

DE RATIONE COMPUTANDI

De ratione computandi is one of three pre-Carolingian Irish textbooks on the science of computus (the other two being the *Computus Einsidlensis* and the *Computus Monacensis* [Munich Computus]; cfr. pp. 119-25 and 126-32). It was briefly mentioned as a potentially interesting pre-Bedan text by Charles Jones in his 1977 edition of Bede's scientific works¹, and subsequently fully introduced into scholarship by Dáibhí Ó Cróinín in 1982²; this Ó Cróinín followed up by an *editio princeps* in 1988, in which he placed this text in close connection to Cummian's Letter of the 630s³. Principally because of shared uncommon sources (Jerome's *Commentarium in Aggaeum*, Ambrosiaster's *Liber quaestionum*, pseudo-Origen's *Homiliae in Leviticum*) between Cummian's Letter and *De ratione computandi*, Ó Cróinín argues that both were compiled in the same circle, and dates the latter around ca. 650 because of its heavy reliance on Isidore and a generally very active computistical scene in Ireland at this time. More recently, a later date in the 720s was prosed for *De ratione computandi* on the basis of the scientific content of this text⁴.

Only one of the three Irish computistical textbooks, the Munich Computus, is securely datable (to 718/9). Together with the *Computus Einsidlensis*, it appears to form one of the first attempts at compiling a textbook of some scale on this discipline of monastic learning. *De ratione computandi* displays a different level of sophistication, with crystal-clear definitions in a well-structured framework. It certainly rivals, if not outshines Bede's more lecturing *De temporum ratione* of 725, of which it had no knowledge⁵.

1. CCSL 123, p. XIII.

2. D. Ó Cróinín, *A Seventh-Century Irish Computus from the Circle of Cummianus*, «Proceedings of the Royal Irish Academy, Section C», 82 (1982), pp. 405-30, repr. in D. Ó Cróinín, *Early Irish History and Chronology*, Dublin 2003, pp. 99-132.

3. M. Walsh - D. Ó Cróinín (edd.), *Cummian's Letter De controversia paschali, together with a Related Irish Computistical Tract De ratione computandi*, Toronto 1988, pp. 99-213.

4. I. Warntjes (ed.), *The Munich Computus: Text and Translation. Irish Computistics between Isidore of Seville and the Venerable Bede and Its Reception in Carolingian Times*, Stuttgart 2010, pp. LIV-LV, CXCI-CCL. Cfr. also earlier M. Smyth, *Isidore of Seville and Early Irish Cosmography*, «Cambridge Medieval Celtic Studies», 14 (1987), pp. 69-102, at pp. 94-5, 100.

5. Bede's *De temporum ratione* is ed. by C. W. Jones, *Beda's opera de temporibus*, Cambridge 1943, with superb commentary, and later in CCSL 123. Cfr. also the translation and commentary by F. Wallis, *Bede: The Reckoning of Time*, Liverpool 1999, and the commentary by R.-P. Pillonel-Wyrsh, *Le calcul de la date de Pâques au Moyen Âge: analyse et commentaires sur De temporum ratione de Bède*, Fribourg 2003.

In this context it is interesting to note that unlike the other textbooks, *De ratione computandi* does not provide any calendrical algorithm, and was therefore more clearly separated from the other substantial genre of computistical texts, the formulary. *De ratione computandi* thus appears to represent a stage in the history of computistical textbooks more mature than its predecessors, which is further accentuated by the fact that it is the only one of the three Irish computistical textbooks that does not include the vernacular. The theory here is that when scientific ideas were first formulated in the second-language, Latin, the author would revert to the mother-tongue, Irish, when it was felt that the complexity of the concept demanded it. Once such ideas became common knowledge, more precise ways of explanation could be formulated in Latin, which is mirrored by *De ratione computandi*'s level of sophistication. If the discussion outlined in chapter 100 can be interpreted as a dating clause, as a recent study by Tobit Loevenich suggests, this would confirm a date in the eighth century, probably the 720s⁶. It may have been used as early as 727 for the composition of one of the earliest Frankish computistical texts, *Dial. Burg*⁷.

