music: not just 'fitting together' but arranging according to numerical proportions».

9. In his description of the propaedeutic sciences, al-Kindī classifies the science of numbers (on the discrete quantities), the science of geometry (planes and surfaces), astronomy (solids in motion), and harmony *'ilm al-ta'līf*, which concerns «finding the ratio of one quantity to another and the nature of the relationship two by two. This is the knowledge of what is in harmonic combination and what is divergent. This research concerns the quantity that is in

mathematical proportions and modalities (*kayfiyya*) through which divine Harmony manifests [itself] and could even be realized in the human arts ("the dexterity in all the arts is attained by both of them the knowledge of proportions and the modalities of Harmony")[10].

Earlier readings

A few observations will probably arise when comparing the above translated excerpt with previous readings[11].

• Earlier interpretations did not properly consider the use of the expression "commonly known", which is a translation of the Arabic word *mulaqqaba*;

• Earlier readings did not relate the adjectives *jusmaniyya* (bodily) and *rūḥāniyya* (spiritual) to "matter" (*hayūlā*), which is discussed later in the Brethren's text. This, together with other oversights, likely led to the failure to recognize the exceptional nature of the art of music.

proportion to the other». Al-Kindī, *Risāla fī kammiyyat kutub Arisṭūṭālīs* (On the quantity of Aristotle's books), in *Rasā'il al-Kindī l-falsafiyya*, ed. Abū Rīdāh (Cairo 1950), 377. It is noteworthy that in al-ʿĀmirī's division of sciences (in the fourth/tenth century) reported in *al-Iʿlām bi-l-manāqib al-Islām, ta'līf* (harmony) is placed as the fourth of the propaedeutic sciences, after arithmetic, geometry, and astronomy. «Harmony refers to establishing proofs of what is or is not harmonized among powers and quantities in the heavenly and earthly worlds, that is the spiritual and physical worlds». Abū l-Ḥasan al-ʿĀmirī, *al-Iʿlām bi-l-manāqib al-Islām* (Exposition on the merits of Islam), ed. A. ʿAbd al-Ḥamīd Ghurāb (Riyadh 1988), 86.

10. As already observed, *Epistle* 5, *On Music*, more properly covers the subjects of modalities, while *Epistle* 6, *On Composition*, addresses the topic of proportions and rational numbers separately.

11. The following observations are not meant to be exegetical in any way, but rather to illustrate that *Epistle* 5 *On Music* can be read differently than it has previously been. A brief – hopefully illustrative – explanation is provided without comparing translations directly, which is not the purpose of this paper. Below are translations of the epistle, with the exception of Wright's translation, see note 4: [Ikhwān al-Ṣafā'], «L'Epître sur la musique des Ikhwān al-Ṣafā'», ed. and trans. A. Shiloah, in *Revue des Etudes Islamiques* 32 (1965), 125-62, and 34 (1967), 159-93; [Ikhwān al-Ṣafā'], «The Epistle on Music of the Ikhwān al-Ṣafā'», in *Documentation and Studies* 3 (Tel Aviv 1978), repr. in Shiloah, *The Dimension of Music in Islamic and Jewish Culture* (Aldershot 1993); [Ikhwān al-Ṣafā'], «Die Propaedeutik der Araber im zehnten Jahrhundert», *Die Philosophie bei den Arabern im X Jahrhundert n. Chr.*, vol. 3, ed. and trans. F. Dieterici (Berlin 1865), repr. Hildesheim 1969, 100-53.

It is described as the noblest of the arts (Epistles 5, 6, 21, 36) and an exception among them because it is composed of both spiritual and physical matter[12].

• Earlier readings disregarded the expression *ṣināʿat al-taʾlīf* as being different from *taʾlīf*, the former being the "art of harmony" and the latter being divine harmony, Harmony. This significant distinction occurs throughout the epistle and may go undetected unless a literal reading of the epistle is combined with an understanding of the Brethren's purported attempt to reinterpret the unsystematized Greek material on music as a propaedeutic science, which is on the basis of the epistle. The same Greek tradition underlies the next treatise corresponding to the epistle *On Proportions*.

• Earlier readings translated *nisab* as "ratios" instead of proportions, without considering that when talking of "harmony" the reference could also refer to harmonic proportions. Unfortunately, in Arabic *nisba* (sing.) can also be translated as the two mathematical concepts of "ratio" and "proportion".

• The substantive *kayfiyya*, "modality" was interpreted as a technical musical term, whereas here it refers to the original meaning of the Arabic root k-y-f ("how to"). During the fourth/tenth and the fifth/eleventh centuries, *laḥn* or *ṭarīq* were the two most commonly used terms to indicate musical modality[13]. The Ikhwān do not deal with the subject in the epistle. Thus, *kayfiyya* cannot be considered "musical modality".

• The expression *maʿrifat al-nisab wa-kayfiyyat al-taʾlīf alladhīna bi-himā*, which contains a relative with a dual subject, was grammatically misinterpreted.

• Earlier readings did not recognize the echo of the Greek tradition, supposedly Theon of Smyrna's *Expositio* (first/second century).

12. See *infra*, 347. «Evidence 2: The art of music belongs to the art of harmony».

13. In this regard, refer to L. I. al Faruqi, *An Annotated Glossary of Arabic Music* (London 1981). As an example of «modality» as a specific musical term, refer to (1) al-Kindī who uses *laḥan, Risāla fī khubr ṣināʿat al taʾlīf*, ed. and trans. Y. Shawqī (Cairo 1969), 35-41; (2) al-Ḥasan b. ʿAlī b. Aḥmad al-Kātib uses *ṭārīq, Kamāl adāb al-ghinā'*, ed. Maḥmūd Aḥmad al-Ḥifnī (Cairo 1975), 112-17.

In his treatise, Theon, as the Brethren later do[14], explains that since the purpose of his treatise is "the intellectual understanding of harmony and the music of the cosmos" (ὀρεγόμεθα δὲ τὴν ἐν κόσμῳ ἁρμονίαν καὶ τὴν ἐντούτῳ μουσικὴν κατανοῆσαι, 17_{2-3}), the instrumental and practical aspects of music are not considered. In stating this, Theon was following Plato (Rep. VII 531 a6-7) when he said "there is no need to torture the strings as if chasing a neighbour's voice"[15].

The role of "commonly known" in the Epistles

As observed, it is important to note that *Epistle* 5 is referred to as the "commonly known Epistle on music" (*al-Risāla al-mulaqqaba bi-l-mūsīqī*) rather than by its actual designation, which is the object of the same treatise, namely the art of harmony. The ratio of occurrences of the lexeme *mullaqaba* (Ar., l-q-b) in the 52 epistles supports this conclusion (see appendix, *Frequency of the lexeme 'laqaba' in Rasā'il Ikhwān al-Ṣafā'*)[16].

14. An in-depth analysis of Theon of Smyrna's contribution to the Brethren's *Epistle* 5 would be very interesting, and would require more attention. As such, in this article it is just as pertinent to highlight the way this resonates (more than previously thought) with a richer Greek tradition.

15. «We shall first recall the fundamental notions of arithmetic, to which are inextricably linked those of music in numbers; we do not absolutely need that in instruments, according to Plato's saying that there is no need to torture the strings as if chasing a neighbour's voice. We aspire rather to an intellectual understanding of harmony in the cosmos and music in it». Theon of Smyrna, *Expositio Rerum Mathematicarum ad Legendum Platonem Utilium*, ed. E. Hiller (Leipzig 1878), 16 line 24 to 17 line 3 (trans. L. Tribuzio). There is also a translation of the *Expositio* in English, though at times the translation is loose. Theon of Smyrna, *Mathematics Useful for Understanding Plato*, trans. R. and D. Lawlor, ed. and annotation, Ch. Toulis et al. (San Diego 1979). A more reliable translation, in Italian, is provided by F. M. Petrucci, *Expositio rerum mathematicarum ad legendum Platonem utilium. Introduzione, traduzione, commentario* (PhD thesis, Università degli studi di Pisa 2011/2012).

