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Proclus and the Sciences

Approaches to Understanding the Divine

Edited by
Anna Motta and *Daniela P. Taormina*

SISMEL
EDIZIONI DEL GALLUZZO ~ 2026

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Abbreviations and Critical Editions

Below are abbreviations not found in the aforementioned tools and critical editions used by the authors of the chapters. As a rule, ancient texts are abbreviated according to the guidelines of the *Oxford Classical Dictionary* or *Thesaurus Linguae Graecae*.

Alex. Aphr. *De an*. = Alexander Aphrodisiensis, *De anima; Quaestiones*, *Supplementum Aristotelicum* II.1−2, edidit I. Bruns, Berlin, Reimer, 1887–1892.

Alex. Aphr. *In APr*. = Alexander Aphrodisiensis, *In Aristotelis* Analytica priora *Commentaria*, edidit M. Wallies, Berlin, Reimer, 1883.

Amm. *In APr.* = Ammonius, *In Aristotelis* Analytica, edidit M. Wallies, Berlin, Reimer, 1899.

Arist. = Aristoteles, *Opera*, edidit I. Bekker, 4 vols., Berlin, Reimer, 1831–1870; *Mete*. = *Meteorologicorum libri quattuor*, ed. F. H. Fobes, Cambridge, Harvard University Press, 1919.

Dam. *In Phd.* = Damascius, *The Greek Commentaries on Plato's* Phaedo, Vol. II, edited by L. G. Westerink, Amsterdam / Oxford / New York, North Holland Publishing Co., 1977.

Dam. *In Phlb*. = Damascius, *Commentaire sur le* Philèbe *de Platon*, edition par G. Van Riel, Paris, Les Belles Lettres, 2008.

El. *In Porph*. = Elias, *In Porphyrii Isagogen Commentaria*, edidit A. Busse, Berlin, Reimer, 1900.

Eucl. *El.* = Euclides, *Opera omnia*, ediderunt J. L. Heiberg – H. Menge, Leipzig, Teubner, 1896, vol. IV.

Herm. *In Phdr.* = Hermias Alexandrinus, *In Platonis* Phaedrum *Scholia*, ediderunt C. M. Lucarini – C. Moreschini, Berlin, De Gruyter, 2012 (= P. Couvreur, *Hermiae Alexandrini in Platonis* Phaedrum *scholia*, Hildesheim, 1971).

Hom. *Od.* = Homerus, *Odyssea*, edidit P. von der Mühl, Basel, Helbing & Lichtenbahn, 1962.

Iambl. *Comm. Math*. = Iamblichus, *De communi mathematica scientia*, ediderunt N. Festa – U. Klein, Leipzig, Teubner, 1975.

Iambl. *De anima* = Iamblichus, *De Anima*, edited by J. M. Dillon and J. F. Finamore, Leiden / Boston / Köln, Brill, 2002.

Iambl. fr. = Iamblichus, *In Platonis dialogos commentariorum fragmenta*, edidit J. M. Dillon, Leiden, Brill, 1973.

Iambl. *In Nic*. = Iamblichus, *In Nicomachi* Arithmeticam Introductionem, edidit E. Pistelli – U. Klein, Leipzig, Teubner, 1975.

Iambl. *Myst*. = Jamblique, *Reponse a Porphyre (De Mysteriis)*, edition par H. D. Saffrey – A.-Ph. Segonds, Paris, Les Belles Lettres, 2013.

Iambl. *VP* = *Jamblique*, Vie de Pythagore, edition et tradution par L. Brisson – A.-Ph. Segonds, Paris, Les Belles Lettres, 1996.

[Iambl.] *Theol. Ar.* = [Iamblichus] *Theologoumena Arithmeticae*, edidit V. De Falco, Leipzig, Teubner, 1922.

Marin. *In dat*. = Euclides, *Data cum commentario Marini et scholiis antiquis*, in *Euclidis opera omnia*, edidit. H. Menge, Leipzig, Teubner, 1896, pp. 233–57, vol.VI.

Marin. *V. Pr*. = Marinus, *Vita Procli*, edition par H.-D. Saffrey – A. Ph. Segonds, *Marinus. Proclus ou sur le Bonheur*, Paris, Les Belles Lettres, 2001.

Nicom. *Ar*. = Nicomachus Gerasenus, *Introductionis arithmeticae libri II*, edidit. R. G. Hoche, Leipzig, Teubner, 1866.

Olymp. *In Grg.* = Olympiodorus, *In Platonis* Gorgiam *Commentaria*, edidit L. G. Westerink, Berlin, De Gruyter, 1970.

Olymp. *In Mete.* = Olympiodorus, *In Aristotelis* Meteora *Commentaria*, edidit G. Stüve, Berlin, Reimer, 1900.

Or. Chald. = *Oracles chaldaïques*, edition par É. des Places, Paris, Les Belles Lettres, 1971 (Paris, Les Belles Lettres, 2010) (= W. Kroll, *De Oraculis Chaldaicis*, Breslau, Koebner, 1894).

Orph. fr. = *Orphicorum Fragmenta*, edidit A. Bernabé, in *Poetae Epici Graeci. Testimonia et fragmenta*, pars II, fasc. 1–2, München / Leipzig, De Gruyter, 2004–2005; *Orphicorum fragmenta*, edidit O. Kern, Berlin, Weidman, 1922, pp. 20–7.

Phlp. *In APo*= M. Wallies (ed.), Joannes Philoponus, *In Aristotelis* Analytica posteriora, edidit M. Wallies, Berlin, Reimer, 1909.

Pl. = Plato, *Opera*, edidit J. Burnet, 5 vols., Oxford, Clarendon Press, 1900–1907; *Res Publica* edidit S. R. Slings, Oxford, Oxford University Press, 2003.

Plot. *Enn*. = Plotinus, *Opera*, 3 vols., ediderunt P. Henry – H.-R. Schwyzer, Oxford, Clarendon Press, 1964–1982.

Porph. *In Harm*. *Ptol*. = Porphyrius, *Kommentar zur* Harmonielehre *des Ptolemaios*, edidit I. Düring, Göteborg, Elander, 1932 (Hildesheim / New York, George Olms Verlag, 1978).

Porph. *Plot*. = Porphyrius, *Vita Plotini*, in *Plotini Opera*, vol. 1, ediderunt P. Henry – H.-R. Schwyzer, Oxford, Clarendon Press, 1964, pp. 1–38.

Porph. *Sent*. = Porphyrius, *Sententiae ad intelligibilia ducentes*, edidit E. Lamberz, Leipzig, Teubner, 1975.

Porph. *V. Pyth*. = Porphyre, *Vie de Pythagore, Lettre à Marcella*, edition par É. des Places, Paris, Les Belles Lettres, 1982.

Procl. *De dec*. *dub*. = Proclus, *De decem dubitationibus circa providentiam*, in *Procli Tria Opuscula*, edidit H. Boese, Berlin, De Gruyter, 1960, pp. 3–108.

Procl. *De prov*. = Proclus, *De providentia et fato et eo quod in nobis*, in *Procli Tria Opuscula*, edidit H. Boese, Berlin, De Gruyter, 1960, pp. 109–71.

Procl. *Hyp.* = Proclus, *Hypotyposis astronomicarum positionum*, edidit C. Manitius, Leipzig, Teubner, 1909.

Procl. *El. Theol*. = Proclus, *The Elements of Theology*, edited by E. R. Dodds, Oxford, Clarendon Press, 1963.

Procl. *In Alc*. = Proclus, *In Platonis* Alcibiadem, edidit L. G. Westerink, Amsterdam, North-Holland Publishing Co., 1954.

Procl. *In Cra*. = Proclus, *In Platonis* Cratylum *Commentaria*, edidit G. Pasquali, Leipzig, Teubner, 1908.

Procl. *In Eucl.* = Proclus, *In Primum Euclidis* Elementorum *Commentarii*, edidit G. Friedlein, Leipzig, Teubner, 1873.

Procl. *In Or. Chald.* = Proclus, *In Oracula Chaldaica* (*excerpta Michaelis Pselli*), in *Oracles Chaldaïques*, edition par É. des Places, Paris, Les Belles Lettres, 1971, pp. 202-12.

Procl. *In Prm*. = Proclus, *In Platonis* Parmenidem *Commentaria*, edidit C. Steel, 3 vols., Oxford, Oxford University Press, 2007–2009; C. Luna – A.-Ph. Segonds, *Commentaire sur le* Parménide *de Platon*, 7 vols., Paris, Les Belles Lettres, 2007–2021 (V. Cousin, *Procli philosophi Platonici opera inedita.* Part III: *Commentarius in* Parmenidem, Paris, A. Durand, 1864).

Procl. *In R.* = Proclus, *In Platonis* Rem publicam *Commentarii*, edidit W. Kroll, 2 vols., Leipzig, Teubner, 1899–1901.

Procl. *In Ti.* = Proclus, *In Platonis* Timaeum *Commentaria*, edidit G. Van Riel, 5 vols., Oxford, Oxford University Press, 2022 (= E. Diehl, *Procli Diadochi in Platonis* Timaeum *Commentaria*, 3 vols., Leipzig, Teubner, 1903–1906).

Procl. *Theol. Plat.* = Proclus, *Théologie platonicienne*, edition par H. D. Saffrey – L. G. Westerink, 6 vols., Paris, Les Belles Lettres, 1968–1997.

Ptol. *Syntax. Math.* = Claudius Ptolemaeus, *Syntaxis Mathematica*, edidit J. L. Heiberg, Leipzig, Teubner, 1898–1903.

Simpl. *In Cat*. = Simplicius, *In Aristotelis* Categorias *Commentarium*, edidit K. Kalbfleisch, Berlin, Reimer, 1907.

Simpl. *In Ph*. = Simplicius, *In Aristotelis* Physica *Commentaria*, edidit H. Diels, Berlin, Reimer, 1882−1895.

Stob. *Anth*. = Ioannes Stobaeus, Anthologium, ediderunt K. Wachsmuth – O. Hense, Berlin, Weidmann, 1884–1912.

Syr. *In Hermog*. = Syrianus, *In Hermogenem Commentaria*, 2 vols., edidit H. Rabe, Leipzig, Teubner, 1892–1893.

Syr. *In Metaph*. = Syrianus, *In Aristotelis* Metaphysica *Commentaria*, edidit W. Kroll, Berlin, Reimer, 1902.

Them. *In APo.* = Themistius, Analyticorum posteriorum *Paraphrasis*, edidit M. Wallies, Berlin, Reimer, 1900.

Theo Sm. *Exp.* = Theon Smyrnaeus, *Expositio rerum mathematicarum ad legendum Platonem utilium*, edidit E. Hiller, Leipzig, Teubner, 1878.

PROCLUS AND THE SCIENCES

APPROACHES TO UNDERSTANDING THE DIVINE

Anna Motta - Daniela P. Taormina

PROCLUS AND THE SCIENCES

A New Volume on Proclus

This volume explores Proclus' thought, emphasizing the interconnected role of mathematics, theology, and other scientific disciplines within his philosophical system. One of the main objectives is to analyse how, in the Platonic tradition, Proclus (410-485 AD) conceived mathematics not only as a science in its own right, but also as a fundamental tool for understanding the divine and metaphysical reality. An innovative aspect of the volume, which unifies the various essays, is the analysis of Proclus' exegetical practices, highlighting the significance of mathematics and the sciences more broadly in relation to the philosophical developments of his thought – and consequently, not only to the emergence of theological science, i.e. Platonic theology approached in a scientific way[1]. This volume stems from a 2024 workshop organized by Anna Motta and Daniela P. Taormina, entitled *Proclus: Science and Theology*, held in Naples (Italy), with support from the 2022 PRIN *Ancient Science, Ancient Philosophy Funds*. The publication continues a broader research project funded by the Italian Science Foundation 2021 and coordinated by Lorenzo Perilli. This project investigates how the status and methodology of science since the classical age have been consistently influenced by philosophical reflection, focusing on the scientific foundation of theological discourse in the imperial age.

Consistent with the workshop's objectives, this volume aims to identify the mathematical aspects of Proclean thought and to define the argumentative and scientific procedures allowing Proclus to provide theology with a universal, necessary, and demonstrable foundation[2]. Recent analyses of the Classical model of science certainly provide valuable insights into the philosophical

We would like to thank the editor of the *Micrologus Library* and the entire staff of the SISMEL editorial office for their support throughout the publication process. Finally, we are grateful to the anonymous referees for their helpful comments, to Sergio Knipe for proofreading, and to Francesco Ferro, Marco Guerrieri, Giulia Rescigno, and Giovanni Trovato for their assistance in the final stage of revising the proofs.

[1]. Greek terms are transliterated without the use of symbols indicating tonic accent or long and short vowels. Critical editions of ancient texts are not indicated within the chapters unless there are ambiguities that cannot be resolved by the bibliography at the end of each essay. Abbreviations of ancient texts are indicated below.
See Chenu 1957 and Saffrey 1996.

[2]. See Cleary 2013.

significance of a number of important issues related to Neoplatonism and mathematics; however, they remain insufficient for explaining how Proclus sought to establish a scientific foundation for theology. Not all the contributions from the invited speakers at the conference have found their way into this volume, which also incorporates the results of the discussions that followed their presentations.

Regarding the collection of studies, the chapters offer significant insights into theology, mathematics, and other sciences across various Proclean works. This approach highlights diverse methodologies and a range of research interests, thereby providing perspectives on the philosophy of mathematics and the philosophical significance of the analytic-synthetic distinction, the axiomatic method, the hierarchical ordering of the sciences, and the status of logic as a science.

This multilingual volume (English, French, Italian) aims to serve as a key resource for scholars and students of Ancient Philosophy and Classics, offering insights into Proclean Platonism and the broader philosophical, scientific, and theological themes it addresses. These advancements in the field of Neoplatonism and Proclean studies offer a valuable foundation for understanding Proclus and his philosophy, particularly his integration of mathematics, science, and theology. Consistent with this research program, the volume investigates how Proclus a) employs mathematics while excluding or reformulating other criteria of scientific analysis; b) formalizes the divine *taxeis* on the basis of a demonstrative structure grounded in the articulation of hypotheses and conclusions established in the *Parmenides*; and c) uses dialectical concepts and arguments as a code to 'translate' elements from other theological traditions – originally expressed through different forms – into a clear scientific system.

Thematic Points in Three Sections

The volume is divided into three main thematic sections (plus a philological appendix), each containing essays that explore specific aspects of Proclus' thought. This structure highlights the importance of engaging with mathematics, science, and theology, offering a renewed and advanced perspective on Proclean Platonism, its commentary practices, and the philosophical landscape in which his works were produced. Many contributors were among the scholars present at the 2024 workshop. As editors, we curated a diverse group of contributors, taking into account their international backgrounds, career stages, and gender representation to ensure the volume's overall balance.

The section “Mathematics: Branches, Ontological Status and Method” investigates the ontological status of mathematical objects according to Proclus, as well as the special faculties of the soul involved in their comprehension. It also explores the use of the theological notion of harmony, which aims to reveal the essence, intelligible relations, and causality of the soul by taking its harmonic structure as a starting point. The chapters show how the demonstrative procedures employed by Proclus follow the structure of mathematical arguments, from which he infers general laws applicable at every level of the ontological hierarchy. The essays further address Proclus’ polemic against the idea that mathematics can be derived from empirical experience, highlighting his purely rational and metaphysical approach.

The section “Mathematics: A Bridge to Theology” examines the correspondences between mathematical objects and the gods in Proclus’ texts. The essays aim to demonstrate how this bridge is constructed and how it surpasses previous positions in the history of Platonic thought. Proclus formalizes the divine *taxeis* through a demonstrative procedure based on axioms and demonstrations. Drawing on Aristotle and Euclid, he equates axioms with common notions, treating them as the premises of science: they require neither proof nor evidence and must be accepted as the starting points of demonstrations, as they are immediate, evident, knowable in themselves, innate, and eternal. The chapters of this section provide a comprehensive presentation of the late Platonic metaphysical/theological system, showing the basis on which Proclus establishes an ‘agreement’ among the various theologies. They also examine the ways in which Proclus employs mathematical and analogical reasoning to explain complex theological concepts. Moreover, the section highlights a notable tension: mathematics may serve more as a confirmation of theological doctrines than as an independent pathway to divine insight.

The section “Other Sciences and the Divine” explores the philosophy of nature as a science and how Proclus establishes a solid foundation by identifying, in the *Timaeus*, a connection between rational and empirical approaches to the natural world. Furthermore, through geometry, Proclus secures a universal, necessary, and demonstrable foundation for all philosophy of nature, while broadening the perspective to encompass other scientific disciplines and their relationship with the divine. This section demonstrates that Proclus’ thought extends beyond mathematics to includes sciences such as astronomy and even “potamology” (the study of rivers), integrating them into a unified cosmological and theological framework. This approach emphasizes how, for Proclus, the entire universe, in all its manifestations, reflects the divine order.

Aligned with the volume’s overarching theme is the philological appendix. This essay examines the transmission and reception of Proclus’ *Commentary on the* Alcibiades and *Commentary on the* Timaeus in Byzantium, comparing the Middle Byzantine period with the

Palaiologan era. The chapter enriches the volume with a philological note highlighting the importance of Neapolitan witnesses for reconstructing Proclean texts. Indeed, we maintain that the dialogue among scholars of ancient philosophy cannot be separated from engagement with philologists and philology. A study of Proclus necessarily reflects the view that philology is not a separate practice from philosophy but a true habitus.

All in all, this volume aims to reveal the intricate connections between mathematics, theology and science in Proclus' thought, offering a detailed analysis of how he employed mathematics to explore and understand metaphysical reality and the divine realm.

A Comprehensive Chapter-by-Chapter Summary

The first section *Mathematics: Branches, Ontological Status and Method* comprises five chapters. It opens with a chapter by Dominic J. O'Meara (*Syrianus and Proclus on Why True Mathematics Cannot Be Empirically Based*). The author presents arguments from Syrianus (*Commentary on Aristotle's* Metaphysics) and Proclus (*Commentary on Euclid's* Elements) that refute Aristotelian explanations for the empirical origin of mathematical concepts and theories (Aristotelian abstractionism). These arguments establish the ontological status of mathematical objects as discursive projections of a priori concepts innate in the human soul (*ousiodeis logoi*), which derive from a transcendent demiurgic intellect. This ontology shows how mathematical objects can be precise, eternal, and universally valid, while still allowing for the creative activity of the mathematician in developing mathematical science. It further illustrates how mathematics functions as a science that foreshadows metaphysics and serves as a paradigm for physics.

The second chapter, by Anne Sheppard (*The Place of Music in Proclus' Conception of Mathematical Exegesis*), is devoted to harmonics. In *Republic* VII 530d–531c, harmonics, together with its 'sister science' astronomy, is described as one of the branches of mathematics that the Guardians must master as they progress toward the highest level of their education: the study of dialectic. In 530d, Plato attributes the connection between harmonics and astronomy to the Pythagoreans. This link reappears in *Republic* X 616b–617d, and the connections between mathematics, music, and astronomy are also evident in the *Timaeus*, particularly in 35b–36b and 38b–39e. The details of Platonic/Pythagorean harmonic theory are developed more fully in later authors such as Nicomachus of Gerasa and Iamblichus. Both *Republic* VII and the *Timaeus* provide the background for Proclus' brief remarks on *mousike* as a branch of mathematics in *Commentary on Euclid* 35.28–36.1 while *Republic* VII also informs his discussion of the four types of *mousike* in

Commentary on the Republic I 56.20–60.13. In such a chapter, the author maintains that Proclus' use of the notion of mathematical exegesis in his *Commentary on the Timaeus* is grounded in the Platonic conception of the mathematical sciences. This framework also underpins his distinction between mathematical, physical, and philosophical modes of interpretation, developed in the context of his analysis of the musical scale in *Timaeus* 35b. Proclus' conception of mathematical exegesis encompasses music as well as astronomy, and he follows the Platonic and Pythagorean tradition of including one type of music within mathematics.

The central question of the third chapter, by Denis Walter (*Incommensurability in Geometrical Entities in Proclus'* In Eucl. *and* In Prm.), is the origin of incommensurability within Proclus' ontology. The chapter begins with an analysis of Proclus' *Commentary on the First Book of Euclid's* Elements. The issue is complex, and scholarly debates have suggested that incommensurability might not be a subject of scientific inquiry at all. Yet how does it arise at a specific ontological level, and how does it relate to a particular mode of cognition? This chapter first shows where geometrical entities are situated – namely, between intelligible and the sensible realm – and then demonstrates why sensible matter, by itself, cannot unqualifiedly account for incommensurability. To this end, incommensurability is contrasted with imprecision, the defining characteristic of the sensible world. However, intelligible matter alone cannot unqualifiedly account for incommensurability either, since some objects projected onto intelligible matter (i.e. the soul) do not exhibit this feature. Instead, the chapter shows that incommensurability is an intrinsic property of specific geometrical entities, belonging to them as other identifiable characteristics do. Drawing on Proclus' *Commentary on the* Parmenides, the chapter ultimately argues for the plausibility of accepting a form of incommensurability.

The fourth chapter, by Álvaro José Campillo Bo (*The Unfolded Syntax of Intellect: Proclus on the Metaphysical Foundations of Elementary Arrangements*), This chapter analyses Proclus' epistemology and ontology of the discursive sciences, focusing on the noetic foundations of dialectic and mathematics. It examines demonstrative reasoning and elementary dispositions as iconic projections of the intelligible structure of Intellect. Drawing on Proclus' philosophy of language, it argues that discursive syntax reproduces the relational articulation of noetic activity. The chapter shows how the dynamism of Intellect is fully expressed in dialectical operations and only derivatively in mathematical demonstration, thereby grounding a hierarchical dependence of mathematics upon dialectic and their coordination within "common mathematics". This dependence is clarified through an analysis of the continuous logical syntax shared by both sciences. As a case study, the chapter demonstrates how propositions 1–4 of the *Elementatio theologica* are analytically derivable from Euclidean arithmetical definitions. It concludes that elementary logical syntaxes in

the contemplative sciences function not merely as methodological devices but as discursive enactments of the noetic order.

In the fifth chapter (*Can the Will Ground Science?*), Antonio Vargas challenges interpretations that view the *Elements of Theology* as a work of dialectic based on self-evident axioms. He argues that Proclus denies any ultimate grounding of knowledge in axiomatic propositions. For the philosopher, the science that grounds all knowledge on an unhypothetical first principle is Parmenidean dialectic, and his *more geometrico* metaphysics, the *Elements of Theology*, cannot be considered an example of this science. Although some have speculated that this might be the case because the *Elements* lacks an initial list of axioms and definitions, Vargas argues that this absence must be viewed within the broader context of the *Elements'* many departures from the geometric ideal, arguing that the lack of such a list is ultimately insignificant. According to Vargas, the *Elements* makes no use of what Proclus identifies in the *Parmenides Commentary* as our access to an unhypothetical first principle – namely, the so-called 'one of the soul', which, based on *De Providentia*, can in certain respects be identified with the will. He complements this negative argument regarding the *Elements* with a positive argument about how the will, in Proclus, can serve as a ground for knowledge. In Proclus, science is not grounded in axioms but in the absolute unity of the soul's will. The *Elements of Theology*, while pedagogical, is not dialectical, since it never begins from the unhypothetical first principle. Instead, true dialectic involves the interplay of will, *nous*, and discursive reasoning (*dianoia*).

The second section, *Mathematics: A bridge to Theology*, includes a further five chapters. The first chapter of this section and the sixth of the volume, by Carlos Steel (*Proclus on the usefulness of mathematics for theology*), begins with the introduction to Proclus' *Commentary on Euclid*, where he declares that "mathematics prepares intellectual insight for theology", as it "reveals in its reasonings the truth concerning the gods". Throughout many other passages, Proclus emphasizes that mathematical science should serve as a "stepping-stone" to elevate the soul to higher realms, rather than being valued solely for the technical discussion of its demonstrations. These programmatic declarations have often led to a neglect of the real import of Proclus' geometrical arguments, making its whole purpose 'theological'. Steel extensively discusses Gouling's study, which argues that "geometry, practiced in a certain way, constituted for Proclus the core of the higher theurgy". He contends, however, that one should not be misled by Proclus' Neoplatonic statements about mathematics as a bridge to theology. The theological speculations in the text are digressions, illustrating how it is possible from our discursive perspective – the proper domain of mathematics – to gain intellectual insight into the divine causes. In particular, Steel examines the difference between the technical 'textual' analysis of Euclid's demonstrations and the

speculative approach (*theoria*) to reality, a distinction present throughout all of Proclus' commentaries.