De ratione computandi survives in two principal recensions, a long version in **B** - Bruxelles, KBR 5413-22, ff. 77v-107v (north-eastern France, s. IX^{3/3}) and a short one in **V** - Città del Vaticano, Biblioteca Apostolica Vaticana, Reg. lat. 1260, ff. 87r-99v⁸ (Saint-Denis or Loire?, s. IX^{2/4} or IX *med.*?). A breakdown of content may serve to illustrate the differences.

c.	content	B	V	R
1	names for sun	x	x	
2	the art of numbers	x	x	
3	names of numbers	x	partially	
4	definitions of these names	x		
5	inventor of numbers	x	x	
6	modes of numbers	x		
7	base number 1	x		
8	cardinal numbers	x		

6. T. Loevenich, *The Date of De ratione computandi*, «Peritia», 34 (2023), forthcoming.

7. *Dial. Burg.* is ed. by A. Borst, *Schriften zur Komputistik im Frankenreich von 721 bis 818*, 3 vols., Hannover 2006, pp. 348-74; for its potential dependency on *De ratione computandi*, cfr. also Warntjes, *Munich Computus* cit., pp. CC-CCI.

8. Note that chapter 1 and the beginning of chapter 2 appear a second time in this manuscript, on f. 124v.

c.	content	B	V	R
9	etymologies of cardinal numbers	x		
10	Greek numbers	x	x	
11	Latin numerical symbols	x		
12	numbers replicate nature	x		
13	14 divisions of time	x	x	
14	increasing order of divisions of time	x	x	
15	relations of units of time	x	x	
16	atom	x		x
17	moment	x		x
18	minute	x		x
19	point	x		x
20	hour	x	x	x
21	quarter-day	x		x
22	day	x		x
23	artificial (12-hour) day	x		x
24	natural (24-hour) day	x	partially	x
25	night	x	partially	x
26	beginnings of the day	x		x
27	week	x		x
28	history of months	x	partially	x
29	names of months	x	x	x
30	marker-days of Roman months	x	x	x
31	classification of months	x	x	
32	names of days of month	x	x	
33	days of months with same weekday	x	x	
34	weekday relation between Calends	x		
35	continued	x		
36	Egyptian months	x		
37	beginnings of Egyptian months	x		
38	length of Egyptian months	x		
39	<i>bissextus</i> in Egyptian calendar	x		
40	Macedonian months	x		
41	Hebrew months	x	x	
42	seasons	x	partially	x
43	length of seasons	x		x

c.	content	B	V	R
44	year	x		x
45	beginnings of year	x	x	x
46	13 types of year	x	x	
47	solstices and equinoxes	x	x	x
48	relation of these to the seasons	x		x
49	continued	x		x
50	increase and decrease of daylight	x		x
51	5 remaining days if year is divided in 4 equal blocks of 90 days	x		x
52	names of <i>bissexthus</i>	x	partially	x
53	date and definition of <i>bissexthus</i>	x		x
54	zodiac	x	x	
55	definition of <i>bissexthus</i> on the basis of course of sun through zodiac	x	x	x
56	history of <i>bissexthus</i>	x		x
57	date of <i>bissexthus</i> , weekday and lunar implications	x		x
58	relation between <i>bissexthus</i> and creation	x		x
59	<i>aetas</i>	x		x
60	<i>saeculum</i>	x		x
61	<i>mundus</i>	x		
62	transition from solar to lunar theory	x	x	x
63	moon	x	partially	x
64	conjunction	x		x
65	different types of moon rise	x		x
66	the hour of moon rise	x		x
67	the lunar age of creation	x	x	x
68	lunar year and epacts	x	partially	x
69	introduction of the following chapters	x		x
70	definition of <i>luna abortiva</i>	x	x	x
71	lunations of 30 and 29 days	x	x	x
72	sequence of lunations	x	x	x
73	<i>lunae abortivae</i> in 19-year lunar cycle	x	x	x
74	difference between <i>luna abortiva</i> and embolism	x	x	x
75	correlation between lunar and weekday calculation	x		