16. This Arabic root is peculiar because it invariably defines a name other than the one originally given. See this repository of Arabic lexicons for evidence: http://arabiclexicon.hawramani.com/search/%D9%84%D9%82%D8%A8. In the fifty-two epistles, l-q-b occurs seventeen times (oversights excluded and hopefully excused) and is used to refer either to generally known names or titles. For example, the original science of prophets (*'ilm al-anbiyā'*) is also known as the *science of theological objects* (*al-mulaqqab bi-l-ʿilm al-ilahiyyāt*). «The Epistle on Intellect and the Objects of the Intellect» (*Risāla al-ʿaql wa-l-maʿqulāt*) is also

According to the analysis of its recurrence, it emerges that this root is only used to indicate that commonly used expressions stand for more appropriate expressions or contents, as follows:

• "The generally known SCIENCE NAME corresponds to the original OTHER NAME" (e.g., the generally known *Science of theological objects* corresponds to *The science of prophets*);

• "The Epistle generally known as BOOK TITLE is about NAME OF THE CONTENT" (e.g., the epistle generally known *as On knowledge of death* is about the "knowledge of death and life");

• "The epistle named after BOOK TITLE from the ARISTOTELIAN TRADITION in ARABIC is about NAME OF THE CONTENT (e.g., the epistle named after [Aristotle's] *On the heaven and the world* is about "planets, constellations and the four elements")[17].

Recovering "Harmony," the Example of "Music": Evidences

According to the literature, the classification of μουσική (music) in μαθήματα (mathematical sciences) can be traced back to Pythagorean speculations (fifth century BCE). Since then the two terms, μουσική or ἁρμονία (harmony) have not been univocally used to define a mathematical science whose main purpose was the study of relations among quantities[18]. These quantities[19] could be identified by num-

commonly known by the abridged title, «The Epistle on the Objects of the Intellect» (*Risāla al-ma'qulāt*). A few times l-q-b refers to titles from the Greek Aristotelian tradition as translated and known in Arabic, for example «The Epistle on Planets, Constellations and the Four Elements» (*Risāla fī l-aflāk wa-l-kawākib wa-l-arkān al-arbi'a*) corresponds to the exposition of Aristotle's *Περὶ οὐρανοῦ* (*De Caelo* or *De Caelo et Mundo*), which circulated in Arabic under the title *On the Heaven and World* (*al-mulaqqaba bi-l-sama' wa-l-ālam*). In the appendix, a list of recurrences of the root l-q-b in the fifty-two epistles is provided with an explanation for each of them.

17. For more details, please refer to the appendix.

18. «It is obvious that music was strategically situated to lend elaboration and structure to the concept of harmony in a way that proved ineradicable. Indeed, the term 'music' becomes at times more or less synonymous with 'harmony' and takes on the same breadth of meaning». A. Lippman, «Hellenic Conceptions of Harmony», in *Journal of the American Musicological Society* 16.1 (Spring 1963), 4.

19. This article does not deal with the distinction between numbers ἀριθμός and magnitude (μέγεθος), nor with the distinction between proportions of num-

bers. As a result, this science, known as "harmony"[20] or "music", dealt with the relationship between numbers, but also with the movements of solids as expressed through numerical relationships. In fact, Plato called it ἁρμονία (harmony) and connected it to astronomy[21].

In our analysis there are a number of evidences that could prompt us to re-evaluate *Epistle* 5 and *Epistle* 6 strictly within this tradition, where "harmony" and "music" were not systematized and were not univocally used to define specific domains.

In contrast to this tradition, the Brethren sought to rethink issues and themes, such as the relationship between "harmony" and "music", that were not previously unanimously organized. Thus they formalized a distinction between "music" and "harmony", which were often intertwined concepts, and between their divine and human dimensions. "Music" is the mathematical expression of "harmony". Through "music" it is possible to recognize and reproduce "harmony" in its divine and human dimensions. However, music is just the most effective manifestation of "harmony"; it does not exclusively coincide with it. As a consequence, "harmony" can be perceived in other phenomena and achieved through other arts. Eventually, the Brethren distinguish between a qualitative approach to the

bers and magnitudes, nor with the question of «real number» from the original Greek mathematical perspective. Instead, a generalization of these concepts has been used, for lack of adequate space. See J. Klein, *Greek Mathematical Thought and the Origin of Algebra*, trans. E. Brann (New York 1992). Among other subjects, this study examines the revival and assimilation of ancient Greek mathematics through Arabic science during the seventh/thirteenth to tenth/sixteenth centuries. I. Grattan-Guinness, «Numbers, Magnitudes, Ratios, and Proportions in Euclid's Elements: How Did He Handle Them?», in *Historia Mathematica* 23 (1996), 355-75.

20. In Plato's Republic VII 522-617b, a sequence of mathematical disciplines is as follows: arithmetic, namely the study of the unit (a point without position); numbers and calculation; geometry, which deals with one-dimensional lines and two-dimensional surfaces; solid geometry (stereometrics); astronomy, in which the solids are in motions, i.e., in the circular revolutions of the heaven; and harmony, which studies the motions according to numerical ratios.

21. Plato's Republic VII 530d argues that they are «sister sciences, as the Pythagoreans admit». They are closely connected because «as the eyes frame the world of astronomy, so are the ears framed by the movements of harmony». This statement recalls Archytas of Taranto (Fragment 1). In his description of astronomy, geometry, arithmetic, and music, Archytas calls the four disciplines «sister studies» (H. Diels-Kranz, *Die Fragmente der Vorsokratiker* (Berlin 1960), bk 1, 1:431-32).

subject of "harmony" and its relationship to "music" (*Epistle* 5) and a quantitative approach (*Epistle* 6) describing rational numbers.

Five of these evidences are discussed in more detail below, preceded by a brief overview.

Overview

The first two evidences (evidence 1 and evidence 2) indicate that the Ikhwān were cognizant of the interchangeable use of the terms "music" and "harmony". This interchangeability stemmed from the fact that "music" fundamentally consists of harmonic proportions, which are regarded as the most perfect proportions, surpassing both arithmetic and geometrical ones. It is with this recognition that the Ikhwān sought to distinguish between divine Music as the expression of divine Harmony and the art of music as the most appropriate example on earth of the art of harmony. Notwithstanding its exceptional nature, because it is essentially composed of perfect harmonic proportions crafted by God (lit., see *infra*), the art of music, like other human arts, has a limited purpose, which corresponds to the harmonization of melodies and tunes. This exceptional condition makes it the noblest and most excellent example of the art of harmony among all the other human arts. This is also shown in the third proof [evidence 3]. Here it is implied that "music", as the most representative of other arts, is the first, but not the only, art to be discussed in *Epistle* 5. All other non-propaedeutic arts are thus treated as related to it by affinities (*munāsabāt*) and considered in terms of their possibility of expressing the art of harmony[22]. Once the perfect artifact is created, the other arts realize "harmony" through the application of harmonic proportions, unlike the art of music, which is intrinsically harmonious.

The same evidence explains the relationship between the art of music and divine Music. The art of music is the most powerful expression of heavenly Harmony on earth, and recalls, through sounds, the divine Music of the spheres and planets, the hearing of which is the ultimate goal of the soul after death.

22. It is important to consider that as a propaedeutic art or science, it does not stand alone, rather it transverses other arts.