In the seventh chapter (*La mathématique générale chez Proclus*), Alain Lernould aims to demonstrate that, above the particular mathematical sciences of arithmetic, geometry, music, and the spherical, Proclus – in the first Prologue of *In Euclidem* – introduces the concept of *general* mathematics, which is distinct from mathematics *in general*. General mathematics constitutes a category that transcends the particular mathematical sciences and goes beyond mathematics in general. This chapter shows how general mathematics constitutes the highest stage within the discursive sciences in the ascent toward the Intellect and the divine. Lernould demonstrates that it functions as an intermediate, unifying science between the particular mathematical disciplines and the higher philosophical sciences (dialectic, intellect), without reducing it to mere logical abstraction. He further underscores the ontological priority of "common theorems" (universals *ante rem*) over their instantiations in arithmetic or geometry, aligning them with productive principles rather than post hoc generalizations.

In the eight chapter (*Relations that Are Given and Established. Proclus' Use of Analogical Reasoning in Theology*), Marije Martjin suggests that one of the main methods of human reasoning is analogical reasoning, in which conclusions about a less-known "target domain" are drawn from its similarity to a better known "source domain". For Proclus, analogy constitutes a key component of scientific reasoning across all sciences and, *a fortiori*, in theology. This chapter examines the role of analogical reasoning in Proclus' understanding of theology, addressing his definition of analogy, its relation to mathematical analogy or proportionality, the various kinds of analogy he distinguishes within the context of theology, and some problematic aspects. A central challenge for analogical reasoning is justifying the assumed similarity between the "source domain" and the "target domain". Martijn discusses how, according to Proclus, the use of different kinds of analogy in theological can be justified. Unsurprisingly, the primary justification is ontological, relying on emanation and reversion, which themselves establish analogical relations to some degree. In addition, in theology, the *via analogiae* merges with the *via eminentiae*, yet is ultimately surpassed by the *via negativa*. Proclus' conceptual toolkit also includes certain problematic forms of analogy, notably symbolic and literary analogies, for which the ontological justification is less apparent. In these cases, he appears to introduce a separate category: a relation of similarity established by an initiate, such as a philosopher or priest. Even here, however, a real – but remote – ontological analogy underlies the relation.

In the ninth chapter (*L'ἀριθμός divino nel sistema metafisico di Proclo*), Miriam Cutino focuses on Proclus's ordered theology, with particular attention to the concept of 'number', the

notion of ‘multiplicity’, and the relationship between the monad and the series in the Neoplatonic ontological hierarchy. The ontological realm presented by Proclus in the *Platonic Theology* is a ‘metaphysical system full of unity’ – i.e. one in which the presence of unity holds fundamental existential significance as the condition of being for each entity. The chapter aims to study the articulation of the *taxis*, namely the universal ontological hierarchy, which itself comprises three different forms of order. It demonstrates how the structure of Proclus’ divine *diakosmoi* is organized according to a mathematical analogy, whereby the Pythagorean divine numbers correspond to the various levels of the ontological reality. The analysis then focuses on the ontological significance of the term *diakosmos* as number and the paradigmatic cause of unification, and of the term *arithmos* as the first ‘otherness’ introduced by the intelligible-intellectual world, relating it to its usage in the works of Iamblichus and in Sirianus’ *Commentary on Aristotle’s* Metaphysics.

In the tenth chapter (*Les structures numériques des ordres divins dans la* Théologie Platonicienne *de Proclus. De la triade à l’hebdomade et à la dodécade*), Philippe Hoffmann explores how Proclus links ‘science’ and ‘theology’ through arithmological structures in his *Platonic Theology*, a comprehensive synthesis of Plato’s doctrines. He focuses on the divine order of the intellective gods, organized according to a hebdomadic (sevenfold) structure, following the triadic patterns of the intelligible and intelligible-intellective orders. The intellective hebdomad consists of two triads plus a separative monad: the triad of the “fathers” (Cronos, Rhea, Zeus), paired with an anonymous “immaculate” triad, and completed by the monad, yielding 6 + 1 = 7. This arrangement relies on earlier triads to maintain mimetic continuity within the processional hierarchy. The structure is based on the topography of Plato’s *Phaedrus* myth (246e–248a), which distinguishes the supercelestial realm, Heaven (supersensible in Proclus’ view), from the subcelestial vault – Heaven occupying the middle position in all three divine orders. Hoffmann concludes that this hebdomad fits into a broader arithmetic progression (1–2–3–7–12), culminating in the hypercosmic gods and intriguingly echoing the start of Hofstadter’s series in modern mathematics.

The section *Other Sciences and the Divine* comprises the eleventh and twelfth chapters. In the eleventh chapter (*Proclus’ Potamology: The Role of Rivers in Eschatology*), Gerd Van Riel examines how Proclus integrates cosmology, psychology, and soteriology into a coherent vision. Van Riel explores Proclus’ interpretation of the role of rivers in Plato’s eschatological myths: Lèthè, Styx, Acheron, Pyriphlegethon, and Cocytus (as set out in *Phaedo* and *Republic* X). Unsatisfied with merely physical or allegorical readings of these rivers, the later Neoplatonists combined a literal interpretation (on the cosmological level) with a moral message concerning the

destiny of the soul. The role they attribute to the rivers ("potamology") thus provides valuable insights into the interaction between body and soul, and the relationship between cosmology, theology, physics, and ethics.

The final chapter (*Proclus' "Theological" Spherical Cosmic Whole and His Scientific Innovative Astronomy*) offers a dense and innovative contribution to Proclus' astronomical and ontological thought, with a theoretical ambition that unites ancient philosophy, modern physics, and Platonic theology into a coherent and interdisciplinary discourse. Emilie Kutash argues that, although Proclus admired Ptolemy's *Almagest*, he rejected Ptolemy's mechanical attempts to reconcile the apparent irregular movements of the planets through epicycles and eccentrics. Guided by Platonic theology, Proclus envisioned the cosmos as a finite, spherical, and thus curved space. He held that all apparent irregularities in planetary motions stem from "self-moving" motion, which inevitably reverts toward circular paths and ultimately toward the One, the spherical center symbolizing eternal unity. In contrast to models based on concentric spheres or epicycles, Proclus conceived of planets as self-moving bodies with independent three-dimensional trajectories (in longitude, latitude, and depth), producing spiral motions as a mean between purely circular and straight lines. This theory explained astronomical anomalies through higher principles and was later cited by Copernicus for its account of combined motions. Proclus also questioned the Euclidean parallel postulate, anticipating the notion of non-linear space. Conceptually, his model bears intriguing parallels to Einstein's curved space and space-time continuum, developed many centuries later.

In the philological Appendix (*Difformità storico-tradizionali in Proclo, Commento all'*Alcibiade *I e Commento al* Timeo *tra il periodo mediobizantino e l'età dei Paleologi: Giorgio Pachimere (Napoli, BN, III.E.17, In Alc.), Giovanni Catrario (Napoli, BN, III.D.28, In Ti.) e lo ὕπατος τῶν φιλοσόφων Michele Psello (Patmos Eileton 897*), Mariella Menchelli provides a philosophical reading of the reception of Proclus' studies on Plato's dialogues in Byzantium, highlighting their close connection with the transmission of Neoplatonic commentaries, some of which were included in the ninth-century "philosophical collection". During the first Palaeologan Age, the same – or other – texts were carefully copied and studied by learned philosophical circles in Constantinople and, for example, in Thessalonike, as manuscript evidence appears to confirm. The collections in Naples provide valuable examples in which it is now possible, in some cases, to identify the hands and scribes at work, offering insights into some important primary witnesses and their role in shaping the reading of the text. Proclus' *Commentary on Plato's* First Alcibiades, Neap. BN III.E.17, is in part also a *codex unicus*. Proclus' *Commentary on Plato's* Timaeus, Neap. BN III.D.28, belongs to the φ-branch of the textual tradition, along with ms. Chigi R VIII 58 and

Patmos Eileton 897, a paper scroll from the eleventh century – the age of Psellus – featuring a rich apparatus of mathematical scholia. BNN III.E.17 was largely copied by Georgius Pachymeres, assisted by two copyists, one of whom one Pantelis Golitsis has identified as Nikephoros (e. g. Laches, initium). Neap. III.D.28 was written by John Katrarios. In the same branch, Patmos Eileton 897 chart 7.747 x 250 mm, ff. 19, *in volumine*, is an exceptional paper scroll, discovered by A. Kominis in the 1980s: its reading of the text is also significant for the exegetical apparatus it contains. The aim of this chapter is to explore these three witnesses and their implications for Proclus' text.

Some Concluding (and Bibliographical) Remarks

In recent years, critical editions and studies on Proclus have proliferated, enabling him to be regarded as the thinker who identified religion with philosophy by theologizing Platonism and rationalizing religion: the fusion of religion and philosophy gave rise to the science of theology – that is, philosophical speculation on the divine. The relationship between philosophy and theology in ancient thought remains a central subject of study, as shown in the recent volume *L'Esprit critique dans l'Antiquité*, II. *La Naissance de la théologie comme science*[3], the foundational studies of André-Jean Festugière and Pierre Hadot remain essential for reconstructing the theoretical background, methods, and significance of this Proclean endeavor[4]. These two scholars emphasized the link between Neoplatonic thought and religious traditions. Their studies should be considered alongside important texts tracing discussions on the origins of theology as a science, within a framework that underscores the centrality of Plato's *Parmenides*[5]. Without merely quoting the Iamblichean statement, Proclus affirms the prominent position of the *Parmenides*, where dialectic is identified with 'theological science' (*episteme theologike*)[6]. Theological truth is expressed in various dialectically intertwined forms. When assessed in light of the overall Proclean system and the Athenian project of a theological science – as Jean Pépin and Stephen Gersh have shown – the modes of discourse on the divine reveal not merely a classificatory ambition, but the presence of a genuine philosophical method.[7] The editorial work on the *Platonic Theology* by Henri Dominique Saffrey and Leendert G. Westerink, the volume *Proclus et la* Théologie Platonicienne edited by

[3]. Boulnois *et al*. 2024.
[4]. See, e.g., Festugière 1966-1968; Festugière 1969; Festugière 1970; Festugière 1971; Hadot 1987.
[5]. Saffrey 1992.
[6]. On the importance of Parmenides in Neoplatonism, see the still fundamental Dodds 1928; Saffrey 1984.
[7]. Pépin 2000; Gersh 2000.

Alain-Philippe Segonds, Carlos Steel *et al*. (Louvain / Paris 2000)[8], and the studies of Philippe Hoffmann all suggest that, in addition to the *Commentary on the* Parmenides, we must take into consideration at least one other Proclean work, namely the *Platonic Theology*[9]. In this text, Proclus provides a scientific description of the regulated procession of realities emanating from the One. He expounds the theology attributed to Plato, as disseminated in the dialogues, and orders it according to the teaching of the second part of the *Parmenides*, which is interpreted as a discourse on theological realities that, once harmonized with ancient doctrines, constitute the 'scientific' identity of the gods of traditional polytheism.

The scientific character of this exposition is evident in the second *kephalaion* of the *Platonic Theology*: "How many propositions must be proved before the number of divine classes can be found? Systematic exposition of these propositions"[10]. This is a "demonstration" of four fundamental propositions that precedes the "discovery" of the plurality of divine classes, the existence of which is then scientifically demonstrated on the basis of these propositions. What emerges is the pursuit of demonstrative perfection, intended to firmly establish the dogma of Greek polytheism – namely, the existence of a plurality of gods. These propositions, which Proclus calls "axioms", correspond to common notions already possessed by the soul and evident to those who – unlike Christians – are not constrained by "atheistic" prejudices[11]. This volume seeks to make a substantial contribution to the scholarly debate on the significance of Proclus' use of the sciences, and of mathematics in particular. Our aim is to show how Proclus, while adhering to a classical model of science, adapts certain features of this ancient paradigm to epistemology, cosmology, ontology, and Neoplatonic metaphysics.

[8]. Saffrey – Westerink 1968-1997; Gavray – Hoffmann 2024.
[9]. Hoffmann 2019.
[10]. Procl. *Theol. Plat*. III 1.10–11.
[11]. Hoffmann 2012.

Bibliography

O. Boulnois – Ph. Hoffmann – C. Lafleur – J.-M. Narbonne (eds.), *L'Esprit critique dans l'Antiquité*, II. *La Naissance de la théologie comme science*, Paris, Les Belles Lettres, 2024.

M.-D. Chenu, *La Théologie comme science au XIII^e^ siècle*, Paris, Vrin, 1957.

J. J. Cleary, *Proclus' Philosophy of Mathematics*, in J. M. Dillon – B. O'Byrne – F. O'Rourke (eds.), *Studies on Plato, Aristotle and Proclus*, Leiden and Boston, Brill, 2013, pp. 201–20.

E. D. Dodds, *The* Parmenides *of Plato and the Origin of the Neoplatonic One*, in «Classical Quarterly» 22.3-4 (1928), pp. 129–42.

A.-J. Festugière (ed.), *Proclus : Commentaire sur le* Timée, 5 vol., Paris, Vrin, 1966–1968.

A.-J. Festugière (ed.), *L'ordre de lecture des dialogues de Platon aux V^e^/VI^e^ siècles*, in «Museum Helveticum», 26.4 (1969), pp. 281–96.

A.-J. Festugière (ed.), *Proclus : Commentaire sur la* République, 3 vols., Paris, Vrin, 1970.

A.-J. Festugière (ed.), *Études de philosophie grecque*, Paris, Vrin, 1971.

M.-A. Gavray – Ph. Hoffmann, *Modes d'exposition de la théologie dans le néoplatonisme athénien (Proclus, Damascius, Simplicius)*, in O. Boulnois – Ph. Hoffmann – C. Lafleur – J.-M. Narbonne (eds.), *L'Esprit critique dans l'Antiquité*, II. *La Naissance de la théologie comme science*, Paris, Les Belles Lettres, 2024, pp. 219–58.

G. Stephen, *Proclus' Theological Methods. The Programme of* Theol. Plat. *I. 4*, in A.-Ph. Segonds – C. Steel (eds.), *Proclus et la* Théologie platonicienne, Leuven / Paris, Leuven University Press / Les Belles Lettres, 2000, pp. 15–27.

P. Hadot, *Théologie, exégèse, révélation, écriture dans la philosophie grecque*, in M. Tardieu (ed.), *Les Régles de l'interprétation*, Paris, Éditions du Cerf, 1987, pp. 13–24.

Ph. Hoffmann, *Un grief anti-chrétien chez Proclus : l'ignorance en théologie*, in A. Perrot (ed.), *Les Chrétiens et l'hellénisme. Identités religieuses et culture grecque dans l'Antiquité tardive*, Paris, Éditions Rue d'Ulm, 2012, pp. 161–97.

Ph. Hoffmann, *L'herméneutique de Proclus et la constitution d'un système théologique dans la* Théologie Platonicienne, in «Comptes rendus des séances de l'Académie des Inscriptions et Belles-Lettres» 3 (2019), pp. 961–1005.

J. Pépin, *Les modes de l'enseignement théologique dans la* Théologie platonicienne, in A.-Ph. Segonds – C. Steel (eds.), *Proclus et la* Théologie platonicienne, Leuven / Paris, Leuven University Press / Les Belles Lettres, 2000, pp. 1–14.

H. D. Saffrey, *La* Théologie Platonicienne *de Proclus, fruit de l'exégèse du* Parmenide, in «Revue de théologie et de philosophie» 116 (1984), pp. 1–12.

H. D. Saffrey, *Accorder entre elles les traditions théologiques : une caractéristique du Néoplatonisme Athénien*, in E. P. Bos – P. A. Meijer (eds.), *On Proclus and his Influence*, Leiden, Brill, 1992, pp. 35–50.

H. D. Saffrey, *Les débuts de la théologie comme science (III^e^–VI^e^ siècle)*, in «Revue des sciences philosophiques et théologiques» 80 (1996), pp. 201–20.

H. D. Saffrey – Leendert Gerrit Westerink (eds.), *Proclus.* Théologie Platonicienne, 6 vols., Paris, Les Belles Lettres, 1968–1997.

ABSTRACT

Anna Motta – Daniela Taormina, *Proclus and the Sciences*

This chapter introduces this new collective volume on Proclus that investigates how late antique Platonism integrates mathematics, theology, and the sciences into a single philosophical project. The book asks how Proclus (410–485 CE) could treat mathematics not only as an autonomous discipline but also as a methodological paradigm for giving theology a universal, necessary, and demonstrable foundation. The editors stress that prevailing accounts of ancient scientific models, though useful, do not fully explain Proclus' attempt to "scientize" theology; therefore, the volume foregrounds his exegetical practices and his formal use of hypotheses and conclusions, especially in relation to Plato's *Parmenides*.

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I

MATHEMATICS: BRANCHES, ONTOLOGICAL STATUS AND METHOD

Dominic J. O'Meara

SYRIANUS AND PROCLUS ON WHY TRUE MATHEMATICS CANNOT BE EMPIRICALLY BASED

Introduction

The ontological status of the objects studied in the mathematical sciences, as conceived by the Platonist philosophers of Late Antiquity, has been examined by modern scholars in a number of publications and is familiar to specialists in the field[1]. In this chapter I would like to examine in more detail one aspect of the subject which, I believe, deserves further attention, namely the arguments given by these Platonists, in particular by Syrianus and by Proclus in Athens in the 5th century AD, in support of their ontology of mathematical objects, arguments designed to refute the position that such objects can be derived empirically, from sense perception. These arguments seek to show that 'true mathematics'[2] cannot be empirically based and lead to an important and novel conception of what mathematics is as a science and how it relates to other sciences, in particular to the science of transcendent divine being, i.e., metaphysics or "theology" (as they called this science).

Before looking at the arguments presented by Syrianus and Proclus, it may be of use to recall briefly the ontological landscape in which these philosophers situated mathematical objects[3]. This landscape, if we take it in its simplest form as we find it in Plotinus, has at its summit the ultimate source of reality, the One, transcending all beings and thought. From the One derives a succession of lower levels of existence: a divine Intellect, which is one with transcendent Forms, and, deriving from Intellect, Soul, which produces the material world.

In Syrianus and Proclus this landscape acquires many more differentiating features, many more intermediate levels. Following Aristotle's reports on Plato, the Platonists of Late Antiquity held that mathematicals occupy an intermediary place in this ontological landscape, between Intellect/the Forms, on the one hand, and the material world, on the other. But in Plato, soul also

Earlier versions of this essay, presented in Amsterdam and Oslo, brought comments and questions, as did this version, presented in Naples, for which I am grateful.

[1] See for example Mueller's Forword to Morrow 1970; Cleary 2000; O'Meara 2016.
[2] I indicate what is meant by this expression below, in section 4.
[3] I will refer to these objects in the following pages as "mathematicals", term that can include both mathematical objects such as numbers or geometrical figures and the relations which obtain between them.

seems to occupy this intermediate position. What then is the relation between mathematicals and soul? It is this question which will be pursued in the pages which follow.

In Syrianus and Proclus, and before them in Iamblichus, the theory of the ontological status of mathematicals replaces a position which derives these objects from material bodies, the objects of sense perception. The claim that mathematicals are taken from sensible objects goes back in particular to Aristotle, in his criticism of Platonism in *Metaphysics* Books M and N, more precisely in Book M, chapters 2 and 3. The great commentator on Aristotle of the second century AD, Alexander of Aphrodisias, commented on these books of the *Metaphysics*, a commentary which is now lost but which was probably used by Syrianus in his commentary on *Metaphysics* MN[4], where he sought to refute Aristotle's claim in arguments which we can also find, along with other such arguments, in the work of Syrianus' student and successor, Proclus, in particular in Proclus' *Commentary on Euclid's* Elements[5]. In the following pages I propose examining the arguments presented by Syrianus and Proclus refuting the derivation of mathematicals from the objects of sense perception, arguments supporting thereby their Platonist theory of the ontological status of mathematicals and their relation to soul − a theory which I will describe in section 5 of this chapter.

Mathematical Objects Are Not to Be Found in the Sensible World

Syrianus points out that many of the objects studied in the mathematical sciences are not to be found among the objects of sense perception (*In Metaph*. 91.25−29):

> We have never seen any polygons of such size and quality as geometry concerns itself with by way of plane figures, nor such a variety of many-sided figures as are examined in stereometry, or divisions of angles or sides and surfaces[6].

Aristotle would agree with this statement, as indeed would Alexander[7]. However, both would maintain that mathematicals correspond to aspects to be found in the objects of sense perception, that are derived from sense perception by certain procedures in the thinking subject.

[4] See below, note 37.
[5] Longo 2010 has shown how Proclus, in his commentary on Euclid, takes up arguments given by Syrianus in his commentary on the *Metaphysics*. Helmig 2010 discusses these arguments and especially similar arguments given by Proclus in his commentary on Plato's *Parmenides.*
[6] See Syr. *In Metaph*. 91.25−29, commenting on Arist. *Metaph.* M 2: μηδὲ ἑωρακέναι ἡμᾶς ποτε μήτε πολύγωνα τοσαῦτα καὶ τοιαῦτα, οἷα ἐπίπεδα ἡ γεωμετρία θεωρεῖ, μήτε πολύπλευρα οὕτω ποικίλα <οἷα> ἡ στερεομετρία ἐπισκέπτεται, ἢ γωνιῶν διαιρέσεις ἢ πλευρῶν ἢ ἐμβαδῶν; tr. Dillon and O'Meara 2006. See also Syr. *In Metaph*. 176.26−31; Procl. *In Eucl.* 12.19−23; 49.12−17; 139.26−140.2.
[7] Arist. *Metaph*. M 2.1077b15; Alex. Aphr. *In APr*. 4.7−9.

Alexander speaks of the "help" provided by the thinking subject in deriving mathematicals from sense perception[8]. But what is this "help", and what are these procedures? In his *Commentary on Euclid*, Proclus distinguishes between two procedures, "abstraction" (ἀφαίρεσις) and (what I will translate as) "accumulation" (ἄθροισις)[9]. I will take each of these procedures and the refutation of them by Syrianus and Proclus insofar as they might constitute possible means for the derivation of mathematicals from sense perception.

Abstraction

According to Aristotle and his commentator Alexander, abstraction, as a procedure, allows for the derivation of mathematicals from the objects of sense perception[10]. Against this possibility Proclus gives two arguments[11], of which this is the first (*In Eucl*. 12.26−13.3):

> How then will we give stable being to unchanging [mathematical] *logoi*, if they are derived from things that are ever changing from one state to another? For they agree [axiom 1] that all that which results from changing beings receives from them a changeable being[12].

If mathematicals (*logoi*) are unchanging, and the objects of sense perception, i.e., bodies, are in constant flux, then the former cannot derive from the latter, on the principle (which I label as "axiom 1"[13]) that all that comes from changing beings is itself changing. The principle is one to which "they" – the Aristotelians, I take it − subscribe[14], who consequently cannot derive mathematicals from the objects of sense perception without violating a principle to which they themselves adhere.