c.	content	B	V	R
76	types of lunar years	x	x	x
77	definition of embolism	x	x	x
78	placement of embolisms	x	x	x
79	necessity of embolisms in 19-year lunar cycle	x		x
80	sequence of common and embolismic years	x	x	x
81	definition of <i>ogdoas</i> and <i>hendecas</i>	x	x	x
82	list of the 19 years of the <i>cyclus decemnovennis</i> , with specification of the embolisms	x		x
83	Length and beginning and end of lunar years	x		x
84	definition of <i>pascha</i>	x	x	x
85	the first <i>pascha</i> (Exodus)	x		x
86	the historic year of the passion	x		x
87	early development of Easter celebration	x		x
88	date of the passion	x		x
89	chronological data for the historic year of the passion	x		x
90	list of Easter new moons	x		x
91	importance of the spring equinox for Easter	x		x
92	Julian calendar limits for Easter full moon	x	partially	x
93	list of Easter full moons	x	x	x
94	origin of Easter full moon and Julian calendar limits of Easter Sunday	x	partially	x
95	difference between Roman and Greek practice	x		x
96	lunar limits of Easter Sunday	x		x
97	continued	x		x
98	history of these lunar limits	x		x
99	lunar limits for Easter Sunday and Lent in different traditions	x		
100	correlation between Easter and Lent lunar limits, and impact of <i>bisextus</i> and <i>saltus</i>	x	x	
101	<i>luna 2</i> that regulates Lent	x	x	
102	<i>bisextus</i> and <i>saltus</i> together in Lenten period	x	x	
103	Julian calendar limits for beginning of Lent, and description of Easter table	x	x	

c.	content	B	V	R
104	difference between <i>cyclus lunaris</i> and <i>decem-novenalis</i>	x		
105	<i>saltus</i> in these cycles	x		x
106	history of <i>saltus</i>	x	x	x
107	definition of <i>saltus</i>	x	x	x
108	construction of <i>saltus</i>	x	x	x
109	<i>saltus</i> in nature	x		x
110	names of <i>saltus</i>	x		x
111	continued	x		x
112	placement of <i>saltus</i>	x		x
113	interval of <i>saltus</i>	x		x
114	definition of cycle	x		x
115	history of Easter cycles	x	x	x

As highlighted by bold lines in the table above, this text falls into three principal sections, on numbers, solar theory, and lunar theory including the calculation of Easter and Lent. The consistency in style of the long version suggests that the short version is best understood as an abridgement rather than the long version as an expansion. The breakdown above highlights the method applied in condensing this text: all themes are covered, but reduced to the essentials, while retaining the overall order of content. After a shortened introduction on numbers, the solar theory focusses of the 14 divisions of time only on the crucial units hour, day, month, season, and year, plus the complex calendrical feature of the intercalated day every four years (*bissextus*). In lunar theory, essential explanations were kept but longer discourses about the history of certain phenomena or rules were suppressed.

The relationship between the two recensions is important, especially for the sources used. *De ratione computandi* is the earliest witness for the reception of Boethius' *De institutione arithmetica* in Ireland, who is cited by name only in chapters 11 and 12 exclusive to the long version⁹. Across both recensions, Virgilius Maro Grammaticus features promi-

9. Cfr. especially P. P. Ó Néill, *Boethius in Early Ireland: Five Centuries of Study in the Sciences*, in *Music and the Stars: Mathematics in Medieval Ireland*, curr. M. Kelly - C. Doherty, Dublin 2013, pp. 21-43, at pp. 24-5.

nently¹⁰, as does Augustine. More standard for Irish computistical textbooks were Isidore's *Etymologiae* and *De natura rerum*, the Macrobius excerpt *Disputatio Chori et Praetextati*, and late antique Easter letters and prologues.

Was the short version created at the time of the full text in Ireland, or later on the Continent? The omission of chapter 112 may be indicative in this context. The abridger was very careful not to lose data essential for understanding the technicalities of the lunar calendar. By omitting chapter 112, the important discussion about the placement of the *saltus lunae* was lost. The date favoured is 21 March, a specifically Irish custom. On the ninth-century Continent, the *saltus* was placed in November or in July. Therefore, this outdated practice may have fallen victim to ninth-century editing.