The structure of *Epistle* 5 and the contents of the chapters [evidence 4] are mainly concerned with explaining how harmonic proportions identify natural phenomena and human activities (arts), and the influences of the art of music on man's soul. Apart from rhythms and their cycles, which are part of the fundamentals of the art of music and are discussed extensively in the epistle, little attention is paid to other specific musical components.

Complementing *Epistle* 5, *Epistle* 6 offers a numerical, quantitative view of harmonic proportions as they relate to ratios and proportions (rational numbers). *Epistle* 5 and *Epistle* 6 are therefore both related to the Greek tradition of the science of music (mathematical relationships between quantities) and should therefore be considered as two parts of the same treatise.

This perspective is further reinforced by the list of propaedeutic sciences reported in the epistle *On the Theoretical Speculative Arts* (*Epistle* 7). Here "proportions" are not mentioned except as part of "music", which is indeed "the science of harmony, that is, to knows the quiddity of proportions".

By considering the epistle *On Proportions* [evidence 5] as a separate treatise but following the so-called epistle *On Music*, the Brethren could have restored an intimate connection between the qualitative and quantitative dimensions of the science of music. In doing this they could have revised the role and positioning of the section on rational numbers proposed by Nicomachus. Unlike Nicomachus' *Introduction to Arithmetic*, on which part of the epistle *On Proportions* is based (Book II, ch. 21 further on), rational numbers are not part of arithmetic but are directly related to music. In this way, the Brethren could have revised a contradiction in Nicomachus' treatise. Despite Nicomachus' understanding of rational numbers as part of the propaedeutic science of music, he placed this topic in arithmetic, leading to an apparent inconsistency that has been overlooked in recent publications[23].

23. See *Brill's Companion to the Reception of Pythagoras and Pythagoreanism in the Middle Ages and the Renaissance*, ed. I. Caiazzo, C. Macris, and A. Robert (Leiden 2021).

Evidences

Evidence 1: "Harmony" can be named "music"

وإنما أردنا نحن أن نذكر من كل جنس من الموجودات مثالا، ليكون دالا علي شرف علم
النسب الذي يعرف بالموسيقى وأن هذا العلم محتاج إليه في بالصنائع كلها. وإنما خص هذا العلم
باسم الموسيقى، الذي هو تأليف الألحان والنغم، لأن المثال فيه أبين
وذلك أن القدماء والفلاسفة والحكماء إنما استدلوا أصول الألحان والنغم من المعرفة بالنسبة
،العددية والهندسية، لما جمعوا بينها
خرجت لهم النسبة الموسيقية كما بينا في الفصل الذي في استخراج النسب.

We wanted to mention for every species of existents (*min kull jins al-mawjūdāt*) an example, in order that it can serve as a signifier of the excellence of the science of proportions that is known as music (*yuʿraf bi-l-mūsīqā*). For this science (the science of proportions) is needed in all the crafts. And this science has been designated by the name "music" (*khuṣṣa bi-l-ʿilm al-mūsīqā*) due to its harmonization of melodies and tunes, since it is a lucid exemplar (*mithāl*) herein, and because the ancients, the philosophers, and the sages derived the principles of melodies and tuning from their knowledge of arithmetic and geometric proportions. For when these are combined, a musical proportion (*al-nisba al-mūsīqiyya*) is revealed through them as we have shown in the chapter on the derivation of proportions[24].

The first evidence corresponds to an excerpt from *Epistle* 6. Here the Brethren explain that "the science of proportions" is generally known or defined as "music" (*yuʿraf bi-l-mūsīqā and khuṣṣa bi-l-ʿilm al-mūsīqā*), but "music" – they claim – corresponds properly with the harmonic combination of melodies and tunes (*taʾlīf al-alḥān wa-l-nagham*) wherein proportions, with their best expression in harmonic proportions, can be found in every species of existents (*min kull jins al-mawjūdāt*). This once again[25] proves that the Brethren acknowledged the lexeme "music" as a general term that refers to another more appropriate term. The examplar (*mithāl*) of music for the application of harmonic proportions explains why the term is used to refer generally to harmonic proportions and even more generally to all propor-

24. [Ikhwān al-Ṣafāʾ], *On Composition and the Arts*, Ar. 32; Eng. 63, with slight revisions.

25. See *supra*, para. «The role of 'commonly known' in the Epistles», in which the meaning of *mulaqqab* (commonly known) is discussed.

tions. Thus "music" is often used as a synecdoche (the particular for the general). Additionally, this lexical phenomenon is associated with the adjective "musical proportion" (*al-nisba al-mūsīqiyya*) to refer to "harmonic proportion"[26].

It must be noted that even original Greek sources were aware of the mutual sense expressed by the word "music." As an example, Iamblicus (third/fourth century CE) called the harmonic proportions μουσική[27].

Evidence 2: The art of music belongs to the art of harmony

بداند که موسیقی تألیفی است که وضع ونهادن آن ارحواح حیوانی راست نه اجسام را که جسم را از آن نصیبی نیست. وموسیقی صناعتی است مرکب از جسمانی وروحانی، وتألیف غناء والحان از وی است.

You should know that Music is a kind of Harmony (*ta'līfī ast*) whose domain is that of vital spirits and not bodies because Harmony has nothing to do with the body.

As a form of art (*ṣinā'at al-mūsīqī*), music consists of both bodily and spiritual [matter]; as a result, the harmony of playing (*ta'līf-i ghinā'*), and melodies belong to it[28].

The second evidence comes from a seventh to eighth-/thirteenth to fourteenth-century Persian compendium, *Mujmal al-ḥikma*, that was intended, as mentioned above, to explain the epistles. At the beginning of the epistle *On Music*, there is a clear explication of what "music" is and how it is related to "harmony". Music is a type of Harmony. It is made by vital spirits. In this way, it differs from its human counterpart, which is the art of music (*ṣinā'at al-mūsīqī*) composed of both spiritual and physical matter[29].

26. For example, ch. 5 from [Ikhwān al-Ṣafā'], *On Composition and the Arts*, reports the title «On the merit of arithmetic, geometric and musical proportions».

27. Iamblichi, *In Nicomachi Arithmeticam introductionem liber*, ed. H. Pistelli (Leipzig 1975; repr. 1894), 118 line 20.

28. Anonymous, *Mujmal al-ḥikma*, 77.

29. Being made of spiritual and physical matter is at the core of the uniqueness and excellence of the art of music, refer *supra*, 339. «Earlier readings».

Evidence 3: Divine and human dimensions of "harmony" and "music"

An important evidence is provided by the statement at the beginning of chapter 12 of *Epistle* 5, which clarifies the content of the treatise. The epistle is devoted to explaining how Harmony is related to the cycle of life and death.

Thus, the Ikhwān begin the chapter by stating that the previous chapters were meant to demonstrate that all artifacts are arranged according to proportions (*bi-an fī l-i'tibār hādhihi al-mithalāt allatī tuqaddamu dhikruhā fī hādhihi al-fuṣūl*). Among them, the most perfect are those made by God, the finest Harmonizer (lit., *mu'allafan laṭīfan*), who arranged them according to harmonic proportions. These artifacts reveal Harmony and materialize in "the structure of the celestial spheres and their heavenly bodies, the dimensions of their parts, the measures of the elements and what is derived from them".

> You should know, dear brother, may God aid you and us with His spirit, that the considered examples mentioned above in these chapters (*bi-an fī l-iʿtibār hādhihi al-mithālāt allatī tuqaddamu dhikruhā fī hādhihi al-fuṣūl*) were meant to demonstrate that the best-made artifacts, the most masterfully structured, and the most beautiful of those harmoniously composed are those in which the structure of their constitution is arranged according to the perfect proportion (*al-nisba al-afḍal*)[30] and, likewise are the harmonious composition of its parts. So it is a demonstration and an analogy for everyone who understands, reflects, and considers that the structure of the heavenly spheres and their heavenly bodies, the dimensions of their parts, the measures of the elements, and what is derived from them, are also made one in relation to the other according to the most perfect proportion, and likewise the order of the distances of these spheres and stars, and their motions, correspond to the perfect proportions and are harmonious according to them[31].