Proclus then offers a second argument intended to refute Aristotelian abstractionism (*In Eucl*. 13.3−5):

[8] See Alex. Aphr. *De An.* 87.28.
[9] Procl. *In Eucl.* 12.4−7.
[10] Arist. *APo.* 74a33−b1; *Metaph*. Z 11.1036a34−b3; b22−23; on Alexander see below, note 37 and Mueller 2016 who presents Alexander as championing "abstractionism". On Alexander's abstractionism see also De Libera 1999 ch. 1.
[11] On these issues see Helmig 2012, p. 212; Iambl. *Comm. Math.* 4.9−10 (a cue inspiring Proclus' approach?).
[12] Tr. Morrow 1970, modified as in the following quotations. πῶς οὖν τοῖς ἀκινήτοις λόγοις ἐκ τῶν κινουμένων καὶ ἄλλοτε ἄλλως ἐχόντων αὐτὴν τὴν μόνιμον οὐσίαν δώσομεν; πᾶν γὰρ τὸ ἀπὸ κινουμένων οὐσιῶν ὑφιστάμενον καὶ ὕπαρξιν μεταβλητὴν ἔχειν ὁμολογεῖται παρ' αὐτῶν. See 49.17−24.
[13] The principle is formulated as a universal affirmative proposition: "All that which results (…)", reminiscent of the universal affirmative propositions of Proclus' *El. Theol.*
[14] See Procl. *El. Theol*. 76 (part of which corresponds word-for-word to axiom 1). In his edition Dodds 1963, p. 345 refers us to Arist. *Ph.* Θ 6.259b32−260a10. See Proclus' commentary on Plato's *Parmenides* III 786.17−18.

> And how can we give precision to forms which are precise and irrefutable from things which are not precise? For [axiom 2] all that is cause of unadulterated knowledge must itself be such to a greater degree[15].

The second argument has the same structure as the first. A principle is stated, which I label as "axiom 2"[16], which the Aristotelians are taken to uphold[17]. Mathematicals are precise and certain, but deriving their precision and certainty from things which are imprecise, impure, and unstable is to violate this principle.

In reading Proclus' arguments, we might make the following observations. Proclus assumes that mathematicals are of an unchanging, eternal nature: they are precise and clear in their nature, whereas bodies are forever changing, confused, imprecise. He also seems to consider *solely* the derivation of mathematicals from the objects of sense perception, without taking into account the role which the thinking subject, i.e., the mathematician, might have in generating mathematicals from sense perception, even if the concept of abstraction would seem to include such a role. However, by ruling out derivation solely from sensible objects, the arguments turn the focus on the role played by the thinking subject, the rational soul, as we see in the conclusion Proclus gives to his two arguments against abstractionism (*In Eucl.* 13.6−8):

> Soul must therefore be what generates mathematical forms and *logoi*[18].

The role played by soul in generating mathematical science will therefore require examination: can it be reduced to a procedure of abstraction? Or can soul be the only source of mathematicals, as Proclus appears to claim? I will come back to these questions below.

Accumulation

A second procedure is mentioned by Proclus as put forward by those who seek to derive mathematicals from sense perception: "accumulation". Who might be the proponents of "accumulation"? What precisely is this procedure and how does it relate to the procedure of abstraction? Is it an alternative procedure, as Proclus' argumentative strategy seems to imply, or is

[15] πῶς δὲ τοῖς ἀκριβέσικαὶ ἀνελέγκτοις εἴδεσιν ἀπὸ τῶν μὴ ἀκριβῶν τὴν ἀκρίβειαν προσθήσομεν; πᾶν γὰρ τὸ τῆς ἀκιβδήλου γνώσεως αἴτιον μειζόνως ἐστὶν αὐτὸ τοιοῦτον. See also 139.26−140.4. See Syr. *In Metaph*. 91.25; 95.29−32; 96.31−33; see Pl. *Phd*. 75a−b.
[16] Again, this principle is formulated as a universal affirmative proposition: "All that is cause (…)". See Procl. *El. Theol*. 7.
[17] See Alex. Aphr. *De An.* 88.17−89.8.
[18] ψυχὴν ἄρα τὴν γεννητικὴν ὑποθετέον τῶν μαθηματικῶν εἰδῶν τε καὶ λόγων.

it in some way complementary to abstraction? I would like to take up these questions before coming to the analysis of Proclus' arguments against the procedure of accumulation.

It might be thought that "accumulation" refers to a Stoic theory[19], which would mean that in attacking abstraction Proclus criticizes Aristotelianism, and in refuting accumulation he is rejecting a Stoic theory of mathematicals. However, I will argue here that accumulation is a procedure which can also be found in Aristotelianism[20], in particular in Alexander of Aphrodisias, and that Proclus' arguments against the procedure of accumulation, like his arguments against abstraction, are addressed to an Aristotelian opponent, most likely Alexander.

According to a report in Simplicius, the verb "to accumulate" was used by Alexander (*In Ph.* 1075.7−8):

> This, then, is the way Alexander explained [it], wishing the universal and the cognition of the universal to be assembled from the particulars, and he said that it was stated that 'it somehow knows the particulars through the universal' [Arist. *Ph.* H 3.247b5−7] as a sign that the cognition of the universal is accumulated (ἀθροίζεσθαι) by means of the particulars, since the universal knowledge is of each of the things [falling] under the universal and it has been accumulated (ἠθροίσθη) from these[21].

If this is good evidence that the term "accumulation" would have been used by Alexander in a relevant context, we can ask if the procedure which the term designates was used by him to refer to the generation of mathematicals. This procedure, as Proclus describes it, seems to involve the accumulation from sensible particulars of what is common to them, which includes mathematicals[22]. A comparable procedure can be found in the extant works of Alexander, when he speaks of intellect putting together (σύνθεσις) similar particulars, grasping the universal through the similarity in sensible particulars. Intellect sees (συνιδών) what is common in particulars, grasping this common form, separating form from matter[23]. If this synthetic (or synoptic) procedure corresponds to what Proclus refers to as "accumulation", then I think that we can say that the procedures of abstraction and of accumulation, as Alexander seems to have seen them, were not alternatives, as Proclus' critique might lead one to think, but were linked: to accumulate perceptions of sensible particulars, seeing what is similar in them, what is common to them, allows abstraction of the form common to them, i.e., the universal.

[19] See *SVF* II 841 (Chrysippus, as reported in Galen), Sext. Emp. *Pyr.* III 188; see Helmig 2012, p. 31 (a very useful collection of references to the use of the term "accumulation") and 280.
[20] See, for example, Sext. Emp. *Math.* VII 224.
[21] Tr. Hagen 1994, slightly modified.
[22] *In Eucl.* 12.6−7; 14.1; 15.17−18. The relation between universals and mathematicals cannot be discussed here. In speaking of what is common to sensible particulars, Proclus seems to associate them, as had Alexander of Aphrodisias (*De An.* 90.2−11). Proclus also speaks of the "accumulation", i.e., the combination of theorems as premises in order to constitute demonstrations (71.13−17); see Alex. Aphr. *In APr.* 17.24−25; Ammon. *In APr.* 26.4−6.
[23] See *De an.* 83.11−15; 85.18−20; *Quaest.* 79.16−18; Helmig 2012, p. 161.

To confirm that Proclus is indeed attacking an *Aristotelian* procedure of accumulation, and not, for example, a Stoic theory, we can observe that this procedure is mentioned by Late Antique commentators on Aristotle's *Posterior Analytics* when they deal with the famous chapter on induction (II 19). Both Themistius, in his paraphrase of this chapter, and the author of a commentary on it attributed to Philoponus[24], discuss the topic of the "principles" (ἀρχαί) of demonstration. Themistius speaks in this connection of how experience (ἐμπειρία) is "accumulated" (ἀθροίζεται) from many perceptions and from memories by the soul's "collecting" (συλλέγουσαν) what is similar in sensible particulars, the universal[25]. 'Philoponus' also speaks of the accumulation (he uses the verbs συναθροίζειν and ἐπισυναθοίζειν) of perceptions and memories leading to knowledge of the universal[26]. Now it is precisely with regard to the topic of the principles of demonstration that Proclus' two arguments against the procedure of accumulation are formulated, seeking to show that to derive such principles through accumulation is to violate rules for demonstration laid down in the *Prior Analytics*[27]. Here then is the first of Proclus' two arguments (*In Eucl.* 13.27−14.15):

> If we accumulate mathematical *logoi* from below, from sensibles, must we not say that demonstrations which are constituted from sensibles are superior to those constituted from ever more universal and simpler forms? For we say that everywhere the causes for demonstrations should be appropriate to the hunt for what is sought[28]. If therefore particulars are causes of the universal and sensibles of the objects of discursive thought, how can the term[29] in a demonstration refer to the what is universal rather than to what is particular[30] and the being of objects of discursive thought be declared more related to demonstrations than the being of sensibles? For the man, they say,[31] who demonstrates that the isosceles triangle has the sum of its angles equal to two right angles and that the same is true of the equilateral and the scalene triangles does not properly understand these propositions; rather it is he who demonstrates about any triangle, without qualification, that knows in the strict sense of the term[32].

[24] On the question of the attribution to Philoponus, see most recently Lagnerini 2023.
[25] Them. *In APo.* 63.19−64.2; see *In Ph.* 206.15.
[26] *In APo.* 435.16−23. Philoponus' distinguishes between accumulating perceptions and (furthermore) collecting (ἐπισυνάγεται) the universal from these.
[27] The connection of Proclus' argument with the *An. post.* is noted by Helmig 2010, pp. 51−2.
[28] See Arist. *APo.* 71b19−33.
[29] ὅρος, the subject or predicate in a premise (Arist. *APr.* 24b16).
[30] *APo.* 85b23−27.
[31] *APo.* 73b28−74a3; 74a25−26.
[32] εἰ κάτωθεν καὶ ἀπὸ τῶν αἰσθητῶν ἀθροίζομεν τοὺς τῶν μαθημάτων λόγους, πῶς οὐκ ἀνάγκη τὰς ἀποδείξεις ἀμείνους λέγειν, ὅσαι ἀπὸ τῶν αἰσθητῶν συνίστανται, καὶ οὐ τὰς ἀπὸ τῶν καθολικωτέρων ἀεὶ καὶ ἁπλουστέρων εἰδῶν; τὰ γὰρ αἴτια πανταχοῦ ταῖς ἀποδείξεσιν οἰκεῖα πρὸς τὴν τοῦ ζητομένου θήραν εἶναι φαμέν. εἰ οὖν τὰ μερικὰ τῶν καθόλου καὶ τὰ αἰσθητὰ τῶν διανοητῶν αἴτια, τίς μηχανὴ τὸν ὅρον τῆς ἀποδείξεως ἐπὶ τὰ καθόλου μᾶλλον ἀναφέρειν ἀντὶ τῶν μερικῶν καὶ τῶν διανοητῶν τὴν οὐσίαν πρὸ τῶν αἰσθητῶν ταῖς ἀποδείξεσιν συγγενεστέραν ἀποφαίνειν; οὐδὲ γὰρ εἴ τις,

Sharing with Aristotelians fundamental premises concerning scientific demonstration as laid out in Aristotle's *Posterior Analytics*, Proclus ("we") argues that this is incompatible with the procedure of generation of mathematicals by accumulation. For "they" (the Aristotelians) agree that the universal is prior and superior to particulars and is the principle of scientific demonstration, not the particulars.

The same point is made in Proclus' second argument against accumulation (*In Eucl.* 14.15−20):

> Again [they say] that the universal is better than the particular for demonstration and consequently that demonstrations from what is universal are more demonstrative. But those [things] from which demonstrations are constituted are prior and naturally superior to particulars and the causes of what is demonstrated[33].

Proclus' arguments against "accumulation" can already be found in large part in Syrianus' commentary on Aristotle's *Metaphysics* Book M[34]. Although Syrianus does not mention the term "accumulation" in this context, I think that we can take it that there is a very good chance that he is arguing against a position he found in Alexander's (now no longer extant) commentary on Aristotle's Book M[35], where Proclus would have read the term[36]. At any rate, we are now in a position to explain why in particular the topic of the principles of demonstration comes up in Proclus' argument against accumulation and to see that Aristotelianism is the target of the arguments against both abstraction and accumulation. Attention to Alexander's views also suggests that abstraction and accumulation are not alternative procedures for generating mathematicals, contrary to what seems to be suggested by the structure of Proclus' critique.

As in the case of Proclus' arguments against abstraction, we note that no account is taken of the activity of the thinking subject, i.e., the rational soul, in generating mathematicals; sensible particulars alone are considered as the source of mathematicals. Yet, the rejection of sensible objects as sources of mathematicals allows Proclus to turn now to soul and to its role in constituting mathematicals. Before coming to this subject, I would like to introduce two descriptions of what an empirically based mathematical science might look like, in the eyes of Syrianus and Proclus.

φασὶν, ἀποδείξειεν ὅτι τὸ ἰσοσκελὲς δυεῖν ὀρθαῖς ἴσας ἔχει τὰς γωνίας, καὶ ὅτι τὸ ἰσόπλευρον ὠδὶ καὶ τὸ σκαληνὸν ἐπίσταται κατὰ τρόπον, ἀλλ' ὁ πᾶν τρίγωνον καὶ ἁπλῶς ἀποδείξας ἔχει τὴν ἐπιστήμην καθ' αὑτό.

[33] καὶ πάλιν ὅτι τὸ καθόλου βέλτιον τοῦ κατὰ μέρος πρὸςἀπόδειξιν, καὶ ἐξῆς ὅτι αἱ ἀποδείξεις ἐκ τῶν καθόλου μᾶλλον, ἐξ ὧν δὲ αἱ ἀποδείξεις, ταῦτα πρότερα καὶ τῇ φύσει προηγούμενα τῶν καθ' ἕκαστα καὶ αἴτια τῶν δεικνυμένων. See Arist. *APo*. 71b25−72a7; 86a5−30.

[34] Syr. *In Metaph.* 90.17−91.2; 164.4−12.

[35] Alexander certainly spoke there of abstraction, as we can see in Syrianus' report (*In Metaph*. 96.14−19), as quoted below, note 37.

[36] Syrianus (*In Metaph.* 54.12−13) regarded Alexander's commentary on Aristotle's *Metaphysics* as adequate for use, except of course where Platonism and Pythagoreanism were the object of criticism, which Syrianus undertook to refute.

Syrianus gives us the following description of such a mathematics (*In Metaph*. 96.14−19):

> But how could it [soul] achieve that, if it concerned itself with the products of abstraction? (...) if in the words of Alexander of Aphrodisias, it moulds for itself certain objects of intellection, which are not by their own nature intelligible, and then vainly plays shadow battles (ἐσκιαμάχει) with them[37].

Syrianus is quoting, it appears, Alexander's (lost) commentary on *Metaphysics* M 3, on the topic of abstraction. Syrianus regards a mathematics which produces its objects by abstraction from sensible particulars (which in themselves are not intelligible objects) as a sort of shadow-battle. The reference is to the image of the cave in Plato's *Republic*, according to which prisoners contend with each other in elaborating conceptions and theories on the basis of the shadows that they see moving on the wall of the cave[38]. To construct such a chimerical world from shadows is what it is to construct a mathematical science from the shadows of reality which are sensible particulars.
Such a mathematical science cannot be considered as true, as is suggested by Proclus in another description of an empirically based mathematics (*In Eucl.* 13.13−21):

> But if it [soul] weaves this great immaterial structure and generates such a great knowledge without knowing or having previously known these *logoi*, how can it judge whether the offspring it bears are fertile or wind eggs [Pl. *Tht.* 151e6], whether they are phantoms instead of truth? What criteria could it use for measuring the truth in them? And how could it even produce so great a variety of *logoi*, if they did not have their being in it? For then we would be making their being come about at random, without reference to any standard[39].

If rational soul is entirely dependent on what it experiences of material things through sense perception, what independent criteria could it have for judging of the truth of its mathematical thoughts, what standard? Without such a standard, soul's mathematical constructions would be unverifiable and random, and the result of chance encounters in sense experience. Proclus also refers to the fertility of mathematical thought, namely the great variety of mathematical truths elaborated by the rational soul. Can this soul's creativity be simply reduced to the procedures of abstraction/accumulation? We note that, in Proclus' view, soul's contribution to the constitution of

[37] ὃ περὶ τὰ ἐξ ἀφαιρέσεως διατρίβουσα πότ' ἂν κατεπράξετο (…) εἰ καθάπερ φησὶν ὁ Ἀφροδισιεὺς Ἀλέξανδρος, νοητὰ ἄττα ἑαυτῇ ἀναπλάσασα, οὐκ ὄντα τῇ οἰκείᾳ φύσει νοητά, περὶ αὐτὰ ταῦτα μάτην ἐσκιαμάχει.
[38] Pl. *Resp*. VII 515a−b, 516c−d, 520c−d (σκιαμαχούντων).
[39] εἰ δὲ μὴ ἔχουσα μηδὲ προειληφυῖα τοὺς λόγους τοσοῦτον ὑφαίνει διάκοσμον ἄϋλον καὶ τοσαύτην ἀπογεννᾷ θεωρίαν, πῶς τὰ γεννηθέντα δύναται διακρίνειν, εἴτε γόνιμα τυγχάνει ὄντα εἴτε ἀνεμιαῖα καὶ εἴδωλα ἀντ' ἀληθῶν, ποίοις δὲ κανόσι χρωμένη τὴν ἐν τούτοις ἀλήθειαν παραμετρεῖ; πῶς δὲ καὶ μὴ ἔχουσα τὴν οὐσίαν αὐτῶν ἀπογεννᾷ τοσαύτην ποικιλίαν λόγων; ηὐτοματισμένην γὰρ οὕτω τὴν ὑπόστασιν αὐτῶν ποιήσομεν καὶ πρὸς οὐδένα ὅρον ἀναφερομένην.

mathematical science is far greater and of another nature than what could be produced by the procedures of abstraction or accumulation.

This point is made clear in Proclus' final argument against those who have the soul derive mathematicals from sensibles[40]. To subordinate the soul to sensibles and to make it fabricate (διαπλάττει) images derived from them is to disvalue the soul and to make it inferior to matter which receives forms from nature, forms which the soul would then artificially separate from matter in making images of them.

Soul, Intellect and the Generation of Mathematicals

From Proclus' arguments in his *Commentary on Euclid*, it follows that if mathematicals do not derive from sensible particulars, then their source must be sought elsewhere. In the metaphysical landscape of the Late Antique Platonist, this means, in the first place, in soul and/or in divine Intellect. As regards the soul, Proclus dismisses soul as the sole origin of mathematicals on the grounds that mathematicals are images of the paradigmatic Forms which inspire divine Intellect, as demiurge, as maker of the sensible world. Nor can this Intellect, however, be the sole source of mathematicals in Proclus' view, for this would reduce soul to a merely passive role, receiving mathematicals from Intellect as if it were matter, depriving it of the activity and the dynamic creativity it displays in developing mathematics[41]. From this argument it thus follows that mathematicals must be located in and derive from both soul and Intellect. But how?

The solution to this problem might be briefly summarized as follows[42]. According to the cosmological account presented in Plato's *Timaeus*, divine Intellect, as it is the demiurge of the world, constituted the nature of soul out of mathematical structures (*Ti.* 35a−d). For both Syrianus and Proclus, these mathematical structures constituting the soul, which they call "substantial reasons" (*ousiodeis logoi*)[43], derive from and are images of the paradigmatic Forms which the demiurge uses in ordering the material world. Soul, in developing mathematical science, elaborates, i.e., unfolds ("unrolls") in discursive thought, these "substantial reasons" so as to bring out their implications. Accordingly, for example, the geometer "projects" these reasons in extension, which is provided by the representative faculty (*phantasia*), so as to grasp them more easily and make manifest what they imply. Sensible particulars, imaging the paradigmatic Forms, may serve to

[40] Procl. *In Eucl.* 14.24−15.15.
[41] Procl. *In Eucl.* 15.21−16.4.
[42] See Procl. *In Eucl.* 16.4−18.4.
[43] Procl. *In Eucl.* 17.23−26 (quoted below, note 47); 37.11−26; Syr. *In Metaph.* 91.29−92.5; 95.15−16; 179.8.

remind soul[44] of the innate knowledge (the substantial reasons) which it already possesses as constitutive of its own nature[45]. In mathematical thought, these substantial reasons are explored by the rational soul by means of various methods proper to discursive thought. Geometry, for example, conceptually articulates the substantial reasons in the form of first principles or hypotheses: these are precisely definitions, postulates, and axioms (common notions) of Euclid's *Elements*. By means of the methods of division, definition, demonstration, and analysis, the first principles are elaborated to form a logically rigorous deductive chain of truths and a system of seamless progression of conclusions based on demonstrations, whose conclusions, by the method of analysis, can also lead back in stages to the first principles of the science[46]. Proclus describes this dynamic progression and return of mathematical thought as follows (*In Eucl.* 17.22−18.4):

> So the mathematical *logoi* which fill souls are substantial and self-moving. By projecting them and unfolding them discursive reason brings into existence the mathematical sciences in all their variety; and it will never stop, forever generating and discovering more and more of them as it explicates the indivisible *logoi* within itself. For it contains in advance all of them as being their principle, and by virtue of its infinite power projects manifold theorems from the principles pre-contained in it[47].

Rational soul thus exhibits immense creativity in developing the mathematical sciences and an ability to generate an elaborate system of universal, necessary, and eternal truths which is anchored in the canons of truth represented by the substantial reasons innate in soul, which are themselves rooted in the truth of the transcendent Forms known by divine Intellect. This creativity and the system it generates stand in strong contrast to a contribution of the rational soul which would consist in abstracting or accumulating generalities from sensible particulars[48]. Both the character of rational soul's methods and the objects which it elaborates are of a different order than that which the Aristotelians envisage as the abstracting/accumulating activity of soul and the

[44] Procl. *In Eucl.* 18.17−18; 45.7−15; 140.15.
[45] This theory of the ontological status of mathematicals appears to go back to Iamblichus (*Comm. Math.* 43.15−23; 44.4−9) and has its roots in Plotinus (*Enn.* IV 3 [27]. 30; VI 6 [24]. 16.38−39); see Sheppard 1997; Maggi 2010, pp. 178−84; Kalligas 2015.
[46] Procl. *In Eucl.* 18.10−19.20; 42.15−43.10; 69.13−19.
[47] οὐσιώδεις ἄρα καὶ αὐτοκίνητοι τῶν μαθημάτων εἰσὶν οἱ λόγοι <οἱ> συμπληροῦντες τὰς ψυχάς, οὓς δὴ καὶ προβάλλουσα ἡ διάνοια καὶ ἐξελίττουσα πᾶσαν τὴν ποικιλίαν ὑφίστησι τῶν μαθηματικῶν ἐπιστημῶν, καὶ οὐ μή ποτε παύσηται, γεννῶσα μὲν ἀεὶ καὶ ἀνευρίσκουσα ἄλλα ἐπ' ἄλλοις, τοὺς δὲ ἀμερεῖς αὐτῆς λόγους ἐξαπλοῦσα. πάντα γὰρ προείληφεν ἀρχοειδῶς καὶ κατὰ τὴν ἄπειρον ἑαυτῆς δύναμιν ἐκ τῶν προειλημμένων ἀρχῶν παντοδαπῶν θεωρημάτων ποιεῖται προβολάς.
[48] These generalities are what Late Antique Platonists term "later-generated" (ὑστερογενῆ): as universals post rem they are seen as ontologically inferior to the sensible objects from which they are derived and epistemologically inferior to ante rem Forms such as are known by the substantial reasons innate in soul; see Syr. *In Metaph*. 171.16; Herm. *In Phdr*. 171.9–11 Couvreur = 178.31–179.3 Lucarini – Moreschini; Procl. *In Eucl*. 14.25-15.3. For Aristotelians, the form abstracted from matter is purely intelligible and identical (see Alex. Aphr. *De an.* 85.16-18), whereas for Platonists form is diminished by its presence in matter, and the conceptual form derived (as abstracted) from this enmattered form is even more degraded.

conceptions which it distils from sensibles. Proclus is convinced that his view of mathematical science is what we find in the mathematicians, in particular in Euclid's *Elements*[49].

Mathematics and the Science of the Divine (Theology) and of Nature

The Late Antique Platonist theory of the nature of the mathematical sciences has implications concerning the relations between these sciences and other sciences, in particular metaphysics and physics. As in mathematics the first principles of science (the "substantial reasons") are images of the transcendent paradigmatic Forms thought by divine Intellect, then, by articulating and reaching a better grasp of these first principles, the mathematician comes nearer to reaching ~~a~~ knowledge of transcendent divine beings, that is the domain of metaphysics (theology). Mathematics thus prepares the way for the study of metaphysics, and the mathematical world which it constructs images the transcendent hierarchy of the divine. Similarly, since mathematics has innate knowledge imaging the paradigmatic Forms which inspire the divine maker of the world, it provides access to the rational structures which underlie the organization of the material world while also supplying physics with scientific first principles and methods. These themes, however, are the subject of other investigations[50].