There is no agreement on where to best place the codex containing the short version. The mid-ninth-century Loire valley, probably Fleury, seems to be the preferred location¹¹. From a computistical perspective, this is certainly an attractive proposition, as mid-ninth-century Fleury generally shows strong interest in *De ratione computandi* and its concepts. Jacopo Bisagni has drawn attention to the fact that a four-book encyclopedia in Città del Vaticano, Biblioteca Apostolica Vaticana, Reg. lat. 123 contains 79 of the 115 chapters of *De ratione computandi* (R in the table above), mostly with some slight variations to the edited text¹². The codex was written by the monk Oliva in Ripoll in 1055/6¹³. This does not mean, however, that he is the author of this encyclopedia. The first book deals with solar theory (73 chapters), the second with lunar theory (47 chapters), the third with cosmology (63 chapters), and the fourth with astronomy (122 chapters). Except for eleventh-century additions between books 2 and 3 by Oliva, and twelfth-century additions between books 3 and 4, the encyclopedia is congruent with

10. Cfr. especially D. Ó Cróinín, *The Date, Provenance and Earliest Use of the Works of Virgilius Maro Grammaticus*, in *Tradition und Wertung: Festschrift Franz Brunhölzl*, cur. G. Bernt - F. Rädle - G. Silagi, Sigmaringen 1989, pp. 13-22, repr. in Ó Cróinín, *Early Irish History* cit., pp. 191-200, at pp. 198-9.

11. B. Munk Olsen, *L'étude des auteurs classiques latins aux XI^e et XII^e siècles*, 4 vols., Paris 1985-9, vol. I, p. 534; Walsh - Ó Cróinín, *Cummian's Letter* cit., pp. 107-9; M. Mostert, *The Library of Fleury: A Provisional List of Manuscripts*, Nijmegen 1989, p. 279.

12. J. Bisagni, *The Newly-Discovered Irish and Breton Computistica in Città del Vaticano*, BAV, MS Reg. Lat. 123, «Peritia», 82 (1982), pp. 13-34.

13. For this codex, cfr. the summary by Borst, *Schriften* cit., pp. 300-2 with further literature; for the date, cfr. especially H. M. Bannister, *Signs in Kalendarial Tables*, in *Mélanges offerts à M. Émile Chatelain*, Paris 1910, pp. 141-9, at p. 147.

mid-ninth-century Fleury intellectual culture that evidently informed eleventh-century Ripoll¹⁴, and some argumenta suggest original composition in the 850s/860s¹⁵. The first two books of this encyclopedia are clearly inspired by Irish computistical thought, as the transition from one to the other borrows the phrase from *De ratione computandi* on the end of solar and the beginning of lunar theory. It is in these two books that not only the chapters from *De ratione computandi* can be found, but also other Irish material. It will be the task of future scholarship to extract all Irish elements of these two books and then decide whether these could have constituted a single and cohesive computistical textbook that was included more or less in full into this encyclopedia, or if the Fleury compilers drew on a variety of Irish sources.

Certainly the evidence of *De ratione computandi* chapters 70 and 73 in the Fleury encyclopedia seem to point rather to the latter scenario. These two chapters deal with the concept of the *luna abortiva*, which is arguably the most original contribution of *De ratione computandi*. Chapter 70 defines this concept as lunations falling between the Calends of two successive months (e.g. in the ninth year of the *cyclus decemnovenalis*, one lunation ends with *luna* 29 on 1 July; the following lunation, of 30 days, falls on 2 to 31 July, i.e. between 1 July and 1 August, the Calends of July and August). Chapter 73 then discusses every single occurrence of this phenomenon in the 19-year lunar cycle. The concept was implicit in earlier Irish computistical texts, but the author of *De ratione computandi* was the first to spell it out in detail, and its originality made it attractive right into the eleventh century¹⁶. These two chapters were copied in abridged form in the early ninth-century *Liber Commonei* (817 × 835), a wide-ranging excerpt collection of a Welsh scholar later incorporated in St Dunstan's classbook (Oxford, Bodleian Library, Auct. F.4.32, ff. 19r-36v, here f. 22r-v). In mid-ninth-century Fleury, the two chapters were considerably expanded in order to reconcile them with contemporary computistics¹⁷. This updated version was then included in the Fleury encyclopedia, demonstrating that the encyclopedist did not necessarily copy the Irish computistica directly and slavishly from one early source.