30. In the Epistles the expression *al-nisba al-afḍal* essentially refers to harmonic proportions as superior, and thus perfect, as compared to the arithmetic and geometric proportions contained in them. This is the reason harmonic proportions are also considered three-dimensional. See, Nicomachus of Gerasa, *Introduction to Arithmetic*, ed. and trans. M. L. D'Oodge (London 1926), 2:39: «It remains for me to discuss briefly the most perfect proportion, that which is three-dimensional and embraces them all, and which is most useful for all progress in music and in the theory of the nature of the universe. This alone would properly and truly be called harmony». Nicomachus of Gerasa, *Introductionis arithmeticae*, ed. R. Hoche and G. B. Teubner (Leipzig 1866), 284.

31. [Ikhwān al-Ṣafā'], *On Music*, *Epistle* 5, ch. 12, Ar. 137-38.

These harmonic proportions, the perfect ones, are not set by humans but are intrinsic to the heavenly movements of the planets and serve as a defining characteristic of divine Harmony. In the same way heavenly movements define heavenly music, harmonic proportions describe heavenly music mathematically.

Thus, God is "the wise craftsman who crafted the harmonic proportions, the skilled artificer who constructed them, the finest harmonizer"[32] who, instead of creating out of nothing, used Harmony to manipulate and organize the chaotic matters through a process. This process consists of arranging opposite natures (lit., *mutaḍāddāt*) and quantities according to harmonic proportions, handling and giving forms and concepts to undifferentiated matters. As a result of Harmony, God preserves forms and proportional dimensions during emanations until the word of generation and corruption is reached.

Fortunately, in the world of earthly matters, man is able to recreate the harmonic proportions that originally describe heavenly musical sounds, through the art of harmony. This is because philosophers have been able to derive them (*istakhrajūhā*) by observing divine heavenly movements. They then transmitted these proportions to masters, who in turn transmitted them to disciples and common people in the form of the art of harmony. In this way people can duplicate the Harmony of God through a human art.

The philosophers extracted all the arts (*istakhrajūhā*) with their philosophy. After that, the people (*al-nās*) learned the arts from the sages (*al-ḥukamāʾ*) and passed them on to each other, so that they became a heritage that was passed on from the sages (*al-ḥukamāʾ*) to the community (*al-ʿāmma*), from the scholars (*al-ʿulamāʾ*) to the disciples (*al-mutaʿallimīn*), and from the masters (*al-asātīdh*) to the pupils (*al-talāmidha*). The same happened with music. The sages extracted it through their philosophy, and the people learned it from them and applied it in their activities and behavior, as well as for the other arts according to their various purposes[33].

32. [Ikhwān al-Ṣafāʾ], *On Music*, *Epistle* 5, ch. 12, Ar. 139.
صانعا حكيما صنعا ومركبا متقنا ركبها ومؤلفا لطيفا ألفها
Note the Arabic verbs and adjectives used to describe the stages of God's involvement in manipulating (*ṣanaʿ*), combining (*rakkaba*), and harmonizing (*allafa*) through mathematical proportions at the end.

33. [Ikhwān al-Ṣafāʾ], *On Music*, *Epistle* 5, ch. 2, Ar. 13.

The art of music, to which *Epistle* 5 is devoted, is the most effective and perfect expression of the art of harmony since it is intrinsically based on harmonic proportions, the same proportions that characterize the movement of the heavenly bodies. For this reason, the art of music serves as a model for all other arts, in achieving the human equivalent of divine Harmony. Each man, through his craft (calligraphy, prosody, cooking, mechanics, medicine, alchemy, magic, construction, architecture, painting), can achieve the perfect harmonic proportions that human music intrinsically possesses and create the most perfect artifact[34].

Evidence 4: *Epistle* 5 and the quality (*kayfiyya*); Epistle 6 and the quantity (*kammiyya*)

Epistle 5, affinities and how "harmony" is expressed

Epistle 5 consists of seventeen chapters, which can be divided into three main parts. The first part contains chapters 1 to 7. The second part contains chapters 8 to 14, with the exception of chapter 13. The third part contains the chapters 15 to 17. The first and third parts deal with the art of music as an expression of the art of harmony; it is described more for its ethos than for its theoretical fundamentals, which are limited to sound and rhythm topics. In the second part, the Brethren of Purity begin to draw a universe characterized by affinities (*munāsabāt*) and connections between natures, substances, spirits, heavenly and earthly words, human arts and crafts. These affinities describe a relationship of similarity between matters (whether spiritual or bodily matter) based on numbers in a mathematical relationship. This mathematical relationship is explained in the following *Epistle* 6.

With the exception of chapters 1 and 2, which deal with the influences of the art of music, chapters 3, 4, and 5 deal with acoustics and the lute, chapter 6 presents the universality of the art of music, chapter 7 deals with prosody and rhythm in relation to music, and chapters 8, 9, 10, and 11 describe, in no particular order, how proportions can be found as we move from heavenly bodies to the earth. Proportions are found everywhere, from the heavenly bodies to the

34. As later emerged in the *Mujmal*, this ability could be dangerous and is the reason some arts were hidden from the common people.

earth and its elements, nature, and temperaments; to the human body and newborn children; to calligraphy[35], rhythm, and prosody; to artifacts created through sculpture, painting, construction, and composition[36].

Chapter 12 examines the life cycle of the individual soul and how harmonic proportions relate to it. Picking up on one of the many themes discussed in chapter 9, this chapter describes the ultimate goal of the individual soul, which yearns to bear witness to the celestial harmonic music when it leaves the earth. During its earthly life, the soul has already been reminded of heavenly music by earthly melodies[37].

Chapter 13 deals with the rule of Arabic rhythms. Chapter 14 concerns the opposing and concordant tetrads that exist in nature, and how pragmatically craftsmen can overcome their opposites by using harmonic proportions to create the most useful of artifacts. Chapter 15 is about the skillful musicians, those who know the modality for influencing souls. Chapter 16 reports on the wise sayings of the philosophers concerning "music". Chapter 17 deals with the variety of effects of the musician's tones (*naghamāt*) on the souls of the listeners.

The narrative structure of *Epistle* 5 confirms that the core of the epistle lies in tracing affinities to demonstrate how Harmony can be observed and the art of harmony achieved; accordingly, Music and the art of music are treated as the most representative among harmonic phenomena (divine Music is displayed by heavenly movements) and the most exemplary of the crafted arts, so they are discussed as the first and most perfect examples to be considered. The relationship between the art of music and the other arts is realized when

35. An intriguing exploration of the later reworkings in Ottoman literature that incorporate certain images drawn from the Brethren's epistle *On Music*, particularly those related to the human body and calligraphy, is presented by F. S. Eryilmaz and G. De Callataÿ in their work titled «Following the Steps of the Ikhwān al-Ṣafā' in the Ottoman World I: Insights from Three Universal Histories», in *Journal of Islamic Studies* (2023), 20-28.

36. It is according to the same analogy and model that skilled artists produce their artifacts, whether shaped, sculpted, or painted, so as to be proportionate to one another in construction, composition, and arrangement. *On Composition and the Arts*, paraphrasing *Epistle 6* from the English translation, 60.

37. In conjunction with chapter 9, this chapter provides a comprehensive analysis of how harmony, the life cycle, and death are interrelated, thus complementing the content of the treatise *On the Quiddity of Death and Life* (*Epistle 29* in the critical edition).

they respect and reproduce the harmonic proportions, which the art of music exemplarily displays, by producing the most perfect of artifacts.