Bibliography

J. J. Cleary, *Proclus' Philosophy of Mathematics*, in G. Bechtle – D. J. O'Meara (eds.), *La Philosophie des mathématiques de l'Antiquité tardive*, Fribourg, Editions Universitaires, 2000, pp. 85−101.

P. Couvreur (ed.), *Hermiae Alexandrini in Platonis* Phaedrum *scholia*, Hildesheim, Olms, 1971.

A. De Libera, *L'Art des généralités. Théories de l'abstraction*, Paris, Aubier, 1999.

H. Diels (ed.), *Simplicii in Aristotelis* Physica *Commentaria*, Berlin, Reimer, 1882–1895.

J. M. Dillon – D. J. O'Meara (eds.), *Syrianus On Aristotle* Metaphysics *13−14*, London, Bloomsbury, 2006.

E. R. Dodds (ed.), *Proclus.* The Elements of Theology, Oxford, Oxford University Press, 1963.

[49] Proclus' views on mathematical science, which were to be of interest to Renaissance mathematicians and philosophers, correspond only in part to modern versions of Platonism in the philosophy of mathematics as described, for example, by Linnebo 2023.

[50] See Procl. *In Eucl.* 19.20−20.6; 62.1−26; O'Meara 1989 chs. 9−10, Martijn 2014 and the present volume.

C. Hagen (ed.), *Simplicius on Aristotle* Physics *7*, London, Bloomsbury, 1994.

C. Helmig, *Proclus' Criticisms of Aristotle's Theory of Abstraction and Concept Formation in* Analytica Posteriora *II 19*, in F. de Haas – M. Leunissen – Marije Martijn (eds.), *Interpreting Aristotle's* Posterior Analytics *in Late Antiquity and Beyond*, Leiden, Brill, 2010, pp. 27−54.

C. Helmig, *Forms and Concepts. Concept Formation in the Platonic Tradition*, Berlin, De Gruyter 2012.

P. Kalligas, *Plotinus on Number as Quantity*, in «Philosophical Inquiry», 39 (2015), pp. 207−16.

A. Lagnerini, *The dual Account of Induction in Philoponus' Commentary on the* Posterior Analytics, in «Philosophie antique», 23 (2023), pp. 117−32.

Ø. Linnebo, *Platonism in the Philosophy of Mathematics*, in *The Stanford Encyclopedia of Philosophy*, 2023, https://plato.stanford.edu/entries/platonism-mathematics/ (Seen July 15 2025).

A. Longo, In Euclidem, *Prologue I, chapitre 6: ce que Proclus doit à son maître Syrianus dans les arguments avancés contre la thèse de l'existence 'postérieure' des objets mathématiques*, in A. Lernould (ed.), *Études sur le Commentaire de Proclus au premier livre des* Éléments *d'Euclide*, Lille, Presses Universitaire de Septentrion, 2010, pp. 45−54.

C. Maggi, *Sinfonia matematica. Aporie e soluzioni in Platone, Aristotele, Plotino, Giamblico*, Napoli, Loffredo, 2010.

M. Martijn, *Proclus' Geometrical Method*, in P. Remes – S. Slaveva-Griffin (eds.), *The Routledge Handbook of Neoplatonism*, London and New York, Routledge, 2014, pp. 145−59.

G. R. Morrow (ed.), *Proclus. A Commentary on the First Book of Euclid's* Elements, Princeton, Princeton University Press, 1970 (1992^2).

I. Mueller, *Aristotle's Doctrine of Abstraction in the Commentators*, in R. Sorabji (ed.), *Aristotle Transformed*, London, Bloomsbury, 2016^2, pp. 501−20.

D. J. O'Meara, *Pythagoras Revived. Mathematics and Philosophy in Late Antiquity*, Oxford, Clarendon Press, 1989.

D. O'Meara, *Mathematics and the Sciences*, in P. d'Hoine – Marije Martijn (eds.), *All from One. A Guide to Proclus*, Oxford, Oxford University Press, 2016, pp. 167−82.

A. Sheppard, *Phantasia and Mathematical Projection in Iamblichus*, in «Syllecta classica», 8 (1997), pp. 113−20.

ABSTRACT

Dominic J. O'Meara, *Syrianus and Proclus on Why True Mathematics Cannot Be Empirically Based*

This chapter presents arguments by Syrianus (in his *Commentary on Aristotle's* Metaphysics) and Proclus (in his *Commentary on Euclid's* Elements) to refute the Aristotelian explanation of the empirical origin of mathematical concepts, known as abstractionism. The authors seek to establish a higher ontological status for mathematical objects. According to Proclus and Syrianus, mathematical objects do not derive from sensible experience but are discursive projections of a priori concepts (*ousiodeis logoi*) innate in the human soul. These concepts, in turn, originate from a transcendent demiurgic intellect. This Neoplatonic ontology demonstrates how mathematical objects maintain qualities of precision, eternity, and universal validity, while simultaneously allowing for the creative activity of the mathematician. Crucially, this perspective positions mathematics as a fundamental science that serves as a paradigmatic model for physics and a crucial foreshadowing of metaphysics, highlighting its vital role in Proclus's overall philosophical system.

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THE PLACE OF MUSIC IN PROCLUS' CONCEPTION OF MATHEMATICAL EXEGESIS

The Platonic Background

In book VII of the *Republic*, in the account of what is sometimes called the "higher education" of the Guardians, Plato treats music as a branch of mathematics. Mathematics plays a key role as an essential preliminary to the highest kind of study, that of dialectic. Socrates describes five branches of mathematics: arithmetic, geometry, solid geometry or stereometry, astronomy and finally, at 530d−531c, harmonics, the mathematics of musical intervals. At 530d Socrates calls harmonics and astronomy "in some sense sister sciences … as the Pythagoreans say"[1]. He claims that both sciences suffer from the same problem, namely that their practitioners are too focused on the physical world – the visible movements of the heavenly bodies in the case of astronomy, audible musical intervals in the case of harmonics – and the proper study of both requires abstraction from physical phenomena so as to think about numbers in themselves. As often, Plato here appears to be picking up on an idea which he found in Pythagorean sources and using it for purposes of his own.

The link between harmonics and astronomy reappears in *Republic* X where the myth of Er contains a description of the rotation of the heavenly bodies around "the spindle of Necessity" (see 616b−617d). Famously, this description includes a reference at 617b to the music of the spheres, another Pythagorean idea. Harmonics also plays an important role in the *Timaeus*, especially in the account of the composition of the world soul at 35b−36b. That is described in terms of a structure which is both mathematical and musical, according to the divisions of a musical scale. The result is the incorporeal circles of the Same and the Different, of which the latter is subdivided into seven concentric circles. Timaeus goes on to describe how the demiurge, having put the world soul together, forms the body of the world in accordance with the same structure, so that seven inner circles carry the visible heavenly bodies of the sun, the moon and five planets (Venus, Mercury,

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[1]. Pl. *Resp.* VII 530d6−8: κινδυνεύει [...] καὶ αὗται ἀλλήλων ἀδελφαί τινες αἱ ἐπιστῆμαι εἶναι, ὡς οἵ τε Πυθαγόρειοί φασι.

Mars, Jupiter and Saturn) while an undivided outer circle carries the fixed stars (see 38b−39e). So it turns out that once again mathematics, music and astronomy all go closely together[2].

Republic VII 530d−531c was picked up and developed by later writers on Platonic/Pythagorean harmonic theory such as Nicomachus of Gerasa and Iamblichus, often in connection with the music of the spheres alluded to in *Republic* X[3]. My concern here is with Proclus and with some of the ways in which he draws on the passage on harmonics in *Republic* VII, combining it not only with *Republic* X on the music of the spheres but also with exegesis of the *Timaeus*. Proclus was of course very familiar with the long tradition of Platonic interpretation that he inherited but he also looks directly to Plato. I shall briefly discuss Proclus' mention of music as one of the mathematical sciences in his *Commentary on Euclid* and his use of *Republic* VII in his *Commentary on the* Republic, especially, but not only, in the fifth essay of that commentary, before turning to his notion of mathematical exegesis in his *Commentary on the* Timaeus. I will examine the role of music in two passages of that *Commentary* in which Proclus contrasts mathematical exegesis with exegesis of other kinds, namely his comments on *Ti.* 31c−32a and 40c−d. In the last part of the chapter I will address some aspects of Proclus' discussion of the musical scale of *Ti.* 35b and will attempt to clarify how he understands the distinctions between mathematical, physical and philosophical exegesis in that part of his *Commentary.*

Music as a Mathematical Science in the Commentary on Euclid *and the* Commentary on the *Republic*

As part of the prologue to his *Commentary on Euclid* Proclus discusses different views about the classification of the mathematical sciences, contrasting the Pythagorean classification with that of Geminus. When he presents the Pythagorean view, he offers a division of mathematics into four branches which may well be drawn from Nicomachus (*Ar.* I 3.1) but also echoes the five−fold division of *Republic* VII: "Arithmetic […] studies quantity as such, music the relations between quantities, geometry magnitude at rest, spherics [i.e. astronomy] magnitude inherently moving (35.28−36.3)"[4]. Solid geometry and astronomy are rolled up together here, but harmonics is picked out as a distinct branch and labelled not ἁρμονία but μουσική. A few lines further on Proclus

[2]. On the use of this harmonic structure in later passages of the *Timaeus* see Barker 2007, pp. 323−6.
[3]. See O'Meara 2007, pp. 147−61.
[4]. καὶ τὴν μὲν ἀριθμητικὴν τὸ καθ' αὑτὸ τὸ ποσὸν θεωρεῖν, τὴν δὲ μουσικὴν τὸ πρὸς ἄλλο, γεωμετρίαν δὲ τὸ πηλίκον ἀκίνητον ὑπάρχον καὶ τὴν σφαιρικὴν τὸ καθ' αὑτὸ κινούμενον. Translations from the *Euclid Commentary* are taken from Morrow 1970. The echo of Nicomachus is pointed out in Baltzly 2009, p. 179 note 329.

develops the contrast between arithmetic and music in a way explicitly derived from the *Timaeus* (*In Eucl.* 36.17−38.1):

> As the *Timaeus* has taught us, it is by virtue of her difference (...) that the understanding (...) projects numbers and the knowledge of numbers which is arithmetic; and by virtue of the unity of plurality in her and the community of bond that binds her together, she projects music. Hence arithmetic is elder than music, since the soul was first divided by the Demiurge and then bound together by ratios in the fashion explained by Plato[5].

Harmonics and music are not mentioned in this *Commentary* otherwise.

The *Commentary on the* Republic would seem to be the obvious place to look to find out more about Proclus' understanding of the treatment of music in *Republic* VII. None of the essays which make up this *Commentary* focus on the "higher education" of the Guardians. However, the approach to music that we find in *Republic* VII is picked up when Proclus discusses Plato's views on music in the fifth essay of the *Commentary.* That essay discusses ten problems in the interpretation of Plato's views of poetry and music. The fifth of these problems addresses the inconsistencies between Plato's various remarks on μουσική both in different dialogues and within the *Republic* and tries to solve the problem in a way characteristic of later Neoplatonism, by separating out four different types of μουσική. The first type is philosophy, as in *Phaedo* 61a; the second is the inspired poetry described at *Phaedrus* 245a; the third is the one which "leads up from imperceptible harmonies to the perceptible beauty of the divine harmony (I 58.28−59.1)"[6]; and the fourth is the one which educates the character and the passions.

I have discussed Proclus' fourth type of μουσική elsewhere; it clearly derives from the discussion of music in the earlier passage on education in the *Republic,* in Book 3, where the focus is on the moral education of the young Guardians[7]. My interest here is in the third type of μουσική. This one is surely the μουσική of *Republic* VII, in which the proper mathematical study of harmonies can lead to an understanding of numbers in themselves, as a stepping stone to the supreme study of dialectic. Proclus stresses that someone who is μουσικός in this sense is reminded of the beautiful through hearing, rather than through sight, describing such a person at I 59.8−11 as someone whose "activities concern the beauty that is in harmonies and rhythms. From these, he ascends to the imperceptible harmonies and the rhythms that are never known through hearing, but

[5]. ὡς ὁ Τίμαιος ἡμᾶς ἀνεδίδαξεν, λεκτέον ὅτι κατὰ μὲν τὴν ἑτερότητα τὴν αὐτῆς καὶ τὴν διαίρεσιν τῶν λόγων καὶ τὸ πλῆθος ἡ διάνοια στᾶσα καὶ νοήσασα ἑαυτὴν ἓν καὶ πολλὰ οὖσαν τούς τε ἀριθμοὺς προβάλλει καὶ τὴν τούτων γνῶσιν, τὴν ἀριθμητικήν, κατὰ δὲ τὴν ἕνωσιν τοῦ πλήθους καὶ τὴν πρὸς ἑαυτὸ κοινωνίαν καὶ τὸν σύνδεσμον τὴν μουσικήν. δι' ὃ καὶ ἡ ἀριθμητικὴ πρεσβυτέρα τῆς μουσικῆς, ἐπεὶ καὶ ἡ ψυχὴ διαιρεῖται πρῶτον δημιουργικῶς, εἶθ' οὕτως συνδέδεται τοῖς λόγοις, ὡς ὁ Πλάτων ὑφηγεῖται.

[6]. ἀναγωγὸν δὲ ὅμως ἀπὸ τῶν φαινομένων ἁρμονιῶν εἰς τὸ ἀφανὲς τῆς θείας ἁρμονίας κάλλος. Translations from the fifth essay of the *Republic Commentary* are taken from Baltzly *et al*. 2018, with some modifications.

[7]. See Sheppard 2005.

are instead apparent to the reasoning of discursive thought"[8]. Proclus' emphasis here is on the anagogic power of this type of μουσική, rather than on its mathematical basis.

The thirteenth essay of the *Commentary on the* Republic, a lengthy treatment of the speech of the Muses in *Republic* VIII 546a−547a, includes a couple of passages which present the same conception of μουσική as a mathematical science with anagogic power. At II 36.3−46.17 Proclus interprets the mysterious nuptial number, described by the Muses in *Resp.* VIII 546c7 as a "geometric number" (ἀριθμὸς γεωμετρικός), first arithmetically, then geometrically, then "musically" (μουσικῶς), i.e. in terms of harmonics, and finally in terms of mathematical astronomy, using the division of the mathematical sciences from *Republic* VII. In a later passage of the same essay, II 68.3−16, Proclus contrasts the harmony (ἁρμονία) of the Muses with the harmony of the Sirens: the former saves souls and situates them among the gods while the latter is associated with the realm of becoming (γένεσις)[9].

Proclus' conception of music as one of the mathematical sciences is further developed in his *Commentary on the* Timaeus, to which I now turn.

Mathematical Exegesis in Proclus' Commentary on the *Timaeus I*

Proclus' fullest presentation of a mathematical approach to music can be found in his exegesis of the musical scale of *Ti.* 35b. Before addressing this, it may be helpful to consider two other passages of the *Timaeus Commentary* where Proclus contrasts mathematical exegesis with exegesis of other kinds and to examine the role of music in these passages. The first one is *In Ti.* III 28.13−31.26 Van Riel (= II 20.19−23.8 Diehl). Here Proclus is discussing *Ti.* 31c5−32a7. At this point in the dialogue Timaeus is describing how the body of the world is put together from the four elements; 31c5−32a7 sets out the formula for the geometric proportion of this composition. The passage of Plato is difficult and its interpretation is disputed[10]. My interest here is not in how we should understand Plato or even in exactly how Proclus understands him but in the way in which Proclus characterizes the exegetical method he is using. At III 28.13−14 Van Riel (= II 20.19−20 Diehl) Proclus says "It is first necessary to speak about what is said in a mathematical manner, and

[8]. τὸν μὲν περὶ τὶ καλὸν ἐνεργοῦντα τὸ ἐν ἁρμονίαις καὶ ῥυθμοῖς καὶ ἀπὸ τούτου πρὸς τὰς ἀφανεῖς ἁρμονίας ἀνιόντα καὶ ῥυθμοὺς ἐκείνους οὐκέτι δι' ἀκοῆς ὄντας γνωστούς, ἀλλὰ τῷ τῆς διανοίας λογισμῷ καταφανεῖς.

[9]. On the interpretation of the Sirens in this passage and its connection with the four types of music distinguished in the fifth essay, see Moro Tornese 2013.

[10]. For a summary of the modern interpretations of this difficult passage of Plato and of Proclus' view see Baltzly 2007, pp. 11−8.

then in a physical manner, since it is the latter in particular that is under discussion"[11]. His mathematical interpretation runs through until III 31.26 Van Riel (= II 23.8 Diehl) and is then followed by his physical interpretation, from III 32.1−38.17 Van Riel (= II 23.9−28.7 Diehl). The physical interpretation is, as one might expect, concerned with the four elements as they are found in the physical world, although it is worth noting that Proclus emphasizes the divine origin of the elements and concludes with a reference to the Neoplatonic allegorical interpretation of the binding of Ares and Aphrodite by Hephaestus in *Odyssey* 8 (see III 37.16−38.4 Van Riel = II 27.17−27 Diehl). The mathematical interpretation begins with a discussion of mathematical proportion but then moves into an examination of how these proportions apply in music: see III 30.5−6 Van Riel (= II 21.31−2 Diehl): "And of course one could do the same thing with musical values"[12]. Proclus sets out how, in his view, first arithmetic and then geometric proportion are instantiated in music. As Baltzly points out,

> The tradition that the arithmetic, geometric and harmonic means are instantiated in music goes back to Archytas". Our text of Proclus says nothing about the harmonic mean, perhaps because there is a lacuna or perhaps, as Balztly suggests, following Winnington Ingram, because he "set this difficult point to one side for the moment[13].

The way in which Proclus concludes his discussion of the mathematical interpretation makes it very clear that he considers music, understood here as harmonics, as an integral part of mathematics. At III 31.4−8 Van Riel (= II 22.16−20 Diehl) he says:

> All these [ratios] may be seen in numbers as well as in volumes and in musical values, but number has been particularly adapted to the arithmetic [proportion], volume to the geometric and musical values to the harmonic. It is for these reasons that Plato has made use of these three things: numbers, volumes and musical values[14].

He continues at III 31.20−22 Van Riel (= II 23.2−5 Diehl):

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[11]. πρῶτον εἰπεῖν χρὴ περὶ τούτων μαθηματικῶς, ἔπειτα, ὅπερ μάλιστα πρόκειται, φυσικῶς. All translations of passages from the *Timaeus* Commentary are taken from Baltzly 2007, 2009, 2013, with some modifications.

[12]. καὶ δὴ καὶ ἐν δυνάμεσι τὸν αὐτὸν τρόπον. It is clear from what follows that δυνάμεις here means 'musical values'.

[13]. See Baltzly 2007, p. 66 note 85 and 67 note 92. Cf. also Van Riel 2022, note on III 31.15 Van Riel (= II 22.27 Diehl).

[14]. πᾶσαι ἄρα καὶ ἐν ἀριθμοῖς ὁρῶνται καὶ ἐν ὄγκοις καὶ ἐν δυνάμεσι, μᾶλλον μὴν ᾠκείωται ὁ μὲν ἀριθμὸς τῇ ἀριθμητικῇ, ὁ δὲ ὄγκος τῇ γεωμετρικῇ, ἡ δὲ δύναμις τῇ ἁρμονικῇ. καὶ διὰ ταῦτα μὲν οὖν τοῖς τρισὶν ἐχρήσατο τούτοις ὁ Πλάτων, ἀριθμοῖς, ὄγκοις, δυνάμεσιν.

> One *might* say that, strictly speaking, sameness is set over the geometric proportion – for the ratio [between the terms] is that of sameness or identity – but equality is set over the arithmetic proportion, while similarity is set over the harmonic[15].

Proclus also discusses the distinction between arithmetic, geometric and harmonic ratios in the immediately preceding passage of his commentary, III 26.1−28.3 Van Riel (= II 18.21−20.9 Diehl), where the statement in *Ti.* 31c4 that the bonding between the elements is best brought about by proportion (ἀναλογία) leads him to expound these three types of proportion, explicitly focusing on Plato and setting aside the complex accounts of different types of proportion found in philosophers such as Nicomachus and Moderatus (see III 26.15−16 Van Riel = II 19.4−5 Diehl). The three types of proportion are derived from what follows in the *Timaeus* at 34a−36a, as Baltzly explains in the introduction to his translation of Proclus[16]. Overall, then, music, in the sense of harmonics, forms an integral part of Proclus' mathematical interpretation of this part of the *Timaeus.* The mathematical interpretation of the text, in terms of different types of proportion, appears to be at a higher level than the physical interpretation. This is in accordance both with the treatment of mathematics in *Republic* VII and with the part played by mathematics in structuring the world soul in the *Timaeus.* In both dialogues mathematics, including harmonics, operates at a higher, more abstract level than perception of the physical world.

Proclus again contrasts mathematical exegesis with exegesis of another kind at *In Ti.* IV 188.17−193.21 Van Riel (= III 145.32−149.28 Diehl). This time the contrast is not between mathematical and physical exegesis but between mathematical and philosophical exegesis. The passage under examination here is *Ti.* 40c−d where the heavenly bodies are treated as visible gods whose regular movements form a cosmic dance. Proclus offers first a mathematical interpretation of the passage, according to which the movements of the stars are to be understood in astronomical terms, and secondly a philosophical one, according to which the passage is concerned not so much with the heavenly bodies as with the divine souls which animate them. He begins his lengthy exposition of the mathematical interpretation by saying (III 188.17−19 Van Riel = II 145.32−146.2 Diehl): "If first you wish to proceed in the mathematical [mode of interpretation], let us interpret their dances as their well−ordered and harmonised revolutions"[17]. The word "harmonised" (ἐναρμονίους) might lead us to expect that the mathematical interpretation will also involve

[15]. Λέγοιτο δ' ἂν κυρίως ἐπὶ μὲντῆς γεωμετρικῆς ἡ ταυτότης (ὁ γὰρ αὐτός ἐστι λόγος), επὶ δὲ τῆς ἀριθμητικῆς ἡ ἰσότης, ἐπὶ δὲ τῆς ἁρμονικῆς ἡ ὁμοιότης.
[16]. See Baltzly 2007, pp. 8−11.
[17]. καὶ πρότερον, εἰ βούλει, μαθηματικῶς {τε καὶ φιλοσόφῶς} χορείας μὲν ἀκούσωμεν τὰς εὐτάκτους αὐτῶν καὶ ἐναρμονίους περιφοράς. The words τε καὶ φιλοσόφῶς ("and also in the philosophical [mode of interpretation]") seem out of place here and are treated by both Van Riel and Diehl as an interpolation that should be deleted. I follow Festugière and Baltzly in not including them in my translation of the text.

something about music (harmonics). That is not the case: what we get is a very detailed discussion of astronomy which does include references to *Resp.* X 616−17 as well as to *Ti.* 36−39 (see IV 191.20−191.4 Van Riel = III 147.21−31 Diehl). But we should remember that for a Platonist astronomy and music are "sister sciences" and also that the stars are dancing. Dancing is usually accompanied by music; the music to which the stars are dancing is surely not music audible to human ears but the music of the spheres. The philosophical interpretation to which Proclus turns at IV 193.17 Van Riel (= III 149.23 Diehl) is introduced by a reference to another dialogue which is important for the tradition of the cosmic dance, namely the *Phaedrus*: "Surely then the dances of the souls are the Bacchic ones that they make around the intelligible, and the intellectual periods and complete cycles for they [*scil*. souls] also dance when they are following the more divine among their leaders, as Socrates says in the *Phaedrus*"[18]. The interpretation which follows offers a philosophical allegory of the cosmic dance.