14. Cfr. Bisagni, *The Newly-Discovered Irish and Breton Computistica* cit., especially pp. 22-4 with further literature.

15. The *argumenta* ff. 39v and 45v refer to AD 852 + indiction.

16. Cfr. Warntjes, *Munich Computus* cit., pp. LXX-LXXI, CLXV-CLXVI, 248-9; I. Warntjes, *Irische Komputistik zwischen Isidor von Sevilla und Beda Venerabilis: Ursprung, karolingische Rezeption und Forschungsperspektiven*, «Viator Multilingual», 42 (2011), pp. 1-32, at pp. 16-7.

17. This mid-ninth-century Fleury version of *De ratione computandi* chapters 70 and 73 is ed. in Warntjes, *Munich Computus* cit., pp. 333-6.

The mid-ninth-century Loire valley, and especially Fleury, was therefore the centre of the reception of *De ratione computandi* on the Continent. It incorporated more than half of this textbook into a new scientific encyclopedia, updated its most original concept, and may have produced the short version of the text. For the codicological unit containing the short version, however, Bernhard Bischoff hinted at Saint-Denis of the second quarter of the ninth century as a possible place of composition¹⁸. This would place the short-version not only into the ambit of the Irish computist Dúngal, who corresponded with Charlemagne about solar eclipses¹⁹, but also of the most celebrated computistical manuscript in Irish script, the famous Carlsruhe Bede (Karlsruhe, Badische Landesbibliothek, Aug. Perg. CLXVII; Soissons/Laon?, 830s/840s) with its substantial Old Irish glosses²⁰.

Interestingly, excerpts from *De ratione computandi* also feature prominently among the glosses to Bede's scientific works in the late ninth-century Breton manuscript Angers, Médiathèque Toussaint 477 (461)²¹, which may suggest that the long recension of this Irish computistical textbook reached the Loire valley through Breton circles (in Brittany or northern France)²². From the Loire valley, not only *De ratione computandi*'s concepts were transmitted further south and east, but also the first section of the text, the first 12 chapters on numbers (plus the following chapter 13 on the division of time), which were copied in a tenth- to twelfth-century codex from Auch (Paris, Bibliothèque nationale de France, n.a. lat. 456, ff. 189v-190v)²³.

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18. B. Bischoff, *Katalog der festländischen Handschriften des neunten Jahrhunderts (mit Ausnahme der wisigothischen)*, 4 vols., Wiesbaden 1998-2017, vol. III, p. 439.

19. For Dúngal, cfr. now the 2020 Toronto dissertation by J. Warnes, *Dúngal: A Study of his Life and Works*; for his *Epistola ad Carolum*, cfr. B. Eastwood, *The Astronomy of Macrobius in Carolingian Europe: Dungal's Letter of 811 to Charles the Great*, «Early Medieval Europe», 3 (1994), pp. 117-34.

20. Especially the origin of this manuscript is heavily debated; the computistica of this codex make composition on the Continent much more likely than Ireland; cfr. A. Borst, *Der karolingische Reichskalender*, 3 vols., Hannover 1998, pp. 228-30; Borst, *Schriften* cit., pp. 234-5 with further literature. Cfr. now also J. Bisagni, *From Atoms to the Cosmos: The Irish Tradition of the Divisions of Time in the Early Middle Ages*, Cambridge 2020, pp. 35-47.

21. Walsh - Ó Cróinín, *Cummian's Letter* cit., pp. 115-8, 163, 175, 177; Warntjes, *Munich Computus* cit., pp. CV-CVI, n. 307.

22. Cfr. D. Barbet-Massin, *Le manuscrit 477 (461) d'Angers: étude codicologique et textuelle*, «Britannia Monastica», 19 (2017), pp. 15-43.

23. A. Borst, *Das Buch der Naturgeschichte: Plinius und seine Leser im Zeitalter des Pergaments*, Heidelberg 1995, p. 97, n. 48.