Epistle 6, rational numbers and the quantity of "harmony"

If *Epistle* 5 explains through affinities how the reader can effectively recognize on which modalities Harmony is really effective (*kayfiyyat al-ta'līf*) in the natural and supernatural worlds and how man himself can reproduce it through the art of harmony, *Epistle* 6 describes the mathematical relations (rational numbers) at the basis of affinities. In doing so, the Ikhwān again carefully re-examined the Greek material and reconsidered the proper place for rational numbers as the subject of "harmony" and "music"[38].

Rational numbers, as a subject belonging to the propaedeutic science of music, has its tradition in Greek literature, probably from the very beginning of the Pythagorean speculations, since Philolaus' (fifth to fourth century BCE)[39] speculations on mathematics, and continuing up to Nicomachus and Theon of Smyrna (second century CE) and beyond. As a propaedeutic science concerned with proportions and ratios as well as commensurability and incommensurability, "music" has been a crucible of mathematical questions and answers, even up until the sixteenth century, when irrational numbers were still discussed on the basis of musical reasoning[40].

As Arpad Szabo has shown, the concepts of ratio (λόγος) and proportion (ἀναλογία), and even the expressions for operations on ratios, were all developed from considerations and experiments in music theory and then applied to arithmetic and geometry[41].

38. See *infra*, 355-57. «An assumption on the Brethren's approach to revise Nicomachus on rational numbers and 'music'».

39. For more on the mathematical connotation of harmony starting from Philolaus, see C. A. Huffman, *Philolaus of Croton, Pythagorean and Presocratic. A Commentary on the Fragments and Testimonia with Interpretive Essays* (Cambridge 1993).

40. See P. Pesic, «Hearing the Irrational: Music and the Development of the Modern Concept of Number», in *Isis* 101.3 (September 2010), 501-30.

41. The fact that «the original theory of proportions was concerned exclusively with music and that it was subsequently applied to arithmetic and geometry» is extensively explained and proved by Arpad Szabo in the second part of his compelling work, *The Beginnings of Greek Mathematics* (Dordrecht 1978), 170. Szabo's evidence was later expanded by L. Borzacchini in «Incommensurability, Music, and Continuum: A Cognitive Approach», in *Archive for History of Exact Sciences* 61 (2007), 273-302. Peter Pesic's article is in the same vein.

When we read *Epistle* 6, we must bear in mind this close connection between these two realms that seem so distant to us today.

Finally, we should consider another point, namely the absence of any reference to the subject of proportions in the epistle *On the Theoretical Speculative Arts* (*Epistle* 7), except to define the propaedeutic science of music, which is described as: "…the science of harmony, that is, to know the quiddity of proportions[42], the way to harmoniously combine (*kayfiyyat al-ta'līf*) things different in substances". Later in the same passage, the Brethren say: "For each of these arts, we have produced an epistle, [to serve] as an introduction or preliminaries"[43].

If *Epistle* 7 only refers to proportions in connection with the science of music, the question arises as to why there is an epistle *On Proportions* at all.

Given the tradition of Greek music on the subject of rational numbers as part of musical speculation, the content and structure of *Epistle* 5 and *Epistle* 6, as well as the fact that *Epistle* 7 mentions proportions in relation to the science of music, it seems reasonable to identify *On Music* and *On Proportions* as being part of the same treatise.

Evidence 5: Recovering the connection between proportions and the art of harmony

Epistle 6 is simply organized: Chapter 1 describes ratios (*nisab*) and their five types; chapter 2 covers proportions (*nisab*)[44] which are divided into three types (arithmetic, geometric, and harmonic); chapter 3 deals with the extraction of continued proportions; chapter 4 concerns proportionality; chapter 5 is "on the merit of the science of arithmetic, geometric, and musical proportions"[45]; and an

42. Please refer *supra*, para. «Earlier readings» and *infra* «A way to reassess the Epistles 5 and 6 and their topics».

43. [Ikhwān al-Ṣafā'], *On Composition and the Arts*, *Epistle* 7, Eng., 109-10 with slight revisions, Ar. 76.

44. Λόγος (ratio) and ἀναλογία (proportion) are rendered by the Brethren with the same word, *nisba*, pl. *nisab*. In Thābit b. Qurra's *Madhkhal*, λόγος (ratio) is translated as *nisba*, ἀναλογία (proportion) is translated as *tusāwi al-qiyās*, and a definition of the two arithmetical relation is given. *Ṯābit b. Qurra, Madkhal ilā ʿilm al-ʿadad: Arabische Übersetzung der* Ἀριθμητικὴ εἰσαγωγή *des Nikomachos von Gerasa*, ed. W. Kutsch (Beirut 1959), 93.

45. As noted, «music» and «musical» can also refer to «harmony» and «harmonic».

appendix on the rule for finding the cube root is added in the critical Oxford edition[46].

Notwithstanding the limited evidence we have of the Greek tradition on "music/harmony" transmitted into Arabic up to the third/ninth century, it remains to be proved that, with the exception of Euclid's *Elements* (Book V)[47] and Nicomachus' *Introduction to Arithmetic* (Book I, chs. 17-23; Book II, chs. 21 onward)[48], the Ikhwān may not have considered other Greek texts such as Theon of Smyrna's *Mathematics Useful for Understanding Plato* or Ptolemy's *Harmonics*, or even Nicomachus' *Manual of Harmony*[49].

46. The cube is the expression of «harmony» by some Pythagorean authors. As also noted by El-Bizri, the appendix should echo book II, ch. 26, but also – it is worth adding – ch. 29, where harmony is defined as a cube (ο κύβος αρμονία). Further comparisons with Nicomachus' *Introduction* should be made in the context of Philolaus' fragments on geometric harmony represented by a cube.

47. According to tradition, it seems that there existed two versions of Στοιχεῖα (Elements) by al-Ḥajjāj b. Yūsuf b. Maṭar and one by Isḥāq b. Ḥunayn edited by Thābit b. Qurra. For more detail and a new perspective, see S. Brentjes, «Who Translated Euclid's Elements into Arabic?», in *Translation and Transmission: Collection of Articles*, ed. J. Hämeen-Anttila and I. Lindstedt (Munster 2018), 21-54.

48. Three Arabic versions of Nicomachus are known to have existed: Ḥabīb b. Bahrīz's translation from the Syriac, now lost; al-Kindī's revision of the latter, lost in Arabic but extant in a Hebrew translation by Qalonymos ben Qalonymos; and Thābit b. Qurra's translation from the Greek. See M. Zonta and G. Freudenthal, «Nicomachus of Gerasa in Spain, ca. 1100: Abraham bar Hiyya's Testimony», in *Aleph* 9 (2009), 189-224. For the edition of the Arabic translation of Thābit b. Qurra, see *Ṯābit b. Qurra, Madkhal ilā ʿilm al-ʿadad*. Hebrew translation by Qalonymos ben Qalonymos, G. Freudenthal, and T. Levy, «De Gerase a Bagdad [De Gérase à Bagdad]», *De Zénon d'Élée à Poincaré: Recueil d'études en hommage à Roshdi Rashed*, ed. R. Morelon and A. Hasnawi (Leuven 2004), 479-544. See also G. de Vaulx D'Arcy, *Aḥmad b. al-ṭayyib al-Saraḫsī*, «Réviseur de l'Introduction Arithmétique de Nicomaque de Gérase et rédacteur ses Rasā'il Ikhwān al-Ṣafā'», in *Arabic Sciences and Philosophy* 29 (2019), 261-83.