If we put these two passages of the *Timaeus Commentary* together, we can see that for Proclus a mathematical interpretation is superior to a physical interpretation but inferior to a philosophical one. We can also see that a mathematical interpretation can include either harmonics or astronomy or both. None of this is very surprising, but it is deeply Platonist and reflects both the *Republic* and the *Timaeus.*

Mathematical Exegesis in Proclus' Commentary on the *Timaeus II*

Finally, I come to Proclus' interpretation of the musical scale of *Ti.* 35b. In this context too Proclus distinguishes between mathematical, physical and philosophical interpretations but, as we shall see, he also distinguishes more than one level of both mathematical and physical interpretation, partly in order to structure his presentation of the interpretations of earlier commentators. At *In Ti.* III 236.20−237.12 Van Riel (= II 174.15−28 Diehl), Proclus introduces what he describes as the mathematical interpretation of 35b4−6 with these words:

> Mathematical theory ought neither to be entirely scorned, nor sought after for its own sake. The first option means that Plato will not be able to indicate to us, as he wishes to, the things in their images, while the second option makes the whole exegesis of the text as unstable as a ship without ballast, for it is necessary to be firmly anchored, as it were, when one is heading off after the essence of those realities with which the dialogue is concerned. We, however, will steer a middle course, as we said before, having first set out the text

[18]. οὐκοῦν χορεῖαι μέν εἰσι τῶν ψυχῶν αἱ περὶ τὸ νοητὸν βακχεῖαι καὶ αἱ περίοδοι καὶ ἀποκαταστάσεις αἱ νοεραί· συνεπόμεναι γὰρ τῷ θειοτέρῳ τῶν ἡγεμόνων καὶ αὐταὶ χορεύουσιν, ὥς φησιν ὁ ἐν Φαίδρῳ Σωκράτης. Cf. *Phdr.* 246e4−247a7, 250b6 and 252d1.

> mathematically, then, after this we will provide an exegesis of the proposed division [of the psychic essence] that is appropriate to the subject. Now the Pythagoreans were doubtless very wise in as much as they discovered the division of the monochord, but it was *Plato* who imparted the division of the soul in these terms – disclosing the substantial causes and the generative *logoi* of mathematical theorems[19].

The mathematical interpretation of this difficult passage of Plato in which mathematics and music are combined takes Proclus a long time. He starts with an extended account of the numbers and musical intervals involved, including discussion of the views of some earlier commentators. At III 260.25 Van Riel (= II 193.9 Diehl) he turns to what is described as πραγματειώδης ἐξήγησις, which I suggest should be translated as "the exegesis that correlates the text with reality". The word πραγματειώδης, taken from Plato's *Parmenides* (137b2), is used a number of times in Proclus' *Commentary on the* Parmenides to contrast a purely logical interpretation of that dialogue with an interpretation in terms of metaphysical reality. In this passage of the *Timaeus Commentary* there is a parallel contrast between two levels of mathematical interpretation, as described in the text quoted above: one is the purely mathematical interpretation, which would be unstable on its own, while the other is the interpretation that "discloses the substantial causes and the generative *logoi* of mathematical theorems"[20]. The distinction between two levels of mathematical interpretation is expressed in musical terms at III 263.11−19 Van Riel (= II 195.11−24 Diehl):

> Let the manner of exegesis of the soul's essence be naturally consonant with the essence of the soul itself. You must free yourself from the [mere] appearance of harmony, and lift yourself up to the substantive and immaterial harmony, and be led back from images to the paradigms [of these images]. For the concordances that flow through our ears and which consist in soundings and strikings differ entirely from the concordance of what is life−giving and intellectual. Therefore, let no one stop at the point of the mathematical consideration of the subject at hand. Instead he should urge himself on to examine this subject in a manner fitting to the essence of the soul[21].

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[19]. τὴν μαθηματικὴν θεωρίαν οὔτε παντάπασιν ἀτιμαστέον οὔτε μόνην αὐτὴν καθ' ἑαυτὴν ζηλωτέον· τὸ μὲν γὰρ οὐ δείκνυσιν ἡμῖν, ὥσπερ ὁ Πλάτων βούλεται, τὰ πράγματα ἐν ταῖς εἰκόσι, τὸ δὲ ἀνερμάτιστον ποιεῖ τὴν ὅλην ἐξήγησιν· δεῖ γὰρ ὡς ἐπ' ἀσφαλοῦς πείσματος ὁρμεῖν τῆς τῶν πραγμάτων περὶ ὧν ὁ λόγος ἐστὶν οὐσιας. ἐν μέσῳ τοίνυν, ὥσπερ εἴπομεν καὶ πρότερον, καταστήσαντες τὸν λόγον μαθηματικῶς μὲν πρῶτον, οἰκείως δὲ τοῖς ὑποκειμένοις μετὰ ταῦτα τὴν προκειμένην ἐξηγησώμεθα κατατομήν. οἱ μὲν οὖν Πυθαγόρειοι μέγα φρονοῦσιν, ὡς τὴν τοῦ κανόνος κατατομὴν ἀνηυρηκότες· ὁ δέ γε Πλάτων ψυχῆς κατατομὴν ἐν τούτοις παραδίδωσι τὰς αἰτίας τὰς οὐσιώδεις καὶ τοὺς λόγους τοὺς γεννητικοὺς τῶν μαθηματικῶν θεωρημάτων ἀναπτύσσων.

[20]. τὰς αἰτίας τὰς οὐσιώδεις καὶ τοὺς λόγους τοὺς γεννητικοὺς τῶν μαθηματικῶν θεωρημάτων ἀναπτύσσων. On the meaning of πραγματείωδης and the content of this part of the commentary, cf. Baltzly 2009, pp. 23−4 and 168 note 302. Baltzly translates πραγματειώδης ἐξήγησις as "the exegesis that correlates the text with things". I am doubtful about his suggestion that πραγματειώδης refers specifically to 'correspondences between levels'. Morrow–Dillon 1987 sometimes translate the word by "substantive" or "important", as he says, but at other times use phrases such as "concerned with the reality behind the text", "concerned with the underlying realities" and "from the perspective of metaphysical reality", all of which effectively capture the contrast with a purely logical (or purely mathematical) interpretation. Cf. also Procl. *In Eucl.* 82.13−20.

[21]. ὁ δὲ τρόπος τῆς περὶ αὐτὴν ἐξηγήσεως ἔστω τῇ οὐσίᾳ συμφυής, ἀπὸ μὲν τῆς φαινομένης ἁρμονίας ἀπολυόμενος, ἀνάγων δὲ ἑαυτὸν ἐπὶ τὴν οὐσιώδη καὶ ἄυλον ἁρμονίαν καὶ ἀπὸ τῶν εἰκόνων ἐπὶ τὰ παραδείγματα ἀναπεμπόμενος· ἡ

After expounding both levels of mathematical interpretation, Proclus finally moves at III 285 Van Riel (= II 212 Diehl) to other types of interpretation, introducing them as follows (III 285.23−286.5 Van Riel = II 212.3−9 Diehl):

> As we said earlier, it is necessary to interpret what has been said by Plato not only mathematically, but also physically or philosophically. For the soul's essence has not been composed from mathematical numbers and ratios, but instead all these ratios and numbers are images of the soul's real essence and the demiurgic and life − generating divisions within it[22].

The second sentence here once again expounds the two levels of mathematical interpretation which Proclus has treated at length. In what follows he presents physical interpretations of the same lemma as before, *Ti.* 35b4−6, and then, at the end of his discussion, the philosophical interpretation. Proclus organises the physical interpretations into two groups, at two different levels, just as he did with the mathematical interpretations, starting with a brief account of interpretations of the musical scale in terms of the seven heavenly spheres and the movement of the heavenly bodies. However, this kind of interpretation purely in terms of the physical world is swiftly rejected and he moves to discussion of the views of first Amelius and then Porphyry, Iamblichus and Theodore of Asine with another reference to πραγματειώδης ἐξήγησις, introducing their views by saying "After these people [i.e. those who offer a purely physical interpretation of the text] there is yet another crowd of interpreters who hold views that are more in keeping with reality (III 287.15−16 Van Riel = II 213.8−9 Diehl)"[23]. At this second level of physical interpretation, physical phenomena are correlated in various ways with numbers and ratios. For Proclus none of the philosophers discussed under this heading offers a fully satisfactory account of the text, although he expresses approval for some aspects of the interpretations of Porphyry and Iamblichus[24]. At III 295.11 Van Riel (= II 218.20 Diehl) he comes at last to his preferred interpretation, that of his teacher Syrianus, introducing it as "another mode of reasoning (ἄλλον τρόπον λόγων)". This interpretation sets the

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γὰρ δι' ὥτων εἰσρέουσα συμφωνία καὶ ἐν ἤχοις καὶ πληγαῖς ὑφισταμένη τῆς ζωτικῆς καὶ νοερᾶς ἐξήλλακται. μὴ τοίνυν τις ἄχρι τῆς μαθηματικῆς τῶν προκειμένων ἱστάσθω θεωρίας, ἀλλ' ἐπὶ τὸν πρέποντα τρόπον τῇ οὐσίᾳ τῆς ψυχῆς ἑαυτὸν ἐγειρέτω.

[22]. ὅτι μὲν οὐ μόνον μαθηματικῶς, ἀλλὰ καὶ φυσικῶς ἢ φιλοσόφως ἀκροᾶασθαι δεῖ τῶν ὑπὸ τοῦ Πλάτωνος λεγομένων ἐν τούτοις, καὶ πρότερον εἴπομεν· οὐ γὰρ ἐκ μαθηματικῶν ἀριθμῶν ἐστι καὶ λόγων ἡ οὐσία τῆς ψυχῆς, ἀλλ' οὗτοι πάντες οἱ λόγοι καὶ οἱ ἀριθμοὶ τὴν ὄντως οὐσίαν αὐτῆς ἀπεικονίζονται καὶ τὰς ἐν αὐτῇ διαιρέσεις τὰς δημιουργικάς τε καὶ ζῳογονικάς.

[23]. μετὰ δὴ τούτους ἄλλο πλῆθός ἐστιν ἐξηγητῶν πραγματειωδεστέρων λόγων ἀντεχόμενον·

[24]. Theodore of Asine is criticised at III 294.20−295.3 Van Riel (= II 218.8−12 Diehl) for offering an interpretation which is limited to mathematics and fails to explain "how it compares with the reality which is the subject matter of our discourses or how one might interpret this in a Pythagorean manner" (ὅπως δὲ ἀπείκασται τοῖς πράγμασιν ὑπὲρ ὧν οἱ λόγοι, καὶ ὅπως ἄν τις αὐτὰ Πυθαγορικῶς διερμηνεύσειεν). I take this to mean that Theodore's interpretation correlates physical phenomena with mathematical ones but fails to ascend to the higher level of mathematical exegesis, let alone to a fully philosophical interpretation.

understanding of the soul within the big Neoplatonist metaphysical picture of procession from and reversion to first principles and offers a clear example of what Proclus means by a philosophical interpretation. An interpretation of this kind is like the philosophical interpretation of the cosmic dance, i.e. it interprets the text in a way that goes beyond correlations between physical phenomena and numbers and ratios and also beyond correlations between the properties of numbers and ratios and the properties of the soul. At III 299.17−19 Van Riel (= II 221.24−26 Diehl) Proclus argues that such an interpretation offers the best explanation of Plato's text and that by referring to it "we will resolve many of the puzzles (πολλὰ τῶν ἀπόρων διαλύσομεν)".

Conclusion

We have seen that, like Plato, Proclus regards music as one of the mathematical sciences. It follows that Proclus' conception of mathematical exegesis includes music, alongside astronomy. In this he clearly follows the lead given by the text of both the *Republic* and the *Timaeus.* In several places in the *Timaeus* commentary Proclus distinguishes between mathematical, physical and philosophical exegesis; when he comes to discuss the musical scale of 35b he subdivides both mathematical and physical exegesis into two levels to take account of the complexity of the subject matter and the diversity of earlier interpretations. Yet his overall conception of music (μουσική) is much wider, as we can see from the fifth and thirteenth essays of the *Republic* commentary. I argued earlier that the third type of music distinguished in the fifth essay is the mathematical type ("harmonics"). This type of music reappears in the thirteenth essay, both as harmonics and as the anagogic harmony of the Muses, contrasted with the dangerously bewitching harmony of the Sirens. The first two types described in the fifth essay, which are equated with philosophy and with inspired poetry, might have a part to play in philosophical exegesis as we find it in the *Timaeus Commentary* (for example, in the philosophical exegesis of the cosmic dance or in Syrianus' exegesis of 35b4−6 in terms of overarching metaphysical principles). The fourth type, which educates the character and the passions, operates at the same level as physical exegesis, although I have not so far come across any texts which put these two things together. However that may be, Proclus follows Platonic and Pythagorean tradition in including at least one type of music within mathematics.

Bibliography

D. Baltzly (ed.), *Proclus. Commentary on Plato's* Timaeus, Vol. 3, Book 3, Part 1: *Proclus on the World's Body*, Cambridge, Cambridge University Press, 2007.

D. Baltzly (ed.), *Proclus. Commentary on Plato's* Timaeus, Vol. 4, Book 3, Part 2: *Proclus on the World Soul*, Cambridge, Cambridge University Press, 2009.

D. Baltzly (ed.), *Proclus. Commentary on Plato's* Timaeus, Vol. 5, Book 4: *Proclus on Time and the Stars*, Cambridge, Cambridge University Press. 2013

D. Baltzly – J. F. Finamore – G. Miles (eds.), *Proclus. Commentary on Plato's* Republic, Vol. 1, Cambridge, Cambridge University Press, 2007.

A. Barker, *The Science of Harmonics in Classical Greece*, Cambridge, Cambridge University Press, 2007.

A.-J. Festugière, *Proclus, Commentaire sur le* Timée, 5 vols., Paris, Vrin, 1966–1968.

S. F. Moro Tornese, *Music and the Return of the Soul in Proclus' Commentaries on Plato's Timaeus and Republic*, in A. Sheppard (ed.), *Ancient Approaches to Plato's* Republic, London, Institute of Classical Studies, School of Advanced Study, University of London, pp. 117−28. 2013

G. R. Morrow (ed.), *Proclus. A Commentary on the First Book of Euclid's* Elements, Princeton, Princeton University Press, 1970.

G. R. Morrow – J. M. Dillon (eds.), *Proclus' Commentary on Plato's* Parmenides, Princeton, Princeton University Press, 1987 (1992^2).

D. J. O'Meara, *Hearing the Harmony of the Spheres in Late Antiquity*, in: M. Bonazzi – Carlos Lévy – C. Steel (eds.), *A Platonic Pythagoras. Platonism and Pythagoreanism in the Imperial Age*, Turnhout, Brepols, 2007, pp. 147−61.

A. Sheppard, *Music Therapy in Neoplatonism*, in: R. W. Sharples (ed.), *Philosophy and the Sciences in Antiquity*, Aldershot, Ashgate Publishing, 2005, pp. 148−55.

G. Van Riel (ed.), *Procli Diadochi in Platonis* Timaeum *Commentaria*, 5 vols., Oxford, Oxford University Press, 2022.

ABSTRACT

Anne Sheppard, *The Place of Music in Proclus' Conception of Mathematical Exegesis*

This chapter investigates the role of music (*mousike*) within Proclus's concept of mathematical exegesis, drawing upon the Platonic and Pythagorean traditions. In Plato's *Republic* 7 and the *Timaeus*, harmonics is established as the "sister science" of astronomy and a necessary branch of mathematics for the Guardians' education toward dialectic. The author argues that this Platonic conception of the mathematical sciences informs Proclus's application of the notion of mathematical exegesis in his *Commentary on the* Timaeus. The chapter specifically examines passages in *In Tim*. and Proclus's distinction between mathematical, physical, and philosophical interpretation, including his exegesis of the musical scale from *Timaeus* 35b. The core argument is that Proclus's conception of mathematical exegesis intentionally includes music alongside astronomy, adhering to the ancient tradition that incorporates a specific type of music into the domain of mathematics, which is also evident in his discussions in the *Commentary on Euclid* and the *Commentary on the* Republic.

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INCOMMENSURABILITY IN GEOMETRICAL ENTITIES IN PROCLUS' *IN EUCL*. AND *IN PRM*.

Introduction

The central question I aim to explore in this chapter is the origin of incommensurability within Proclus' ontology. This issue is complex, and it has been observed that Proclus might not have regarded incommensurability as a subject of scientific inquiry at all. Yet, how does it emerge at a specific ontological level, and how does it relate to a particular mode of cognition? In this chapter, I will first identify the ontological status of geometrical entities. I will then show why neither sensible matter as such nor intelligible matter can unqualifiedly serve as the cause of incommensurability. I will demonstrate that incommensurability must rather be a property of certain specific geometrical items, belonging to them just as their other identifiable parts do. Finally, I will conclude by advancing what is surely a controversial position: that there might be a form of incommensurability.

Incommensurability in Proclus' Philosophy of Geometry

In his *Commentary on the First Book of Euclid's* Elements, Proclus situates the geometry presented by Euclid within a Neoplatonic framework. According to Proclus, Euclid's arguments in the *Elements* aim at the construction of the five regular bodies known from the *Timaeus*[1], progressively deriving them from the highest principles[2]. The overarching argument begins with a first prologue, where Proclus locates mathematical entities within his ontological hierarchy, demonstrating how the common principles of general mathematics are responsible for the creation of geometrical entities[3]. In a second prologue, he elaborates on the goal of the first book of the *Elements* and the methods of geometry, including a general discussion of its postulates and axioms. After these two prologues, Proclus examines Euclid's definitions, followed by a detailed discussion of specific postulates and axioms, before finally addressing Euclid's propositions. The discussion

[1]. "Solids" would imply their material solidity, which is not a given for geometric entities.
[2]. See Procl. *In Eucl.* 68.21−23; 70.19−71.5; 82.25−83.2.
[3]. See Procl. *In Eucl.* 2.5: τὰς δὲ ἀρχὰς τῆς μαθηματικῆς ὅλης οὐσίας ἐπισκοποῦντες ἐπ' αὐτὰς ἄνιμεν τὰς διὰ πάντων τῶν ὄντων διηκούσας ἀρχὰς καὶ πάντα ἀφ' ἑαυτῶν ἀπογεννώσας, λέγω δὲ τὸ πέρας καὶ τὸ ἄπειρον.

Proclus presents thus becomes progressively more complex and increasingly removed from the higher and simpler causes.

Mathematical entities, generally speaking, occupy an intermediate level between the simple immaterial realities of the highest realm and the extended, composite, and confusedly complex objects of the sensible world[4]. They exist in the vestibule of the forms, being more exact than physical objects yet more complex than the forms themselves[5]. Proclus occasionally emphasizes the complexity of mathematical entities by referring to them as *logoi*[6]. A closer look at mathematical entities reveals that they are further divided into a higher "arithmetical" part and a lower "geometrical" part. The well-known distinction between the two lies in their differing expressions of the relation between Limit and the Unlimited. In the arithmetical part, the Limit ensures that each number remains finite, while the Unlimited accounts for the infinite additivity of numbers. In the geometrical part, the Limit accounts for the boundaries of entities, whereas the Unlimited accounts for their infinite divisibility[7]. As Proclus notes, "If there were no infinity, all magnitudes would be commensurable and there would be nothing inexpressible or irrational"[8]. Magnitudes do not only exist on this intermediate ontological level. They also manifest within the lower realm of the sensible world and are present as *logoi* – though in a distinct, non-extended mode[9]. The *logos* of a geometrical object is the cause of both the geometrical and the sensible entity. The difference between geometrical and sensible objects is that the former are projected onto the soul's *phantasia* – i.e., intelligible matter[10] or *pathetikos nous*[11], a faculty of the soul – whereas the latter are projected onto the lower kind of matter that accounts for sensible objects. One key difference between the geometrical and the sensible projection is the degree of precision each allows, a point which will prove important for our understanding of incommensurability. Clearly, the *phantasia*, unlike the sensible world, can represent a geometrical figure such as a circle with precision. It is this projection onto the soul's *phantasia* that gives geometrical entities their extended nature, just as projection onto lower matter results in the extension of sensible objects[12]. Once the notions of

[4]. See Morrow 1970, pp. XXXIII.
[5]. See Nikulin 2008, p. 154, referring to Procl. *In Eucl*. 5.2−3; 55.2.
[6]. See Nikulin 2008, p. 157.
[7]. See Nikulin 2008, p. 158, referring to Procl. *In Eucl*. 6.21
[8]. See Procl. *In Eucl*. 6.19−21: καὶ τῆς μὲν ἀπειρίας οὐκ οὔσης τά τε μεγέθη πάντα σύμμετρα ἂν ἦν καὶ οὐδὲν ἄρρητον οὐδὲ ἄλογον.
[9]. Procl. *In Eucl.* 54.5−13.
[10]. As Proclus notes, the soul is not a passive receptacle but actively participates in the process of "writing" these geometrical forms. See also Opsomer 2009, p. 215.
[11]. See Nikulin 2008, p. 163.
[12]. See Procl. *In Eucl.* 52.26−53.1: (The soul) thinks the circle as extended, and although this circle is free of external matter, it possesses an intelligible matter provided by the imagination itself (καὶ τόν τε κύκλον διαστατῶς νοεῖ τῆς μὲν ἐκτὸς ὕλης καθαρεύοντα νοητὴν δὲ ὕλην ἔχοντα τὴν ἐν αὐτῇ).

matter, magnitude, and, with them, divisibility are introduced, we might infer that this is also the direct cause of the of incommensurability[13]. Yet, the problem is more complex than that.

The extended yet precise geometrical objects within the *phantasia* represent the mode by which the soul seeks to comprehend the unextended paradigms. Proclus describes this process as follows (*In Eucl*. 54.27−55.4):

> For the understanding contains the ideas but, being unable to see them when they are wrapped up, unfolds and exposes them and presents them to the imagination sitting in the vestibule; and in imagination, or with its aid, it explicates its knowledge of them (…)[14].

Phantasia functions as a tool that enables the soul to grasp items that would otherwise be inaccessible to it[15]. Through the process of unfolding and dianoetic reasoning, the soul can reveal aspects of the *logoi* by systematically working through them. The unfolding, on the one hand, allows us to discover the properties of these entities through discursive reasoning – properties that cannot be apprehended merely by contemplating the *logoi* themselves[16]. Discursive reasoning, on the other hand, expands these properties through construction, employing techniques such as analysis, division, definition, and demonstration[17]. Proclus explicitly states that the relations and properties of geometrical entities are not derived from the lower level of the soul but are instead present in the higher realm, though in a non-extended form. He writes (*In Eucl*. 55.23−56.8):

> Every true geometer should cultivate such efforts and make it his goal to arouse himself to move from imagination to pure and unalloyed understanding, thus rescuing himself from extension and "passive nous" for the dianoetic activity that will enable him to see all things without parts or intervals—the circle, the diameter, the polygons in the circle, all in all and each separately. This is why even in our imagination we show circles as

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[13]. If *phantasia* is furthermore considered an irrational faculty of the soul, incommensurability, being *alogos*, seems to have found its proper place. See also Opsomer 2009, p. 220.

[14]. ἔχουσα γὰρ ἡ διάνοια τοὺς λόγους, ἀσθενοῦσα δὲ συνεπτυγμένως ἰδεῖν ἀναπλοῖ τε αὐτοὺς καὶ ὑπεκτίθεται καὶ εἰς τὴν φαντασίαν ἐν προθύροις κειμένην προάγει καὶ ἐν ἐκείνῃ ἢ καὶ μετ᾽ ἐκείνης ἀνελίττει τὴν γνῶσιν αὐτῶν (…). See also Procl. *In Eucl.* 56.13−23: ἀλλὰ καὶ ὅσα κρυφίως ἐστὶν ἐν ἐκείνῳ, διαστατῶς καὶ μεριστῶς εἰς φαντασίαν προάγεται καὶ τὸ μὲν προβάλλον ἡ διάνοια, τὸ δὲ ἀφ᾽ οὗ προβάλλεται τὸ διανοητὸν εἶδος, τὸ δὲ ἐν ᾧ τὸ προβαλλόμενον παθητικὸς οὗτος καλούμενος νοῦς, ἐξελίττων ἑαυτὸν περὶ τὴν ἀμέρειαν τοῦ ἀληθοῦς νοῦ καὶ διϊστὰς ἑαυτοῦ τὸ ἀδιάστατον τῆς ἀκραιφνοῦς νοήσεως καὶ μορφῶν ἑαυτὸν κατὰ πάντα τὰ ἀμόρφωτα εἴδη καὶ πάντα γιγνόμενος, ἅ ἐστιν ἡ διάνοια καὶ ὁ ἀμερὴς ἐν ἡμῖν λόγος.

[15]. See Bechtle 2006, p. 337: „Wenn die Geometrie also − Proklos' Argumentation gemäß − über den Kreis, seinen Durchmesser, seine Tangenten o.ä. Aussagen macht, dann erfahren wir nichts über Kreise (allgemein *logoi*) in sinnlich wahrnehmbaren Dingen (…) – sagt Proklos − und auch nichts über die *logoi* in der *dianoia*. Denn der Kreis in der Geometrie ist auf der einen Seite immateriell, nämlich im Gegensatz zum Kreis in sinnlich wahrnehm-baren Dingen, auf der anderen Seite multipel, teilbar etc., nämlich im Gegensatz zu den *logoi* in der *dianoia*. Da die Geometrie sich also nur mit den projizierten *logoi* in der Vorstellung (beispielsweise also vorgestellten Kreisen) beschäftigt, sieht der Geometer die eingeborene Form oder Idee (den *logos*) auch nur indirekt als Spiegelbild, obwohl der *logos* selbst (der in der *dianoia*) das eigentliche Ziel seiner Beschäftigung ist". See O'Meara 2005, p. 138.

[16]. See Nikulin 2008, p. 164.

[17]. See Nikulin 2008, p. 165, referring to Procl. *In Eucl.* 42.9−43.21.

inscribed in polygons and polygons as inscribed in circles, in imitation of the proof that the part-less ideas exist in and through one another[18].

Thus, an unextended entity, such as the circle in the higher realm, possesses a diameter and relations to other geometrical entities, albeit in an unextended manner. Now, incommensurability might either result from the weakening power of matter, or it might be a property caused by the higher realm, on a par with the diameter or the angle, which become extended only within the *phantasia*.

Incommensurability and Matter?

Incommensurability and Imprecision

Having set the stage, we may now ask where incommensurability actually originates. The first answer that comes to mind is matter. The unextended *logoi* of geometrical objects are projected onto two distinct recipients: the soul's *phantasia* and the matter of the sensible world[19]. Both of these recipients weaken and extend the *logoi*. However, there is a clear distinction between the two projections. Sensible objects, by their very nature, are imprecise[20]. Everything in the sensible world is extended in three dimensions. For instance, we will never encounter a one-dimensional line or an unextended point in the sensible world[21]. Moreover, in the sensible realm, magnitudes are conflated with one another: what we might identify as a circle displays features of straightness, and what strives to be straight exhibits traits of roundness. No comparison between two entities in the sensible world can ever be exact, owing both to their three-dimensional extension and to the confusion of their contrary properties. The absence of any precise interval in the sensible world makes it impossible to speak meaningfully of commensurability. This does not mean that everything in the sensible world is incommensurable. Rather, the characteristic feature of the

[18]. καὶ ταύτην δεῖ τὴν μελέτην μελετᾶν τὸν ὡς ἀληθῶς γεωμετρικόν, καὶ πρὸς τὴν ἔγερσιν καὶ τὴν ἀπὸ τῆς φαντασίας μετάστασιν εἰς μόνην τὴν διάνοιαν αὐτὴν καθ' αὑτὴν ποιεῖσθαι τέλος, ἁρπάζοντα ἑαυτὸν ἀπὸ τῶν διαστάσεων καὶ τοῦ παθητικοῦ νοῦ πρὸς τὴν διανοητικὴν ἐνέργειαν, καθ' ἣν πάντα ἀδιαστάτως ὄψεται καὶ ἐν ἀμερεῖ τὸν κύκλον, τὴν διάμετρον, τὰ ἐν τῷ κύκλῳ πολύγωνα, καὶ πάντα ἐν πᾶσιν καὶ ἕκαστον χωρίς. διὰ γὰρ τοῦτο καὶ ἐν φαντασίᾳ δείκνυμεν ἔν τε τοῖς πολυγώνοις τοὺς κύκλους ἐγγραφομένους καὶ ἐν τοῖς κύκλοις τὰ πολύγωνα, μιμούμενοι τὴν τῶν ἀμερῶν λόγων δι' ἀλλήλων δεῖξιν.

[19]. See Procl. *In Eucl*. 51.13−15: καὶ τὰ μετέχοντα αὐτὰ διττὰ θέμενοι, τὰ μὲν αἰσθητὰ τὰ δὲ ἐν φαντασίᾳ τὴν ὑπόστασιν ἔχοντα.

[20]. See Nikulin 2019, p. 137. Procl. *In Eucl.* 12.23−26: οὐχ ὁρῶμεν, ὡς ἐν ἀλλήλοις πάντα τὰ αἰσθητὰ συμμέμικται καὶ ὡς οὐδὲν ἐν τούτοις εἰλικρινὲς οὐδὲ τοῦ ἐναντίου καθαρεῦον, ἀλλὰ μεριστὰ πάντα καὶ διαστατὰ καὶ κινούμενα.

[21]. See Procl. *In Eucl.* 49.12−17: ποῦ δὲ καὶ τεθεάμεθα ἐν τοῖς αἰσθητοῖς τὸ ἀμερὲς σημεῖον ἢ τὴν ἁπλατῆ γραμμὴν ἢ τὴν ἀβαθῆ ἐπιφάνειαν ἢ τὴν ἰσότητα τῶν ἐκ τοῦ κέντρου γραμμῶν ἢ ὅλως τὰ πολύγωνα καὶ πολύεδρα σχήματα πάντα, περὶ ὧν ἡ γεωμετρία διδάσκει.

sensible world is imprecision. Thus, there are no geometrical objects in the strict and precise sense in the sensible world[22] – only imprecise magnitudes.

Incommensurability only comes into play when the relationship between two intervals cannot be expressed in terms of a common unit. However – and this point is crucial – it must occur within a realm where *commensurability* is at least possible. Imprecision in the sensible world is not the same as incommensurability. Imprecision precludes any determination of whether something is commensurable or not. The sensible world, which is incapable of providing any precise unit owing to its inherent imprecision, instability, and motion, is therefore characterized neither by commensurability nor by incommensurability, but simply by imprecision. The sensible realm does not even permit the use of the anthyphairetic method to detect incommensurability, since each step of this method requires at least partial commensurability, precision, and a unity, followed by an interval that is not commensurable with the former, and so on. Incommensurability, then, is not a feature of the sensible world but of the *phantasia*. And if this is the case, the direct deduction of incommensurability from lower matter becomes implausible.

Incommensurability and Intelligible Matter

Given this, it makes sense to take not lower matter but intelligible matter as the cause of incommensurability. Incommensurability, then, would not result from the weakening force of lower matter. Proclus, after all, viewed incommensurability primarily as a feature of the geometrical realm of the *phantasia*[23]. However, to take intelligible matter as the cause of incommensurability is also problematic, since only certain items projected upon it exhibit incommensurability, while others do not[24]. Consider the circle, which according to Proclus does not exhibit incommensurability; the

[22]. See Nikulin 2019, p. 137.

[23]. See Procl. *In Eucl.* 6.19−22: καὶ τῆς μὲν ἀπειρίας οὐκ οὔσης τά τε μεγέθη πάντα σύμμετρα ἂν ἦν καὶ οὐδὲν ἄρρητον οὐδὲ ἄλογον, οἷς δὴ δοκεῖ διαφέρειν τὰ ἐν γεωμετρίᾳ τῶν ἐν ἀριθμητικῇ (…). See Procl. *In Eucl.* 278.19−24: ὅταν γὰρ δεικνύωσιν ὅτι ἔστιν τὸ ἀσύμμετρον ἐν τοῖς μεγέθεσι καὶ οὐ πάντα σύμμετρα ἀλλήλοις, τί ἄλλο δεικνύναι φήσει τις αὐτοὺς ἢ ὅτι πᾶν μέγεθος εἰς ἀεὶ διαιρεῖται καὶ οὐδέποτε ἥξομεν εἰς τὸ ἀμερές, ὅ ἐστι κοινὸν μέτρον τῶν μεγεθῶν ἐλάχιστον.

[24]. This distinction was already clearly recognized by Aristotle. See Karasmanis 2011, p. 389 in Anagnostopoulos: "Does he think that incommensurability is a result of infinite divisibility? We do not have evidence that Aristotle thought so. On the contrary, from a passage in the *Prior Analytics* (65b16–20) we see that Aristotle thinks that infinite divisibility and incommensurability are two rather independent properties". See Karasmanis 2011, p. 393: "Now, if we agree that Plato says (25a6–7 [in the *Philebus*, DW]) that apeiron admits opposite characteristics to those of *peras*, then we have to conclude that incommensurability is a further and very important characteristic of *apeiron*. If we define continuity in terms of Zenonian infinite divisibility, as Aristotle does, then all the pairs of the infinite cuts of a continuous thing (e.g., a line) have a specific ratio (a):(b) (where (a) and (b) are numbers). Therefore, according to Plato's conception of *peras*, the Aristotelian continuum belongs to peras and not to apeiron. Hence, we cannot interpret Plato's apeiron just in terms of continuity or infinite divisibility". Commensurability depends on the "cuts" we make in the substrate. If those cuts are in a "Zenonian" ratio, there is no incommensurability. According to Karasmanis, Proclus

same applies to some triangles. These figures can be incommensurable with other entities, but not in themselves. Although all such entities are projected onto intelligible matter, only some manifest incommensurability, while others do not. The difference between the "effect" of lower matter, which causes *everything* to become imprecise and three-dimensionally extended, and that of intelligible matter, where only certain items display incommensurability, requires explanation. For now, it is clear that we must determine why incommensurability arises in some cases but not in others.

Commensurability and Incommensurability in the Cosmos

So far, we have established that incommensurability is a feature of the *phantasia*, yet there must be a reason why it arises in some extended geometrical entities but not in others. If intelligible matter were the sole cause, all such entities would exhibit incommensurability in themselves.

In Ti. II 261.20–262.2 Van Riel (= I 384.29−385.3 Diehl) and *In Eucl.* 6.19−22 suggest a link between matter and incommensurability. However, it is in *In Prm.* VII 1205.9−1206.28 that Proclus develops the concepts of commensurability and incommensurability in greater detail[25].

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did not work this argument into his *Commentary on Euclid*. Nevertheless, it is significant for our investigation, as it highlights yet another reason to distinguish incommensurability from unlimitedness. See *ibid*.: "Let us look at some propositions from Book X of Euclid's *Elements*: Proposition 5: "Commensurable magnitudes have to one another the ratio which a number has to a number". Proposition 6: "If two magnitudes have to one another the ratio which a number has to a number, the magnitudes will be commensurable". Proposition 7: "Incommensurable magnitudes do not have to one another the ratio which a number has to a number". Proposition 8: "If two magnitudes do not have to one another the ratio which a number has to a number, the magnitudes will be incommensurable".

[25]. I quote the entire passage in the footnote and comment on specific lines in the text. The full passage from *In Prm.* VII 1205.1−1206.20 reads as follows: ἐπεὶ γάρ εἰσι καὶ ἐν ταύτῃ πολλαὶ δυνάμεις, αἱ μὲν ἀλλήλαις σύστοιχοι καὶ πρὸς τὸ αὐτὸ τέλος καὶ ἀγαθὸν ἀνατεινόμεναι, αἱ δὲ διαφέρουσαι κατά τε ὑπεροχὴν καὶ ὕφεσιν, ἐκείνας μὲν κατὰ τὴν ἰσότητα χαρακτηρίζεσθαι ῥητέον − τὸ γὰρ αὐτὸ ἀγαθὸν μέτρον ἐστὶν ἑκάστων· τὰ οὖν τῷ αὐτῷ ἀγαθῷ συνηνωμένα τῷ αὐτῷ μέτρῳ μεμέτρηται καὶ ἔστιν ἴσα ἀλλήλοις −, τὰς δέ γε μὴ συστοίχους ἀλλήλαις κατὰ τὸ ἄνισον πεποιῆσθαι τὴν πρόοδον θετέον, ἐπειδὴ τὰς μὲν ὑπερέχειν, τὰς δὲ ὑφεῖσθαι λέγομεν. ἀλλ' ἐπεὶ καὶ τῶν ἀνίσων τὰ μὲν σύμμετρα, <τὰ δὲ ἀσύμμετρα>, δῆλον ὅτι καὶ ταῦτα τοῖς θείοις ἐφαρμόσομεν, τὴν μὲν συμμετρίαν ἐφ' ὧν (10) τὰ δεύτερα συνανακεράννυται τοῖς πρὸ αὐτῶν καὶ ὅλων μετέχει τῶν κρειττόνων_− οὕτω γὰρ καὶ ἐν τοῖς συμμέτροις τὸ ἔλαττον ἐθέλει μέτρον κοινὸν ἔχειν πρὸς τὸ μεῖζον, ὅλον ἑκάτερον τοῦ αὐτοῦ μετροῦντος −, τὴν δὲ ἀσυμμετρίαν ἐφ' ὧν τὰ καταδεέστερα διὰ τὴν τῶν κρειττόνων ἐξῃρημένην ὑπεροχὴν μετέχει (15) μέν πως αὐτῶν, ὅλοις δὲ αὐτοῖς συνάπτεσθαι διὰ τὴν ἑαυτῶν ὕφεσιν ἀδυνάτως ἔχει· ταῖς γὰρ μερικαῖς αἰτίαις καὶ πολλοσταῖς ἀπὸ τῶν πρώτων ἀσύμμετρός ἐστιν ἡ πρὸς ἐκεῖνα κοινωνία καὶ ἡ πρὸς τὸ αὐτὸ νοητὸν ἀγαθὸν ἐκείνοις ἀνάτασις. καὶ ἔοικεν, εἰ τῶν ἐγκοσμίων ἐστὶ θεῶν ταῦτα συνθήματα, τὸ ἴσον (20) λέγω καὶ τὸ ἄνισον, εἰκότως ἐνταῦθα καὶ τὸ σύμμετρον ἀναφανῆναι τοῦτο καὶ τὸ ἀσύμμετρον· ἐν μὲν γὰρ τοῖς ἀσωμάτοις καὶ ἀΰλοις χώραν ἡ τούτων ἀντίθεσις οὐκ ἔχει, πάντων ἐκεῖ ῥητῶν ὄντων καὶ ἐν εἴδεσι καθαροῖς ὑφεστώτων, ὅπου δέ ἐστι καὶ ὑλικὸν ὑποκείμενον καὶ μίξις εἴδους καὶ ἀνειδέου τινός, εἰκότως καὶ συμμετρίας ἐστὶν ἐνταῦθα □καὶ ἀσυμμετρίας□ ἀντίθεσις, ὧν οἱ ἐγκόσμιοι προσεχῶς εἰσι συνεκτικοί, ψυχῶν καὶ σωμάτων, εἴδους καὶ ὕλης. εἰκότως ἄρα πέφυκεν ἐν αὐτοῖς κατὰ τὴν <τοῦ> ἀνίσου διαίρεσιν τὸ σύμμετρον καὶ ἀσύμμετρον, διὸ καὶ τούτων ὁ παρὼν ἐμνημόνευσε (30) λόγος. (1206) Ταῦτα μὲν οὖν καὶ εἰσαῦθις ἐπὶ πλέον ἀνασκεψόμεθα. νῦν δὲ ὅτι γεωμετρικῶς πρὸ τῶν ἀποδείξεων ὅρους παραλαμβάνει τοῦ ἴσου, τοῦ συμμέτρου, τοῦ ἀσυμμέτρου, λεκτέον· ἴσον μὲν λέγων τὸ τοῖς αὐτοῖς μέτροις μετρούμενον, σύμμετρον δὲ τὸ τῷ αὐτῷ μέτρῳ μετρούμενον, ἐὰν μὲν μεῖζον ᾖ, (5) πλεονάκις ἢ τὸ ἔλαττον, ἐὰν δὲ ἔλαττον, ἐλαττονάκις ἢ τὸ μεῖζον, ἀσύμμετρον δὲ τὸ διαιρούμενον εἰς ἴσα μὲν κατ' ἀριθμόν, ἄνισα δὲ κατὰ μέγεθος, εἰ μὲν ἔλαττον εἴη, δῆλον ὡς εἰς ἐλάττονα, εἰ δὲ μεῖζον,

Proclus introduces first equality and inequality as broad principles before coming to our notions (*In Prm*. VII 1204.24−28):

> Now, as we have said, equality and inequality are to be taken as penetrating throughout the whole of divinity in the cosmos, even if the proofs also suit physical equals and mathematic equals and those in reason-principles of soul and those in the intellectual forms[26].

As subclasses of the equal and the unequal, commensurability and incommensurability are also introduced. Unlike in the *Commentary on Euclid's* Elements, Proclus here first discusses what we might call vertical commensurability and incommensurability. He says (*In Prm.* VII 1205.9−28):

> But since even of those that are unequal some are commensurate and others incommensurate, it is plain that we must fit these concepts into the divine realm, (A) commensurability being taken as referring to those cases in which the secondary entities are mixed with those prior to them and participate in the whole of what is superior to them (for in this way also, in entities that are commensurate, the lesser desires to have a common measure with the greater, the same unit of measurement being applied to either of them as a whole); whereas (B) incommensurability applies in those cases where the lesser entities, by reason of the transcendent superiority of the greater, participate in them up to a point, but are unable to be joined to them as a whole because of their own inadequacy. For in the case of partial causes which are at many removes from the primal, communion with them is incommensurate and so is the striving upwards for them towards the same intelligible good, and it would seem, if these things are symbols of the encosmic gods, I mean the equal and the unequal, that it is reasonable that this "commensurate" and "incommensurate" should appear at this point. For among incorporeal and immaterial entities such an antithesis can have no place, since everything there is rationally related and has its basis in pure forms; but where there is also a material substratum and a mixture of form and a formless element, then it is reasonable that there should also be an antithesis of commensurability and incommensurability between those things which the encosmic gods maintain directly—souls and bodies, Form and Matter.

δῆλον ὡς εἰς μείζονα· τὸ μὲν □γὰρ□ ἔλαττον σμικροτέρων ἔσται μέτρων ἴσων (10) κατ' ἀριθμόν, τὸ δὲ μεῖζον μειζόνων· ἐὰν γὰρ λάβῃς τὴν πλευρὰν καὶ τὴν διάμετρον καὶ διέλῃς ἑκατέραν δίχα, δῆλον ὡς ἡ μὲν μεῖζον ἕξει τῶν μερῶν ἑκάτερον, ἡ δὲ ἔλαττον. ἐπ' ἀμφοτέρων δὲ τῶν συμμέτρων καὶ τῶν ἀσυμμέτρων ὁμοίως πρὸς μείζονα καὶ ἐλάττονα τῆς παραβολῆς οὔσης, ἐφ' ὧν μὲν τῶν (15) μέτρων τῶν αὐτῶν ὄντων, ἀλλ' ἢ πλεονάκις ἢ ἐλαττονάκις, ἐφ' ὧν δὲ εἰς ἴσα μὲν τῆς διαιρέσεως γιγνομένης, ἀλλ' ἢ εἰς μείζονα κατὰ τὸ μέγεθος ἢ εἰς ἐλάττονα, ὅπου μὲν κατὰ τὸ πλῆθος τῆς διαφορᾶς οὔσης, ὅπου δὲ κατὰ τὸ μέγεθος □αὐτῶν. καὶ γὰρ ὅπου μὲν κοινὸν μέτρον πάντα μετρεῖ, ὅπου δὲ οὐ δυνατὸν εἶναι πάντων ταὐτὸ μέγεθος□ ἀσυμμέτρων ὑπαρχόντων. τούτων δὴ τῶν ὅρων προειλημμένων ἑξῆς ἐπάγει τὰς ἀποδείξεις, δι' ὧν ἀποδείκνυται πρῶτον μὲν ὅτι οὐκ ἔστιν ἴσον τὸ ἓν οὔτε ἑαυτῷ οὔτε ἄλλῳ, ἔπειτα ὅτι οὔτε ἄνισον πάλιν οὔτε ἑαυτῷ οὔτε ἄλλῳ. λέγει δὲ οὕτως·

[26]. ἀλλ' ὥσπερ εἴπομεν, ἑνὸς ἰσότητα καὶ ἀνισότητα ληπτέον τὴν διὰ πάσης φοιτῶσαν τῆς ἐγκοσμίου θεότητος, εἰ καὶ ἐφαρμόζουσιν αἱ ἀποδείξεις καὶ τοῖς φυσικοῖς ἴσοις καὶ τοῖς μαθηματικοῖς καὶ τοῖς ἐν τοῖς λόγοις τοῖς ψυχικοῖς καὶ τοῖς ἐν τοῖς εἴδεσι τοῖς νοεροῖς.

This passage is highly complex, yet Proclus appears to maintain that commensurability and incommensurability coexist[27] when matter is involved. In the context of his argument, he must mean that these are manifested in the relation between the higher and lower realms within a given object. He does not seem to be referring here to the horizontal commensurability and incommensurability discussed in the commentary on Euclid. By contrast, incorporeal and immaterial entities possess only rational relations to one another, and incommensurability thus has no place among them[28]. From this passage, we might infer that the sensible-material realm is, as such, incommensurable. This would contradict the argument we tried to make about the difference between incommensurability and imprecision. However, the discussion of incommensurability in the *Commentary on the* Parmenides focusses on the relation between the higher and the lower realms. The lower realm is incommensurable insofar as it cannot reach the higher realm. This vertical incommensurability concerns not the entities of the lower realm themselves, but the relation those entities bear to the higher standard.

Incommensurability in Specific Entities

Simple Entities

Given the foregoing, neither lower matter nor intelligible matter can be the sole cause of incommensurability, as shown by the fact some geometrical entities do not exhibit it. Thus, we need to examine these entities more closely. Two straight lines of equal length are clearly commensurable with one another, even though they possess magnitude and extension and are, consequently, infinitely divisible. Thus, while infinitely divisible items exist within the geometrical realm, their ratios may or may not conform to numerical relations. The mere presence of infinity and magnitude does not, by itself, cause incommensurability, but it can create the conditions for incommensurability to arise.[29] At this point, the central question becomes what exactly is being related to what in this specific case. We could cut off a portion of a line such that it becomes incommensurable with another chosen line. But this would be arbitrary. Instead, we must shift the focus to a single geometrical entity and its internal proportions. When we speak of

[27]. (…) εἰκότως καὶ συμμετρίας ἐστὶν ἐνταῦθα □καὶ ἀσυμμετρίας□ ἀντίθεσις (…).
[28]. See Procl. *In Prm.* VII 1205.22−24: "For among incorporeal and immaterial entities such an antithesis can have no place, since everything there is rationally related and has its basis in pure forms" (ἐν μὲν γὰρ τοῖς ἀσωμάτοις καὶ ἀΰλοις χώραν ἡ τούτων ἀντίθεσις οὐκ ἔχει, πάντων ἐκεῖ ῥητῶν ὄντων καὶ ἐν εἴδεσι καθαροῖς ὑφεστώτων).
[29]. See Procl. *In Eucl.* 60.26−61.7.

incommensurability, we are speaking of a property that an item possesses within itself, not of a comparison between two randomly chosen magnitudes.