49. Unfortunately, no evidence has been found for the translation into Arabic of Theon of Smyrna, *Expositio Rerum Mathematicarum ad Legendum Platonem Utilium*, nor for Ptolemy's Ἁρμονικόν (Harmonics) and Nicomachus' Ἐγχειρίδιον ἁρμονικῆς (Manual of harmony). Apart from the absence of Greek material in Arabic, the proximity of Ptolemy's *Harmonics* and Nicomachus' *Manual of Harmony* to the Brethren of Purity's Epistle *On Music* seems at first sight less relevant. Nevertheless, Theon's Expositio seems to be very close in some respects, and its possible tradition in Arabic deserves further investigation. For more information on Ptolemy and Theon in the Arabic tradition, see J. Lippert, «Theon in der Orientalischen Literatur», in *Apollonius, Geminus, Heron, Ptolemaeus, Theon in the Arabic Tradition: Texts and Studies*, coll. and repr. F. Sezgin, Islamic Mathematics and Science, vol. 68 (Frankfurt am Main 1998). A. Jones, «The Ancient Ptolemy», in *Ptolemy's Science of the Stars in the Middle Ages*, ed. D. Juste, B.

In the case of a limited[50] or extended relevance of the Greek materials, which is more plausible when reading *Epistle* 5 and *Epistle* 6 together, we can assume that the Brethren recognized a fundamental contradiction in Nicomachus' *Introduction to Arithmetic* with regard to rational numbers. This understanding could explain the positioning of *Epistle* 6, which deals with rational numbers, not as part of arithmetic, but as directly related to music and harmony. The following paragraph gives a more detailed explanation of the subject.

An Assumption on the Brethren's Approach to Revise Nicomachus on Rational Numbers and "Music"

In earlier Greek speculations transmitted to Arabic philosophical culture from the second/eighth century onward, the concept of "music" as a propaedeutic science and its relation to "harmony", an art originally belonging to God and His creation, were extensively discussed but not unanimously systematized. Then an apparent inconsistency could be envisaged in Nicomachus' *Introduction to Arithmetic* which, we assume, could have been recovered and reinterpreted in *Epistle* 6.

Inconsistencies of this kind could be seen as the physiological result of the fact that the process of identifying and defining various fields of knowledge took time to develop and institutionalize, from early Pythagorean speculations about the science of music to the later formalization of the *quadrivium*.

In fact, the determination of the subjects of what was later called the *quadrivium* does not seem to have been precisely identified and widely shared in the Greek tradition[51].

In his *Introduction to Arithmetic*, Nicomachus of Gerasa, whose influence on *Epistle* 6 has been explained above, disregards throughout

van Dalen, D. N. Hasse, and C. Burnett, Ptolemaeus Arabus et Latinus Studies 1 (Turnhout 2020), 13-34.

50. As El Bizrz reports, the relevance of other Greek sources seems unclear, [Ikhwān al-Ṣafā'], *On Compasition and the Arts*, *Epistle* 6, 16 n. 3.

51. Sometimes it seems that scholars in general have been more concerned with reconstructing unity in the Greek and Arabic traditions than with understanding any form of inconsistency regarding the relationship between music and harmony and their place in the mathematical sciences.

his treatise the original claim made at the very beginning, in which the subjects of the four mathematical sciences are defined (in book I, chapter 3). In this renowned part of the treatise, which is considered the foundation of the Boethius' *quadrivium*, the study of numbers, whether absolute or per se, belongs to arithmetic; numbers in relation to one another, to music; geometry deals with quantities at rest; and astronomy concerns quantities in motion[52].

In truth, Nicomachus seems to have neglected the first two of these definitions; he discusses rational numbers, which should fall under the domain of music, in a long section of the treatise on arithmetic. In addition, numbers are also covered in another section of the *Introduction to Arithmetic*; namely, geometric planes and solids, though it should have been placed in the geometric domain.

Presumably – according to my analysis – Nicomachus could have followed and fixed a tradition on which this apparent contradiction was based. I call this contradiction "apparent" because, as Theon of Smyrna, Nicomachus' contemporary, explains in his discussion and different ordering of the propaedeutic sciences, it is not possible to intellectually grasp the Harmony of the cosmos and the Music in it without acknowledging the numerical rules of sound.

ὀρεγόμεθα δὲ τὴν ἐν κόσμῳ ἁρμονίαν καὶ τὴν ἐν τούτῳ μουσικὴν κατανοῆσαι· ταύτην δὲ οὐχ οἷόν τε κατιδεῖν μὴ τῆς ἐν ἀριθμοῖς πρότερον θεωρητικοὺς γενομένους.

Instead, we seek to grasp Harmony and Music in the cosmos, but Harmony cannot be grasped until a theoretical understanding of numbers is applied[53].

It could be said that Theon, instead of following the same conflicting tradition, tried to find a logical solution – as he himself explains – by incorporating musical numerical rules into arithmetic, by considering the mathematical science of music (the relationship between movement and rest) as the fifth in the mathematical order after arithmetic (numbers), geometry (surfaces), stereometrics (solid objects), and astronomy (solids in motion).

52. The only scholar to suggest this contradiction was M. L. D'Oodge (ed. and trans.) in his translation into English, *Introduction to Arithmetic*, 184 n. 1 (in the 1926 edition).

53. Theon of Smyrna, *Expositio Rerum*, 17 lines 2-4.

For this reason, according to Plato, music is also the fifth discipline and corresponds to the music that is in the cosmos, in the movement, order, and consonance of the stars that move there. However, it is necessary for us, and according to Plato himself, to place it second after arithmetic, because music in the cosmos cannot be conceived unless it is accounted for in numbers and understood intellectually[54].

In his definition of the sciences of mathematics, Theon, following Plato (Republic VII, 522-617b), identifies five sciences in which music is concerned with the relationship between motion and rest, in contrast to Nicomachus, who identified music as the analysis of the relationship between numbers.

Harmony in Music: Not Just "Fitting Together" but Arranging according to Numerical Proportions

As part of a general effort to reconsider *Epistle* 5 together with *Epistle* 6, here it seems necessary to briefly emphasize another point relating the epistles to the Greek elaborations on the propaedeutic sciences of "harmony" and "music".

This point concerns the mathematical conception of "harmony" which, through Nicomachus, must have been transmitted to the Brethren by pre-Socratic authors, in particular Philolaus.

It would seem that this extract from the *Introduction* of Nicomachus was intended to echo Philolaus when he says[55]:

ἁρμονία δὲ πάντως ἐξ ἐναντίων γίνεται ἔστι γὰρ ἁρμονία πολυμιγέων ἕνωσις καὶ δίχα φρονεόντων συμφρόνησις (II.19).

Harmony in every way arises out of opposites. For harmony is the unification of what is a mixture of many ingredients and the agreement of the disagreeing[56].

54. Theon of Smyrna, *Expositio Rerum*, 17 lines 5-11.

55. See also Theon of Smyrna, *Expositio Rerum*, 2 lines 10-12: «καὶ οἱ Πυθαγορικοὶ δέ, οἷς πολλαχῇ ἕπεται Πλάτων, τὴν μουσικήν φασιν ἐναντίων συναρμογὴν καὶ τῶν πολλῶν ἕνωσιν καὶ τῶν δίχα φρονούντων συμφρόνησιν», «Even the Pythagoreans, whom Plato often follows, affirm that music is the harmonious combination of opposites, the unification of the multiplicity as well as the agreement of different understandings».

56. In *Philolahus of Croton*, Theon of Smyrna, *Expositio Rerum*, 416.

The excerpt corresponds to the following Arabic translation by Ibn Qurra:

والتأليف بالجملة إنما يكون عن الأشياء المتضادة وذلك أن التأليف هو اتحاد أشياء مختلفة (نفقة؟) والجمع بين أشياء منفردة بعضها عن بعض.[57]

Harmony in general is about things that are opposites, and that is because harmony is the union of different things (expense?)[58] and the combination of things that are separate from each other.

Philolaus is of great importance for the development of the concept of "harmony" in the Greek tradition, since more than other Pythagorean authors he associated ἁρμονία (harmony), a word previously used to express "fitting together", with numbers, thus becoming quantitatively congruent and beginning to mean the relationship between what is limited and what is unlimited according to numerically specified relationships.

The authenticity of the previous fragment is uncertain, and is more likely to be pseudo-Pythagorean. Nevertheless the concept of harmony it "promotes", which relates it to a mathematical thought in which opposed substances (*mutaḍādd* is the lexeme identifying the concept) are not simply put together but are arranged according to mathematical relationships, must have been of great concern to the Brethren.

In fact, the same claim is reiterated by the Brethren in the epistle *On Proportions*, in the epistle *On Geometry*, and in the epistle *On the Theoretical and Speculative Arts*, where again a conception of harmony as the combination and unification of opposites (*mutaḍāddāt*) according to mathematical proportions (*nisab*) is presented.

Epistle *On Geometry* (*Epistle* 2)

[Music is the knowledge of the] … harmonic composition (*ta'līf or ta'līfāt*) and proportions [ratios] that exist between entities that differ in substance and sustain opposite potencies (*al-mutaḍādda al-quwā*). The origin of this

57. Thābit b. Qurra, *Madhkhal ilā ʿilm al-ʿadad*, 89.

58. Thābit b. Qurra, *Madhkhal ilā ʿilm al-ʿadad*, 89 n. 4. Admittedly, this term does not have mathematical or philosophical connotations.

science lies in the equality of ratios[59], like the ratio of equivalence and qualification, such as the ratio of three to six, which brings the same result as the ratio of two to four[60].

Epistle *On Proportions* (*Epistle 6*)

اعلم أن هذا الأركان متغالبات القوى، متضادات الطبائع، مختلفات الصور، متباينات الأماكن، متعاديات، متنافرات، لا تجتمع ولا تأتلف إلا بتأليف المؤلف لها. والتأليف متى لا يكون على النسبة لم يمتزج ولم يتحد. ومن أمثال ذلك أصوات النغم الموسيقية...

Know that these elements overwhelm each other with their powers and are opposites in their natures (*mutaḍāddāt al-ṭabā'i'*), distinct from each other in their forms, divergent in their places, antithetic, and incongruous wherein they do not come together or get harmonized [combined] except by the composition of a composer. When the composition is in proportion (*'alā l-nisba*), they do not merge and unite; and the examples of this are in the sounds of musical melodies…[61]

Epistle *On the Theoretical Speculative Arts* (*Epistle 7*)

والرابع الموسيقي، الذي هو علم التأليف، وهي معرفة ماهية النسب وكيفية تأليف الأشياء المختلفة الجواهر المتباينة الصور المتضادة القوى المتنافرة الطبائع، كيف يجمع ويؤلف بينهما، كيما لا تتنافر وتأتلف وتتحد وتصير شيئا واحدا فعلا وحدا أو عدة أفعال.

[The fourth is Music, which is the science of harmony [composition], that is, to know the quiddity of proportions [ratios], the way to combine (*kayfiyyat al-ta'līf*) things different in substances, distinct in forms, opposite in powers (*al-mutaḍādda al-quwā*), and incompatible in natures, how they are gathered together and [harmoniously] combined with each other in such a way that they are no longer incompatible but [apt to] be [harmoniously] combined with each other, joined together, and to become one thing, [and apt to] perform one act, or a certain number of acts[62].

59. The equality of ratios means proportion, because proportion is the equality of ratios.

60. [Ikhwān al-Ṣafā'], *On Arithmetic and Geometry: An Arabic Critical Edition and English Translation of Epistles 1 & 2*, ed. and trans. N. El-Bizri (Oxford 2012), Eng. 104; Ar. 84-85.

61. [Ikhwān al-Ṣafā'], *On Composition and the Arts*, *Epistle* 6, Eng. 58, Ar. 29.

62. [Ikhwān al-Ṣafā'], *On Composition and the Arts*, *Epistle* 7, Eng. 109-10 with slight revisions, Ar. 76.

A Way to Reassess Epistles 5 and 6 and Their Topics

As we have seen, a few misconceptions and presumptions underlie the identification of the epistle *On Music* in relation to its actual purpose and content.

Supposedly, a bias in some readings could have been caused by two main factors.

1. The Ikhwān relate "music" and "harmony" according to tradition. Unfortunately, throughout the classical Greek tradition, and in most of the studies devoted to the subject, the concepts of "harmony" and "music" have been intertwined and often confused. Earlier readings of this epistle was probably affected by such confusion.

2. The Ikhwān did not provide a theoretical plan to explain the concepts in this epistle; rather it seems to assume that the reader is already familiar with the concepts and knows how to distinguish between them. The narrative approach of the Ikhwān to the text has probably been overlooked by previous scholars and translators. Since the epistle is listed as propaedeutic, it was probably intended to be expository. As a result, scholars have not paid attention to some of the key concepts implied throughout the epistle.

In this context, the following conclusions can be drawn.

a. Studies on the epistle, references to it, and translations of it have confused divine Harmony with the art of harmony, which is its human counterpart, and similarly divine Music with the art of music. However, if one translates the epistle literally and reports the expressions used by the Brethren accurately, it becomes obvious that they applied such differences to concepts.

b. Researchers have disregarded the real purpose of the epistle, its narrative evolution, and its relation with the epistle *On Proportions*.

In the context of a reassessment of *Epistle* 5 and *Epistle* 6, the following concepts should be considered.

a. Harmony is not the same as its human-crafted and pragmatic realization called the art of harmony. Harmony is divine, its art is human.

b. Music is the most effective realization of divine Harmony, but does not coincide with it completely.

c. Music corresponds with heavenly divine music. Harmony corresponds with a mathematical regulating principle among opposite quantities and qualities; God crafted this principle.

d. Music is not the same as its human-crafted and pragmatic realization called the art of music.

e. The art of music is the most effective and exemplary (*mithāl*) kind of art of harmony.

f. The art of music corresponds with earthly music and has affinities with divine Music (heavenly music). While the art of music is intrinsically perfect, other arts become perfect, and consequently their products are as well, when transforming matter through harmonic proportions.

g. Harmonic proportions are also "appointed" as musical proportions since the art of music is based on them intrinsically, regardless of whether other crafts make use of them.

h. Melodies and rhythms are the artifacts of "music".

i. Music and Harmony belong, originally, to supernatural existence; sages have transmitted them to scholars, scholars to disciples, and masters to men.

j. *Munāsabāt* ("affinities") describe a relation of similarity among things rather than the identity of these things.

k. When referring to the art or science of music the term *nisba* (pl. *nisab*) should be interpreted carefully, since it could refer to ratios at the base of intervals (e.g., 5:4 which is the fourth) but also to "harmony" and to the harmonic proportions at the foundation of it.

l. The lexeme *ta'līf* means not just combining or fitting together but numerical relations; thus, when translated one could use the corresponding lexeme harmony, acknowledging that this suggests a numerical interpretation.

p. Greater consideration should be given to the Greek tradition when reading *Epistles* 5 and *Epistle* 6.

Appendix:
Frequency of the Lexeme laqaba *in the* Rasā'il Ikhwān al-Ṣafā'

Below is a report of the recurrence of the lexeme *laqaba* in the Epistles of the Brethren of Purity. The term appears mostly in the form of adjectives and verbs. The analysis of the recurrence shows that this root indicates that general, common expressions stand for more appropriate expressions or contents.