Let us examine more closely the causes and properties of geometrical entities. Commensurability and incommensurability seem to arise concretely from the interplay of the two principles: Limit and Unlimited. Proclus clarifies this when he explains that material extension is governed by the Unlimited, whereas the higher realm, which is free of extension, is governed by the Limit[30]. The interplay between Limit and Unlimited recurs at every level of being. Proclus states, in *In Eucl.* 151.13−152.5[31], that the *circular line* belongs to the realm and principle of the Limit[32]. Let us compare this with his account of the *straight line*, where he notes that the straight line belongs to the Unlimited (ἡ δὲ εὐθεῖα πρὸς τῆς ἀπειρίας)[33]. This prioritization of the circular over the straight line has significant implications for understanding incommensurability. The *circular line*, as the first among extended geometrical entities, functions as the formative and limiting cause of the *straight line*, such as the diameter. It also holds precedence over the semicircle, which it governs[34]. The circle's priority is further evidenced by its direct dependence on the point rather than on the straight line. When Proclus explains how both circular and straight lines are constructed, he traces each back to the movement of the point[35]. Moreover, he explicitly rejects the claim that the straight line is necessary for the construction of the circular line. I will quote the entire passage (*In Eucl.* 106.20−107.11) below, as it is important for the continuation of the argument:

> One might think that, although both the straight line and the circle are simple lines, the straight line is the simpler. For it contains not even any dissimilarity in thought, whereas concavity and convexity in the circle indicate difference; and the straight line does not suggest the circle, whereas the circular line does bring to mind the idea of the straight line, if not through its mode of generation, at least by its relation to a center. What,

[30]. See Procl. *In Eucl.* 85.14−86.4.

[31]. See Procl. *In Eucl.* 151.13−152.5: ἀλλὰ ταῦτα μὲν μέχρι τούτων διαπεπεράνθω, τὴν δὲ μαθηματικὴν ἀπόδοσιν τοῦ κύκλου θεωρήσομεν εἰς πέρας ἀκριβείας ἥκουσαν. σχῆμα μὲν οὖν αὐτὸν ἔθετο διότι δὴ πεπέρασται καὶ περιέχεται πανταχόθεν ὑφ' ἑνὸς ὅρου καὶ οὐκ ἔστι τῆς ἀπείρου φύσεως, ἀλλὰ τῷ πέρατι σύστοιχος, καὶ ἐπίπεδον δὲ αὖ, καθόσον τῶν σχημάτων ἢ ἐν ἐπιφανείαις ὁρωμένων ἢ ἐν στερεοῖς, ὁ κύκλος τῶν ἐπιπέδων ἐστὶ τὸ πρώτιστον, ἁπλότητι μὲν τῶν στερεῶν ὑπερφέρων, μονάδος δὲ πρὸς τὰ ἐπίπεδα λόγον ἔχων, ὑπὸ μιᾶς δὲ γραμμῆς περιεχόμε νον, ὡς τῷ ἑνὶ προσήκοντα καὶ κατὰ τὸ ἓν ἀφοριζόμενον, τὴν δὲ ποικιλίαν τῶν περικειμένων ἔξωθεν ὅρων οὐ παραδεχόμενον, πρὸς δὲ ταύτην τὴν γραμμὴν ἴσας ἔχοντα πάσας τὰς ἀφ' ἑνὸς σημείου τῶν ἐντὸς αὐτοῦ κειμένων, διότι καὶ τῶν ὑπὸ μιᾶς ὁριζομένων γραμμῆς τὰ μὲν ἐκ μέσου πάσας ἴσας ἔχει, τὰ δὲ οὐ πάσας.

[32]. See also its priority for motion in Procl. *In Eucl.* 17.21−22: πάντων γὰρ τῶν κινουμένων ὁ κύκλος ἀρχὴ καὶ ἡ κύκλῳ κίνησις.

[33]. See Procl. *In Eucl.* 107.15.

[34]. In contrast, the modern approach often seeks to measure the circle by means of an infinite number of triangles, gradually approximating the length of its circumference. Proclus, however, would likely reject this procedure, as it attempts to measure something that belongs to the domain of the Limit by means of something inferior, rooted in the Unlimited. Arguably, he would maintain that incommensurability arises from the inferiority of the straight line, whereas the circular line itself does not exhibit incommensurability. One can, of course, select two segments of a circle that are incommensurable with one another, but this is a matter of chance and has no bearing on the nature of the circle.

[35]. The straight line in Procl. *In Eucl.* 97.6−7: ἀφορίζονται δὲ αὐτὴν καὶ κατ' ἄλλας μεθόδους, οἱ μὲν ῥύσιν σημείου λέγοντες, and Proclus' approval in 13−14: ἡ δὲ ῥύσις τὴν πρόοδον ἐνδείκνυται καὶ τὴν γόνιμον δύναμιν….

then, if someone should say that the circle needs the straight line for its existence? For if one end of a finite line remains stationary and the other moves, it will describe a circle whose center is the stationary extremity of the straight line. Should we not reply that what describes the circle is not the line, but the point that moves about the stationary point? The line only defines its distance from the center, whereas what produces the circle is the point in circular movement[36].

This passage brings us closer to understanding why some geometrical entities exhibit incommensurability while others do not, as will become clear in what follows.

Complex Geometrical Entities

An intricate interplay across different ontological levels is emphasized, for instance, in *In Eucl.* 108−109. There, Proclus explains that the straight represents procession, while the round represents return. These principles govern not only spatial but also spiritual motion. Essentially, the circular line is associated with the Limit, whereas the straight line is associated with the Unlimited[37]. Therefore, the circle defines or limits, whereas the straight line, taken on its own, is indefinitely extensible. However, these principles interact in complex ways. As a diameter within a circle, the straight line depends on the limit imposed by the circle. Its unlimited extension is thus limited and made contingent upon it. With these two basic elements in view, we can now investigate how their interplay informs the construction of geometrical entities in the *phantasia*.

The circular line is not identical with the circular surface: while the circular line represents the Limit in one-dimensional space, the straight line represents the Unlimited in the same space[38]. Beginning with *In Eucl.* 115, we learn that in two-dimensional space another opposition arises – one crucial to our argument – namely, that between round and rectilinear figures (*In Eucl.* 115.3−8):

[36]. δόξειε δ' ἂν ἀμφοτέρων οὐσῶν ἁπλῶν τῶν γραμμῶν, τῆς εὐθείας καὶ τῆς περιφεροῦς, ἁπλουστέρα μᾶλλον ἡ εὐθεῖα εἶναι. ἐν ταύτῃ μὲν γὰρ οὐδὲ κατ' ἐπίνοιάν ἐστιν ἀνομοιότης, ἐπὶ δὲ τῆς περιφεροῦς τὸ κοῖλον καὶ κυρτὸν ἑτεροίωσιν ἐμφαίνει. καὶ ἡ μὲν εὐθεῖα τὴν περιφέρειαν οὐ συνεισάγει κατὰ τὴν ἐπίνοιαν, ἡ δὲ περιφέρεια τὴν εὐθεῖαν, εἰ καὶ μὴ κατὰ τὴν γένεσιν, ἀλλὰ κατὰ τὴν πρὸς τὸ κέντρον σχέσιν. τί οὖν, εἰ λέγοι τις καὶ τὴν περιφέρειαν δεῖσθαι τῆς εὐθείας εἰς τὴν ὑπόστασιν; εἰ γὰρ εὐθείας πεπερασμένης θάτερον μὲν τῶν περάτων μένοι, θάτερον δὲ κινοῖτο, γράψει κύκλον, κέντρον δὲ αὐτοῦ τὸ μένον τῆς εὐθείας πέρας. ἢ τὸ γράφον τὸν κύκλον τὸ σημεῖόν ἐστι περὶ τὸ μένον φερόμενον, οὐχ ἡ εὐθεῖα; τὴν γὰρ ἀπόστασιν αὕτη μόνον ἀφορίζει, τὴν δὲ κυκλικὴν γραμμὴν τὸ σημεῖον ὑφίστησι κινούμενον κυκλικῶς.

[37]. See Procl. *In Eucl.* 107.11−16: ἔοικεν δὲ ἡ μὲν περιφέρεια πρὸς τοῦ πέρατος εἶναι καὶ τοῦτον ἔχειν τὸν λόγον πρὸς τὰς ἄλλας γραμμάς, ὃν πρὸς πάντα τὰ ὄντα τὸ πέρας − καὶ γὰρ ὥρισται καὶ σχῆμα ἀποτελεῖ μόνη τῶν ἁπλῶν—ἡ δὲ εὐθεῖα πρὸς τῆς ἀπειρίας· ἐπ' ἄπειρον οὖν ἐκβαλλομένη οὐδὲ παύεται. See for the priority of circular lines O'Meara 2005, p. 140.

[38]. Proclus does not appear to address the fact that the circular line, though itself one-dimensional, necessitates two-dimensional space.

> For the circle, which is the principle of all curvilinear figures, carries a hidden trinity in its center, diameter, and circumference; and the triangle is the premier of all rectilinear figures, as everyone can see, because it is determined by the number three and formed by it[39].

The circle's surface is, furthermore, the most perfect of all surfaces[40]. This suggests, by analogy with the circular line, its affinity with the Limit in two-dimensional space. The triangle, by contrast, as the principle of the other class of figures, must bear a closer resemblance to the Unlimited in two-dimensional space. This is hinted at, e.g., in *In Eucl.* 214, where Proclus presents the circle as the cause of the construction of a triangle – analogous to the way the circular line gives rise to the straight line, expressed through its diameter[41].

Even though the triangle appears to represent the Unlimited, it remains unclear why some triangles do not exhibit incommensurability within themselves while others do[42]. For instance, the equilateral triangle does not exhibit incommensurability among its sides. Proclus distinguishes three types of geometrical triangles, each reflecting a different degree of proximity to the circle, and thus to the Limit. In *In Eucl.* 213 ff., we learn that the equilateral triangle is the most beautiful and the most akin to the circle[43]. The isosceles triangle stands somewhat further removed from it, while the

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[39]. ὁ μὲν γὰρ κύκλος, ὅς ἐστιν ἀρχὴ τῶν περιφερομένων, ἐν κρυφίῳ ἔχει τὸ τριαδικὸν τῷ κέντρῳ τῇ διαστάσει τῇ περιφερείᾳ· τὸ δὲ τρίγωνον ἁπάντων ἡγεμονοῦν τῶν εὐθυγράμμων παντί που δῆλον, ὅτι τῇ τριάδι κατέχεται καὶ κατ' ἐκείνην μεμόρφωται.

[40]. See Procl. *In Eucl.* 146.24−147.3: "The first and simplest and most perfect of the figures is the circle. It is superior to all solid figures because its being is of a simpler order, and it surpasses other plane figures by reason of its homogeneity and self-identity. It corresponds to the Limit, the number one, and all the things in the column of the better" (τὸ πρώτιστον καὶ ἁπλούστατον τῶν σχημάτων καὶ τελειότατον ὁ κύκλος ἐστί. τῶν μὲν γὰρ στερεῶν πάντων ὑπερφέρει τῷ ἐν ἁπλουστέρᾳ τάξει τὴν ὕπαρξιν ἔχειν, τῶν δὲ ἐν τοῖς ἐπιπέδοις ὑφισταμένων τῇ ὁμοιότητι καὶ ταυτότητι τὴν ὑπεροχὴν ἔλαχεν).

[41]. Procl. *In Eucl.* 214.4−8: "If, then, the circle is the likeness of intelligible being, and the triangle the likeness of the first soul because of the similarity and equality of its angles and its sides, it would seem reasonable to demonstrate it by means of circles as an equilateral middle area cut off in them" (εἰ τοίνυν ὁ μὲν κύκλος εἰκών ἐστι τῆς νοερᾶς οὐσίας, τὸ δὲ τρίγωνον τῆς πρωτίστης ψυχῆς διά τε τὴν ἰσότητα καὶ τὴν ὁμοιότητα τῶν γωνιῶν καὶ πλευρῶν, εἰκότως ἂν καὶ τοῦτο διὰ τῶν κύκλων ἐν αὐτοῖς μέσον ἀπολαμβανόμενον ἰσόπλευρον ἀποδεικνύοιτο).

[42]. Clearly, even though the triangle projected onto the soul represents the Unlimited, its distance from the first cause renders it a mixed entity. By this, I mean that the triangle, possessing a three-sided boundary, also exhibits features of the Limit. See Procl. *In Eucl.* 136; 143. See Procl. *In Eucl.* 142.24−143.5: "Since figure too, like the idea of the angle, exhibits in its own subdivisions the twofold progression of the Limit and the Unlimited, it applies the single boundary and the simple form to the things it bounds when it acts in accordance with the Limit and the many boundaries by virtue of the Unlimited. This is why everything figured has either one or more than one boundary" (ἐπεὶ γὰρ καὶ αὐτὸ τὴν τοῦ πέρατος καὶ ἀπείρου δυοειδῆ πρόοδον ἐν τοῖς οἰκείοις εἴδεσι προτείνει, καθάπερ δὴ καὶ ὁ τῆς γωνίας λόγος, τὸν μὲν ἕνα ὅρον καὶ τὸ ἁπλοῦν εἶδος ἐπάγει τοῖς ὑφ' ἑαυτοῦ περιεχομένοις κατὰ τὸ πέρας, τοὺς δὲ πολλοὺς κατὰ τὴν ἀπειρίαν. καὶ διὰ τοῦτο πᾶν τὸ ἐσχηματισμένον ἢ ἑνὸς ὅρου μετείληφεν ἢ πλειόνων). See also Procl. *in Eucl.* 144.6−18: "But whence comes the idea of figure and from what sort of principles is it perfected? I answer, first, that it owes its being to the Limit and the Unlimited and the Mixture of the two. This is why it generates some kinds by virtue of the Limit, others by reason of the Unlimited, and others according to the Mixed. For circular figures it invokes the idea of the Limit, for rectilinear that of the Unlimited, and for figures derived from both the idea of the Mixed" (ἀλλὰ πόθεν πρόεισιν ὁ τοῦ σχήματος λόγος καὶ ἀπὸ ποίων αἰτίων τελειοῦται; λέγω δή, ὅτι πρῶτον μὲν ἐκ τοῦ πέρατος ὑφίσταται καὶ τοῦ ἀπείρου καὶ τοῦ ἐκ τούτων μεμιγμένου, διὸ καὶ αὐτὸς τὰ μὲν ἀπογεννᾷ κατὰ τὸ πέρας τῶν εἰδῶν, τὰ δὲ κατὰ τὸ ἄπειρον, τὰ δὲ κατὰ τὸ μικτόν, τοῖς μὲν περιφερέσι τὴν τοῦ πέρατος ἰδέαν ἐπάγων, τοῖς δὲ εὐθυγράμμοις τὴν τοῦ ἀπείρου, τοῖς δὲ ἐκ τούτων τὴν τοῦ μικτοῦ).

[43]. See O'Meara 2005, p. 140.

scalene triangle occupies the opposite end of this spectrum, being furthest from the circle. It is in these latter two cases that incommensurability between sides may arise.[44] To be sure, incommensurability may also appear when we arbitrarily compare one geometrical entity with another. However, there are only a limited number of geometrical entities which display incommensurability in themselves and by themselves.

The Form of Incommensurability or the Display of Infinity?

So far, it remains unclear why only some triangles display incommensurability within themselves while others do not. We learned from Proclus that the task of the geometer's *phantasia* is to make visible what the understanding cannot perceive[45]. The entities that exist in an unextended manner, without shape or magnitude in the higher realm, nonetheless have their inherent proper parts. Though unextended and shapeless, the diameter and other elements are present within the form of the circle. Analogously, the angle and the limits of the triangle must likewise be present in an unextended manner within the form of the triangle. But what of incommensurability itself? Could it also be a property like these others? If incommensurability is coextensive with infinity, then clearly infinity is beyond the grasp of the imagination. It suspends any thought about what is immeasurable and thus incomprehensible[46]. It cannot traverse the infinite[47]. Knowing the infinite is akin to seeing darkness in the dark[48]. However, it seems that incommensurability should not be considered coextensive with infinity. In *In Eucl.* 278.19−24, we learn that:

> For when geometers demonstrate that there is incommensurability among magnitudes and that not all magnitudes are commensurable with one another, what else could we say they are demonstrating than that every magnitude is divisible indefinitely and that we can never reach an indivisible part which is the least common measure of magnitudes[49]?

[44]. The elementary triangles of the *Timaeus* which make up the regular bodies are of the latter two classes, as Proclus acknowledges. See Procl. *In Eucl.* 383.17−384.4: "(…) the half-square; and that the scalene half-triangle (…) I do not mention these matters without a purpose, but because they prepare us for the teaching of the *Timaeus*".
[45]. See Procl. *In Eucl.* 55.23−56.8.
[46]. See Procl. *In Eucl.* 285.10−13.
[47]. See Procl. *In Eucl.* 285.18−19.
[48]. See Nikulin 2008, p. 170.
[49]. ὅταν γὰρ δεικνύωσιν ὅτι ἔστιν τὸ ἀσύμμετρον ἐν τοῖς μεγέθεσι καὶ οὐ πάντα σύμμετρα ἀλλήλοις, τί ἄλλο δεικνύναι φήσει τις αὐτοὺς ἢ ὅτι πᾶν μέγεθος εἰς ἀεὶ διαιρεῖται καὶ οὐδέποτε ἥξομεν εἰς τὸ ἀμερές, ὅ ἐστι κοινὸν μέτρον τῶν μεγεθῶν ἐλάχιστον;

We learn that, although all geometrical magnitudes are infinitely divisible, some are nevertheless incommensurable[50]. Infinity and incommensurability are therefore not coextensive. This allows us to identify in incommensurability a different kind of explanation – one that points toward its being something definable. A passage from Proclus' *Commentary on the* Parmenides moves in this direction and offers the following definition of incommensurability. In *In Prm*. VII 1206.4−8, he states:

> (…) commensurate is "that which is measured by the same measure − if it is greater, more times than less, if it is less, less times than the greater"; and the incommensurate is "that which is divided into parts which are equal numerically, but are unequal in size"[51].

It is not common to speak of a form of incommensurability, yet in the context of other forms that Proclus defines in his *Commentary on the* Parmenides, this may be a valid explanation. Likeness and unlikeness, for example, are defined in terms of identity and otherness[52] – and both are forms. Equality and inequality are likewise defined as deriving from identity and otherness[53]. As we have seen above, commensurability and incommensurability appear immediately after Proclus' and Plato's discussion of equality and inequality, and they too are directly defined in terms of identity, equality, and inequality. Thus, even though these notions apparently belong to a lower rank within the realm of mathematical entities – insofar as they are dependent on these other forms – incommensurability, like unlikeness, seems to possess its own definable form. Proclus does not call it a form of incommensurability; therefore, this interpretation is only valid insofar as we accept the analogy with likeness and unlikeness. If likeness and unlikeness are definable in terms of identity and otherness and are themselves forms, then commensurability and incommensurability, too, would appear to be definable forms, though of a lower rank within the two Pythagorean columns. As in the case of likeness and unlikeness, this does not imply that the form of incommensurability

[50]. The notion of infinity discussed a little later in Procl. *In Eucl.* 285−287 also appears to concern infinite extension. See Procl. *In Eucl*. 285.15−24: "It produces it (i.e. the infinite, DW) indeed, because it has an indivisible power of proceeding without end, and it knows that the infinite exists because it does not know it. For whatever it dismisses as something that cannot be gone through, this it calls infinite. So if we supposed the infinite line to be given in imagination, exactly like triangles, circles, angles, lines, and all the other geometrical figures, should we not ask in wonder how a line can actually be infinite and how, being indeterminate, it associates with determinate notions?" (γεννᾷ μὲν οὖν αὐτὸ τῷ δύναμιν ἔχειν ἀμερῆ προϊέναι δυναμένην ἀκαταλήκτως, νοεῖ δὲ ὡς ὑποστὰν ὅτι μὴ νοεῖ τὸ ἄπειρον. ὃ γὰρ ἀφῆκεν ὡς ἀδιεξίτητον, τοῦτο ἄπειρον λέγει, ὥστε τὴν ἄπειρον γραμμὴν τὴν δοθεῖσαν ἐν τῇ φαντασίᾳ θέμενοι, καθάπερ δὴ καὶ τὰ ἄλλα πάντα γεωμετρίας εἴδη, τὰ τρίγωνα, τοὺς κύκλους, τὰς γωνίας, τὰς γραμμάς, οὐ θαυμασόμεθα, πῶς κατ' ἐνέργειαν ἄπειρός ἐστι γραμμὴ καὶ προστίθησιν ἀορισταίνουσα ταῖς ὡρισμέναις νοήσεσιν).

[51]. (...) σύμμετρον δὲ τὸ τῷ αὐτῷ μέτρῳ μετρούμενον, ἐὰν μὲν μεῖζον ᾖ, πλεονάκις ἢ τὸ ἔλαττον, ἐὰν δὲ ἔλαττον, ἐλαττονάκις ἢ τὸ μεῖζον, ἀσύμμετρον δὲ τὸ διαιρούμενον εἰς ἴσα μὲν κατ' ἀριθμόν, ἄνισα δὲ κατὰ μέγεθος.

[52]. See d'Hoine 2014, pp. 1–37.

[53]. Morrow – Dillon 1987, p. 481: "Apart from showing that 'equal' and 'unequal' follow on logically from sameness and otherness, he is chiefly here concerned with the concept of 'measure' and commensurability which Plato introduces here. First, as he notes (1206.1ff.), Plato produces definitions of 'equal', 'unequal', and 'incommensurable', in a proper geometrical manner, before going on to his proofs".

must itself be incommensurable. Rather, analogously to likeness and unlikeness, commensurability and incommensurability are forms of relatives that are themselves not relative. They are "non-relational, though productive of a relation" (ἄσχετος… ἐστιν ὑπόστασις ἐκεῖ, σχέσεως ὑποστατική τινος, *In Prm*. IV 936.25−26)[54]. If incommensurability is analogous to unlikeness – a property that appears as extended in its proper place while depending on a form – and if it is neither merely an "effect" of matter[55] nor identical with infinity, then it can be displayed in the geometer's *phantasia*. In the same way as the diameter in the circle, incommensurability appears as an "element" – and it can be shown in an extended way in the *phantasia* by the triangles in which it is present.

What remains unclear is why there is only one form of the triangle, which nevertheless differentiates into geometrical triangles – some displaying incommensurability, others not. I have not found explicit textual evidence for the claim I wish to propose, but its systematic basis seems to be the following. As noted above, Proclus distinguishes three kinds of geometrical triangles, the equilateral, the isosceles, and the scalene. The last one of these is said to be the most remote from the circle, which I take to mean that it represents the triangle in its purest form. Now, every triangle – whether isosceles, equilateral, or scalene – can be divided by a line drawn from the apex to the base, into two right-angled triangles. Most right-angled triangles exhibit incommensurability, while its absence appears to be the exception (cf. Pythagorean triples). This comes close to the way Plato describes the two elementary triangles in the *Timaeus*.

Conclusion

The argument I have laid out shows that incommensurability in Proclus' philosophy is a complex issue. Scholarly literature rightly emphasizes that intelligible matter plays a crucial role in its manifestation within geometrical entities, just as sensible matter accounts for depotentiation, magnitude, and imprecision in sensible entities. However, we have seen that intelligible matter cannot be the sole cause of the appearance of incommensurability. Some objects display this feature within the geometrical "realm" when projected onto the *phantasia* of intelligible matter, while others do not. The reason, it seems, is that Proclus acknowledges a form of incommensurability – one that is defined and constitutes an inherent of certain entities.

[54]. See d'Hoine 2014, p. 22 in the context of a thorough discussion of Likeness and Unlikeness in Proclus.
[55]. See two forms of dissimilarity, d'Hoine 2014, pp. 31−2, referring to *In Prm.* II 748.12: "The one derives its character from above, the other springs from the indeterminateness of matter (…)".

Bibliography

G. Bechtle, *Die pythagoreisierende Konzeption der Mathematik bei Iamblichos, Syrianos und Proklos. Im Spannungsfeld zwischen pythagoreischer Transposition und platonischer Mittelstellung*, in M. Perkams – R. M. Piccione (eds.), *Proklos. Methode, Seelenlehre, Metaphysik*, Leiden, Brill, 2006, pp. 323−40.

P. d'Hoine, *The Forms of Likeness and Unlikeness in Proclus*, in «Proceedings of the Boston Area Colloquium of Ancient Philosophy», 29 (2014), pp. 1−37.