In the numerical references to the epistles, the critical Oxford University Press publication number (2010-2021) is followed by a slash (/) and the Roman number of the Epistle indicates its position in each of the four sections.

Page numbers refer to the Oxford edition and the slash (/) refers to Khayr al-Dīn al-Zirkalī's edition (Beirut 1928). In cases where one of the editions lacks references, three points are reported as ellipses (...).

Mathematics, Section I

• 1 time in *Risāla* 5/V, p. 5/157: the epistle is commonly known by the title *On Music* (*fī l-mūsīqī*), but it is rather about the art of harmony (*ṣanaʿat al-ta'līf*).

• 2 times in *Risāla* 11/XI (*fī qaṭīghūriās, On Categories*).

First time, p. 47 note/322: used to continue discussing a concept generally known (*laqqabū*) as "matter" (*hayūlā*) which identifies "those things preceding the existence" (*al-ashyā' al-mutaqaddima fī l-wujūd*), and "forms" (*ṣuwar*), as "those things succeeding in existence" (*al-ashyā' al-muta'akhkhira fī l-wujūd*).

Second time, p. 51/323: used to continue discussing a concept generally known (*laqqabū*) as "the comprehensive genre" (*al-jins al-shāmil*).

Natural Sciences, Section II

• 2 times in *Risāla* 16/II.

First time, p. 67/ ...: used to refer to the epistle that is named *On the Auscultation of Being* (*al-Risāla al-mulaqqaba bi-l-simāʿ al-kiyān*) after Aristotle's original treatise *Φυσικὴ ἀκρόασις* (*Physica*, or *Naturales Auscultationes*)

Second time, p. 67/19: the epistle is about the *Planets, the Constellations and the Four Elements* (*al-Risāla fī l-aflāk wa-l-kawākib wa-l-arkān al-arbaʿa*) and is named *On Heaven and World* (*al-Risāla al-mulaqqaba bi-l-samāʾ wa-l-ʿālam*) after Aristotle's original treatise *Περὶ οὐρανοῦ* (*De Caelo or De Caelo et Mundo*).

• 3 times in *Risāla* 17/III.

First time, p. 155/42: *mulaqqaba* is used to refer to an epistle that is named after Aristotle's original treatise *Περὶ οὐρανοῦ* (*De Caelo or De Caelo et Mundo*), *al-Risāla al-mulaqqaba bi-l-samāʾ wa-l-ʿālam* (see *supra*), dealing with celestial bodies (*al-ajsām al-falakiyya*).

Second time, p. 155/42: the epistle is about *Natural Bodies under the Sphere of the Moon* (*al-Ajsām al-ṭabīʿiyya allatī dūn falak al-qamar*) and is named *On Generation and Corruption* (*al-Risāla al-mulaqqaba bi-l-kawn wa-l-fasād*) after Aristotle's original treatise *Περὶ γενέσεως καὶ φθορᾶς* (*De Generatione et Corruptione*).

Third time, p. 183/48: here *mullaqaba* means "named after" Aristotle's Μετεωρολογικά (*Meteorologica* or *Meteora*) a compendium circulated under the title *al-Āthār al-ulwiyya* in Arabic.

• 2 times in *Risāla* 18/IV.

First time, p. 184-186/ …: here *mullaqaba* means "named after" Aristotle's treatise *Περὶ γενέσεως καὶ φθορᾶς* (*De Generatione et Corruptione*) compendium circulated under the title *al-Kawn wa-l-fasād*.

Second time, p. 186/50: the epistle is about *What Happens in the Atmosphere and the Transformations of the Air* (*ḥawādīth al-jaww wa-taghayyurāt al-hawāʾ*) and is named (*al-Risāla al-mulaqqaba bi-l-āthār al-ulwiyya*) after Aristotle's compendium of *Meteorology* known as *al-Āthār al-ulwiyya*.

• 1 time in *Risāla* 20/VI, p. 355/98: concerning the content of the ongoing epistle *On the Arts of Nature* (*al-ṣanāʾiʿ al-ṭabīʿiyya*), refers to an epistle generally known by the title *On Practical Arts* (*al-mulaqqaba bi-l-ṣanāʾiʿ al-ʿamaliyya*) and referring to *Epistle* 8/VIII.

• 1 time in *Risāla* 22/ VIII, p. … /150: referring to *Birāst al Ḥakīm* nicknamed Shāh Murād.

• 1 time in *Risāla* 29/XV, p. 7/367: naming the original epistle *On Knowledge of Death and Life* (*Risāla fī ḥikmat al-mawt wa-l-ḥayāt*) with the abridged title *On Knowledge of Death* (*al-mulaqqaba bi-l-ḥikma al-mawt*).

• 1 time in *Risāla* (no Oxford publication)/XVI, p. 380: naming the original epistle *On Pleasure, Pain, Anguish, Joy, Happiness, Sadness, Rest and Fatigue* (*al-Risāla māhiyyat al-ladhdha wa-l-alam wa-l-ghamm wa-l-surūr wa-l-farah̲ wa-l-ḥuzn wa-l-rāḥa wa-l-taʿab*) with the abridged title epistle *On the Pleasure and Pains* (*al-Risāla al-mulaqqaba bi-l-ladhdha wa-l-ālām*).

Psychology and Rational Sciences, Section III

• 1 time in *Risāla* 32b/II, p. 27/28: explaining that the five names of the days are named after numbers. In Arabic, as in Persian, the names of the days are literally translated as "day one", "day two" and so on.

• 1 time in *Risāla* 35/IV, p. … /46: naming the original epistle *On the Object of the Intellect* [*which are*] *Spiritual Forms* (*al-maʿqūlāt kulluhā ṣuwar rūḥāniyya*), with the abridged title "Epistle on Intellect and Intelligible" (*al-Risāla al-mulaqqaba bi-l-ʿaql wa-l-maʿqūlāt*).

• 1 time in *Risāla* (no Oxford publication)/ IX, p. 131: the original *Science of the Prophets* (*ʿIlm al-anbiyāʾ*) is commonly known as the *Science of Theological Objects* (*al-mulaqqab bi-l-ʿilm al-ilāhiyyāt*).

Theological Sciences, Section IV

No references found.

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Abstract

Laura Tribuzio, *Restoring Harmony through the Propaedeutic Science of Music: A Reconsideration of the Brethren of Purity's Epistle on Music and Its Relation with the Epistle on Proportions*

In the fourth/tenth century, if a novice had wanted to approach the theoretical fundamentals of music, he would have been disappointed by studying the Brethren of Purity's epistle *On Music*. A major objective of the present article is to provide some basis for reassessing the Brethren's views on the art of music in the epistle that is purposely devoted to it. According to the Brethren, who faithfully follow the Greek definition of this art as a propaedeutic science, the subject of the epistle is to present music as the most perfect model (*mithāl*) to represent the art of harmony. For this reason, the epistle does not discuss specific music theoretical elements, but rather it aims to identify the methods (referred to as modalities or *kayfiyya*) through which the concept of harmony can be understood and reproduced by men. Additionally, it explores how the idea of divine harmony is expressed through mathematical principles, which are the foundation for both divine music and its human counterpart, the art of music. This reconsideration enables us to better appreciate the Brethren of Purity's attempt to reconceptualize the important distinction between the two dimensions of "harmony" and "music," the divine and the human, and, consequently, resolve some incon-

sistencies in the Greek literature on the subject. The present contribution highlights the intimate connection between *Epistle* 5 (*On Music*) and *Epistle* 6 (*On Proportions*) that deals with what we could today approximately refer to as rational numbers. Together they represent two aspects of the same propaedeutic art of harmony. The former is based on the qualitative aspects, the latter on the quantitative dimension.

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