V. Karasmanis, *Continuity and Incommensurability in Ancient Greek Philosophy and Mathematics*, in G. Anagnostopoulos (ed.), *Socratic, Platonic and Aristotelian Studies: Essays in Honor of Gerasimos Santas*, Dordrecht / Heidelberg, Springer, 2011, pp. 389−99.

G. R. Morrow (ed.), *Proclus. A Commentary on the First Book of Euclid's* Elements, Princeton, Princeton University Press, 1970.

G. R. Morrow – J. M. Dillon (eds.), *Proclus' Commentary on Plato's* Parmenides, Princeton, Princeton University Press, 1987 (1992^2).

D. Nikulin, *Imagination and Mathematics in Proclus*, in «Ancient Philosophy», 28 (2008), pp. 153−72.

D. Nikulin, *Neoplatonism in Late Antiquity*, Oxford / New York, Oxford University Press, 2019.

D. O'Meara, *Geometry and the Divine in Proclus*, in T. Koetsier – L. Bergmans (eds.), *Mathematics and the Divine*, Amsterdam, Elsevier Science, 2005, pp. 133−45.

J. Opsomer, *The Integration of Aristotelian Physics in a Neoplatonic Context: Proclus on Movers and Divisibility*, in R. Chiaradonna – F. Trabattoni (eds.), *Physics and Philosophy of Nature in Greek Neoplatonism*, Leiden / Boston, Brill, 2009, pp. 189–229.

ABSTRACT

Denis Walter, *Incommensurability in Geometrical Entities in Proclus'* In Eucl. *and* In Prm.

This chapter investigates the origin of incommensurability within Proclus's ontology through an analysis of his *Commentary on the First Book of Euclid's* Elements and his *Commentary on the* Parmenides. The central question concerns how incommensurability arises at a specific ontological level and how it corresponds to a distinct mode of cognition. The chapter first situates geometrical entities in an intermediate position between the intelligible and the sensible realms. It then demonstrates that sensible matter cannot serve as the unqualified cause of incommensurability, since it accounts only for imprecision rather than for incommensurability. Likewise, intelligible matter (the soul) alone cannot explain it, as not all objects within the soul exhibit incommensurability. Instead, the chapter argues that incommensurability is an intrinsic property of certain geometrical entities, comparable to their other identifiable features. Ultimately, the essay supports the interpretation that incommensurability is a definable and necessary feature of some geometrical objects.

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THE UNFOLDED SYNTAX OF THE INTELLECT: PROCLUS ON THE METAPHYSICAL FOUNDATIONS OF ELEMENTARY ARRANGEMENTS

> διὰ γὰρ τὴν ἀλλήλων τῶν εἰδῶν συμπλοκὴν ὁ λόγος γέγονεν ἡμῖν (Pl. *Sph*. 259e5−6).

One of the most remarkable aspects of Proclean epistemology is its account of scientific knowledge. Although human beings are not granted immediate access to eternal realities through their natural faculties alone, Proclus maintains that human sciences, such as theology and mathematics, nonetheless concern such beings.

As is widely known, Proclus explains human temporal engagement with eternal beings by posing that expressions of noetic realities are found within the substance of the soul. The soul accesses these expressions of noetic beings by turning toward its eternal substance, thereby unfolding discursive images of its innate imprints of the Intellect, or "reason-principles", through its temporal powers[1]. This dianoetic activity prepares the soul for the higher cognitive act of intellection (νόησις) − or even for that which lies beyond the soul[2]. Proclus' epistemology of discursive sciences and their apodeictic capacities raises several puzzles. This chapter explores these issues, focusing particularly on Proclus' views concerning the logical structures of dialectic and mathematics, their origins, relationship, and their metaphysical grounding. Its primary aim is to elucidate the metaphysical foundations underlying the logical structure exhibited by dialectic and mathematics and, on this basis, to trace Proclus' conception of the logical and doctrinal continuity among the contemplative sciences through the case study of "common mathematics".

Section 1 examines some aspects of Proclus' philosophy of language, focusing on the idea that the demonstrative structures of dialectic and mathematics are an "icon" or "intellectual paradigm" expressing the architecture of Intellect. Section 2 dwells on the justifications and consequences of this view, scrutinising the relationships between Intellect, the logical methods of dialectic and mathematics, and their elementary syntaxes. Section 3 applies the insights developed

[1]. Procl. *El. Theol.* 191: "Every participated soul has an eternal substance (οὐσίαν αἰώνιον) but a temporal activity (ἐνέργειαν κατὰ χρόνον)". See also Helmig 2016, pp. 183−296; Lernould 2021; Chlup 2012, chapter 4; Steel 1997; O'Meara 2017; O'Meara 1989, chapter 8, 9, and 10; Cleary 1998, pp. 85−102; MacIsaac 2001; MacIsaac 2002.

[2]. On Proclus' views on higher modes of intellection, see Vargas 2025. See also Lernould 2016; MacIsaac 2014; MacIsaac 2010.

in the preceding sections to explore how, according to Proclus, a seamless logical transition between mathematics and theology can be achieved through the intermediary discipline of common mathematics. In this context, a formalisation of propositions 1−4 of the *Elementatio theologica* is proposed, showing them to be deducible from Euclid's arithmetic definitions (Book VII, def. 1 and 2). Finally, Section 4 offers concluding reflections, highlighting how Proclus' foundations of logical functions reveal that the composition of *elementationes* was more than a matter of literary form for the Diadochus: it embodied deep philosophical commitments.

Proclus' Philosophy of Language and the Intellectual Paradigm

In the *Platonic Theology*, Proclus provides some details regarding the science of dialectic, conceived as a demonstrative discourse about divine being (*Theol. Plat.* I 10.45.20−46.9):

> The initial conclusions are immediately evident through very simple and familiar common notions; those that follow are demonstrated through more steps and variations, and the final ones are the most complex. He [Plato] always uses the initial conclusions as demonstrations in his arguments and presents the mutual connection amongst conclusions in the order of geometry or other mathematical disciplines as an intellectual paradigm. If, therefore, definitions bear the image of the things of which they are the explanations, and if, as is the unfolding of demonstrations, such must the order necessarily be of the things demonstrated, then it is necessary, I think, that the things beginning from the most simple principles should be absolutely the most primary and arranged in connection with the One, whereas those that are more numerous and dependent on various demonstrations have come after a considerable interval from the One, if it is permissible to speak in this way[3].

Proclus begins by reflecting on the inferential structures produced by the dialectical chains of demonstrations as found in dialectic. He expresses his views that (i) Plato conducts his arguments according to the method that geometers *also* follow, namely by producing concatenations of proofs that depend on previously demonstrated theorems (this "also" will prove of the utmost importance). As Martijn has shown, *In Timaeum* and *In Parmenidem* provide clear

[3]. τὰ μὲν πρῶτα τῶν συμπερασμάτων δι'ἐλαχίστων ὡς οἷόν τε καὶ ἁπλουστάτων καὶ γνωριμωτάτων καὶ οἷον κοινῶν ἐννοιῶν εὐθὺς κατάδηλα γίνεται, τὰ δὲ τούτων ἐφεξῆς διὰ πλειόνων καὶ ποικιλωτέρων, τὰ δὲ ἔσχατα παντελῶς ἐστὶ συνθετώτατα. χρῆται γὰρ [ὁ Πλάτων] ἀεὶ τοῖς πρώτοις συμπεράσμασιν εἰς τῶν ἐχομένων ἀποδείξεις καὶ τῆς ἐν γεωμετρίᾳ τάξεως ἢ τοῖς ἄλλοις μαθήμασι παράδειγμα προτείνει νοερὸν τὴν τῶν συμπερασμάτων τούτων πρὸς ἄλληλα συνάρτησιν. εἰ τοίνυν οἱ λόγοι τῶν πραγμάτων εἰκόνα φέρουσιν ὧν εἰσὶν ἐξηγηταὶ καὶ ὡς ἔχουσιν αἱ [ἀπὸ] τῶν ἀποδείξεων ἀνελίξεις, οὕτως ἀνάγκη καὶ τὰ δεικνύμενα τάξεως ἔχειν, ἀναγκαῖον οἶμαι τὰ μὲν δι'ἁπλουστάτων ἀρχόμενα πάντως ἀρχοειδέστατα καὶ τῷ ἑνὶ συνενωμένα τετάχθαι, τὰ δὲ ἀεὶ πληθυόμενα καὶ ποικίλων ἀποδείξεων ἠρτημένα πορρώτερον προληλυθέναι τῆς τοῦ ἑνὸς ἀποστάσεως, εἰ θέμις εἰπεῖν. See Solère 2003; Opsomer 2021; Opsomer 2020; Opsomer 2012; Kiosoglou 2022; Nikulin 2003. Unless otherwise indicated, the English translations are mine.

examples of Proclus applying this conviction in his hermeneutics and seeking geometrical-like arrangements within the prose of the divine Plato[4]. To this, Proclus adds an important observation: (ii) that the order and intertwining of propositions constitutes an "intellectual paradigm" (παράδειγμα νοερόν). In other words, the inferential arrangement of theorems in dialectic (much as in mathematics) is itself an image of certain intellectual traits.

This implies that the different modes of theology described by Proclus (*scil*., entheastic, symbolic, mythical, and dialectical) overlap in certain cases[5]. Such an observation accords with Proclus' broader doctrine that (some) linguistic items provide insight into the realities they signify not through direct reference, but through representation − for instance, through etymologies or analogical naming. Proclus maintains that this also applies to complex linguistic items. For example, he holds that the dramatic structure of Plato's dialogues conveys metaphysical teachings. Similarly, in his commentary *In Timaeum*, Proclus accepts Socrates' claim that Timaeus' (demonstrative) cosmogony is, at the same time, a hymn (νόμος) to the Demiurge[6]. Once again tracing overlapping theological insights within a linguistic composition through the iconic function of language, Proclus suggests that the arrangement of demonstrations in dialectic and mathematical speculation reflects certain noetic relations.

How is this supposed to work?

Proclus provides some insight into this issue by introducing a distinctive theory of language, which he derives by fusing two Platonic passages. The first comes from the *Timaeus* (29b3−5), where Plato points to a kinship between explanatory λόγοι and the things of which they are expositors (ἐξηγηταί, συγγενεῖς) − the latter being both exemplars and their images. The second passage is from the *Cratylus* (439a1−3), where Plato suggests that names are images of things (εἰκόνας τῶν πραγμάτων). Combining lexical and philosophical themes from both passages, Proclus formulates a theory of iconic language that unfolds on three levels of complexity: definitions, demonstrations, and elementary arrangements.

First (1), Proclus assumes that explanatory λόγοι bear a likeness to the things they describe. I take λόγος here to mean "definition", although apophantic propositions, such as those employed in demonstrations, could also fit. Proclus' doctrine appears to be that, insofar as a real definition or a true proposition conveys the nature of an entity, its subject-predicate relationship is an image which, in some respects, discursively expresses the defined entity.

4. Martijn 2014.
5. Procl. *Theol. Plat.* I 4.17.9−23.10; See Gersh 2005. I wish to thank Prof. Gersh for bringing this point to my attention.
6. Pl. *Ti.* 27b9; Procl. *In Ti*. II 218.16ff. Van Riel (= I 355.4ff. Diehl); Martijn 2010, p. 11ff. See *Theol. Plat.* V 20.65.10ff.

Second (2), as a consequence of this view, Proclus claims that the arrangement of demonstrations is likewise an image of the ontological order (τάξις) among the referenced entities. He seems to think that if the elements of demonstrations (say, definitions) are images of the defined beings, then the resulting composition must also express the relationships among those beings. Arguably, this means that the way terms are arranged and connected within a syllogism reflects some structural features of the extra-logical being(s) the demonstration deals with.

Third (3), Proclus extends the notion of likeness between logical structures and the metaphysics of the entities they concern to the entire (actual or possible) dialectical system of theorems. He articulates this in a way reminiscent of both his understanding of the series of deductions from the second hypothesis in the *Parmenides*, as the unfolding of the noetic orders, and the layout of his *Elementatio theologica*, which proceeds deductively from the One-itself to the descended soul. Proclus notes that demonstrations concerning prior beings are more directly connected to (propositions about) the One, whereas derivative entities will be logically proven in advanced demonstrations that rely on the previously established theorems. Thus, the demonstrative system of dialectic functions as an informative truth-bearer in two senses: (a) in a purely denotative sense, according to the content expressed in its conclusions; and (b) iconically, insofar as the entities involved in the demonstration are intertwined in a way whose relations of priority and posteriority are discursively represented through the logical structure of dialectic.

In the following sections, we shall explore in greater detail the philosophical justifications and implications of this view, and consider how this aspect of Proclus' philosophy of language – when applied to elementary arrangements – may deepen our understanding of the *Elementatio theologica* and illuminate the connections between dialectic and mathematics.

Unwrapping Intellectual Syntax: The Logical Methods

What are the intellectual traits of the geometrical-like inferential order of dialectic that make it an intellectual paradigm? Whence do such traits come, and how are they philosophically justified? Part of the answer lies in a passage from Proclus' *In Cratylum* (III 5.1−8):

> For Intellect is the projector of dialectic. From itself, as a whole, it generates the entirety of dialectic; and through the progression of all things from the One, it establishes division; through the gathering of each thing into the comprehension of a single property, it establishes definition; through the mutual presence of Forms,

whereby each thing both is what it is and participates in the other Forms, it establishes demonstration; and through the return of all things to the One and to their proper principles, it generates analysis[7].

In this passage, Proclus refers to Intellect as the "projector" (προβολεύς) of dialectic. This designation carries epistemological and metaphysical implications, as the verb προβάλλειν and its derived noun προβολή are employed by Proclus to denote the act of "putting forward" or "projecting" a further reality from a higher principle[8]. As the former passage of the *In Cratylum* concerns the science of dialectic, it is sensible to interpret the term προβολεύς there in a similar sense: it designates the process by which a superior reality is expressed through discursive powers – metaphysically translated into the temporal activity of the soul. Therefore, Proclus maintains that Intellect is projected as the logical powers that enable the soul to engage in demonstrative theology: human science can be understood as the soul's manifestation of its inner reasons according to the distinctive mode of being proper to each kind of knowledge (i.e., metaphysical in dialectic, quantitative in mathematics).

However, it is remarkable that, once again, instead of merely highlighting the doctrinal contents of dialectic as displayed ensouled reasons, Proclus also declares that its logical operations – division, definition, demonstration, and analysis – are likeness-bearers of the Intellect. This detail captures a well-known aspect of his philosophy that is crucial for grasping his notion of the elementary arrangement as an intellectual paradigm. The ontological relations of priority and dependence among intellegible beings translate into determined logical relationships; conversely, the logical connections of dialectic unfold and represent the metaphysical relationships that exist in an undifferentiated manner within the Intellect[9]. Therefore, as the dianoetic expressions of intellection, the methods of dialectic disclose a discursive image of the compressed architecture of Intellect in the form of a web of theorems.

We can illustrate this through Proclus' examples. When the dialectician engages in analysis and determines that a prior proposition must be postulated for the analyzed proposition to hold true, his inferential movement mirrors the activity of (self-)reversion (ἐπιστροφή) within the noetic being under investigation. In this way, the dialectician, through his inferential process, retraces the dynamic ascent toward ontologically and logically prior principles, thereby uncovering a higher noetic being as a necessary, both logical and ontological, precondition of the one under study.

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[7]. ὅτι νοῦς ἐστιν ὁ τῆς διαλεκτικῆς προβολεύς. ἀφ'ἑαυτοῦ ὅλου ὅλην αὐτὴν ἀπογεννῶν· καὶ κατὰ μὲν τὴν ἀφ'ἑνὸς πάντων πρόοδον τὴν διαιρετικὴν ὑφίστησι, κατὰ δὲ τὴν συναγωγὴν ἑκάστου πρὸς μίαν ἰδιότητος περίληψιν τὴν ὁριστικὴν ὑφίστησι· κατὰ δὲ τὴν ἐπ'ἄλληλα παρουσίαν τῶν εἰδῶν, δι'ἣν καὶ ἔστιν ἕ ἐστιν ἕκαστον καὶ μετέχει τῶν λοιπῶν εἰδῶν, τὴν ἀποδεικτικήν· κατὰ δὲ τὴν ἐπιστροφὴν πάντων εἰς τὸ ἓν καὶ τὰς οἰκείας ἀρχὰς τὴν ἀναλυτικὴν γεννᾷ. See also Procl. *Theol. Plat.* I 9.40.5−10.

[8]. Helmig 2012, pp. 291−9.

[9]. Gersh 1973, p. 74ff.

Conversely, when the dialectician uses division, he is logically retracing the outpouring of power (πρόοδος) from a given Form, following its descent from its source to its effects. This movement proceeds from unity and simplicity toward multiplicity and complexity, as the further determinations of a genus are identified and progressively articulated.

In sum, the logical functions that weave together the theoretical corpus of dialectic present an unfolded image of the unitary interconnection of intellegible beings. Moreover, every discourse that employs these logical methods (see below) tend to converge toward an elementary arrangement – the very paradigm of discursive intelligibility.

What, then, of the axiomatic systems of mathematics? In what sense can they be regarded as images of the relationships that obtain among intellectual beings? To answer this, we must first understand how Proclus conceives the relationship between Intellect, dialectic, and mathematics. He addresses this issue in his commentary *In Euclidem* (*In Eucl.* 44.9−23):

> But even higher than it [*scil*. common mathematics, see below], dialectic could be said to be the bond of union among the mathematical sciences or, to repeat Plato's designation in the *Respublica*, their capstone. For this perfects common mathematics and sends it up towards Intellect by means of its peculiar powers, showing that it is truly a science and rendering it steadfast and irrefutable. Yet highest in rank amongst the unifying bonds is that very Intellect which contains in itself all dialectical powers in undifferentiated fashion, combining their variety in simplicity, their partiality in completeness of insight, their plurality in unity. Intellect, then, wraps up the developments of the dialectical methods, binds together from above all the discursiveness of mathematical reasoning, and is the perfect terminus of the upward journey and of the activity of knowing[10].

This section reveals some important details concerning Proclus' theory of discursive science.

First, it shows how the cascade of powers flowing from the noetic realm is mirrored in the soul as a progression of cognitive capacities or sciences. These powers reside initially within Intellect, where the Forms coexist in a unified manner (μονοειδῶς), beyond all discursivity. From there, they are transmitted to the activity of the soul and become differentiated as the logical capacities of dialectic. In turn, these dialectical powers generate (or can be employed as) lower cognitive powers, giving rise to diluted images of themselves. The offspring of dialectic stands as an intermediary science bridging noetic and sensible beings: mathematics. Just as dialectic originates from and reverts to

[10]. ἀνωτέρω δ'ἔτι ταύτης ἡ διαλεκτικὴ τῶν μαθημάτων ἂν εἴη σύνδεσμος, ἢ καὶ θριγχὸν αὐτῶν, ὡς ἔφην, ἐν πολιτείᾳ προσείρηκεν. αὕτη γὰρ καὶ τὴν ὅλην μαθηματικὴν τελειοῖ καὶ εἰς νοῦν ἀναπέμπει ταῖς ἑαυτῆς δυνάμεσι, καὶ ἐπιστήμην ὄντως ἀποφαίνει καὶ μόνιμον καὶ ἀνέλεγκτον ἀπεργάζεται. τρίτην δ'ἂν ἔχοι τάξιν ἐν τοῖς συνδέσμοις ὁ νοῦς αὐτὸς ὁ πάσας τὰς διαλεκτικὰς δυνάμεις ἐν ἑαυτῷ μονοειδῶς περιέχων καὶ τὴν ποικιλίαν αὐτῶν διὰ τῆς ἁπλότητος καὶ τὸν μερισμὸν διὰ τῆς ἀμεροῦς γνώσεως καὶ τὸ πλῆθος διὰ τῆς ἑνώσεως συνάγων. αὐτὸς δὴ οὖν συμπτύσσει μὲν τὰς ἀνελίξεις τῶν διαλεκτικῶν μεθόδων, συνδεῖ δὲ ἄνωθεν πᾶσαν τὴν διέξοδον τῶν μαθηματικῶν λόγων, τέλος δ'ἐστὶ τὸ ἄριστον τῆς ἀναγώγου πορείας καὶ τῆς γνωστικῆς ἐνεργείας; tr. Morrow 1970, slightly modified.

Intellect, so mathematics proceeds from dialectics and attains its epistemic perfection by returning to it.

Second, just as dialectic is a discursive, demonstrative reflection of Intellect, so mathematics constitutes a quantitative reflection of dialectic. In this capacity, mathematics receives the four logical methods in a manner analogous to the way dialectic derives its powers from Intellect (*In Eucl.* 42.12−43.1):

> Our answer is that, as Intellect is set over discursive thinking and dispenses principles to it from above, perfecting it out of its own riches, so in the same way dialectic, the purest part of philosophy, hovers attentively over mathematics, encompasses its whole development, and of itself contributes to the special sciences their various perfecting, critical, and intellectual powers: the procedures, I mean, of analysis, division, definition, and demonstration[11].

Thus, according to Proclus, the four methods by which the axiomatic structure of mathematics is constituted are, in reality, the envoys of dialectic within mathematics – restricted in their intellectual potency by the inferior nature of the mathematical beings with which they operate at this ontological level (see below). Hence, the elementary structures produced in the exercise of dialectic are not imitations of mathematical arrangements. On the contrary, the truth is precisely the reverse: mathematicians appropriate the methods and formal structure that properly belong to dialectic and manifest them in their own science. This ultimately means that if the mathematical sciences can be elementarily arranged at all, it is because mathematics inherits the footprints of the Intellect in the form of its logical movements, which allow (and require) a sort of elementary structure – a twice-removed imitation of noetic syntax. Consequently, when Proclus composed the *Elementatio theologica* or sought deductive, elementary arrangements in Plato's dialogues, he could not have regarded himself as imposing a geometrical-like logical structure upon theology. Rather, from the standpoint of his philosophy, Proclus was attempting to present dialectical theology in its most proper and authentic form: in a logical configuration of which mathematicians are mere vicars. In sum, for Proclus, axiomatized mathematical systems are but attenuated versions of the exemplary dialectical syntax.

But then, in which sense is the elementary structure of mathematics weaker than that of dialectic?

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[11]. καὶ λέγομεν, ὅτι καθάπερ ὁ νοῦς ὑπερίδρυται τῆς διανοίας καὶ χορηγεῖ τὰς ἀρχὰς ἄνωθεν αὐτῇ καὶ τελειοῖ τὴν διάνοιαν ἀφ᾽ἑαυτοῦ, κατὰ τὰ αὐτὰ δὴ καὶ ἡ διαλεκτική, φιλοσοφίας οὖσα τὸ καθαρώτατον μέρος, προσεχῶς ὑπερήπλωται τῶν μαθημάτων καὶ περιέχει τὴν ὅλην αὐτῶν ἀνέλιξιν, καὶ δίδωσι δυνάμεις ἀφ᾽ἑαυτῆς ταῖς ἐπιστήμαις αὐτῶν παντοίας, τελεσιουργοὺς καὶ κριτικὰς καὶ νοεράς, τὴν ἀναλυτικὴν λέγω καὶ τὴν διαιρετικὴν καὶ τὴν ὁριστικὴν καὶ τὴν ἀποδεικτικήν; tr. Morrow 1970, slightly modified.

This follows from the metaphysical principle that the transmission of power from cause to effect entails a diminution of perfection in the recipient's activity. Thus, according to Proclus, the fullest epistemic realization of dialectical methods – and therefore the highest possible form of discursive science – can be attained only within dialectic[12]. Lernould has convincingly argued that such an epistemic superiority lies in the capacity of dialectic to achieve an unhypothetical status[13]. Both dialectic and mathematics employ hypotheses in their analytic phase, the stage of inquiry aimed at uncovering higher principles. This process eventually leads to the first principles specific to each discipline: definitions, postulates, and common notions in mathematics; and the One in dialectic. However, whereas the hypotheses of dialectic culminate in this unhypothetical principle, mathematics, according to Platonic epistemology, can never be freed from hypotheses. The first principles of mathematics remain suppositions of existence[14] and can be justified only when demonstrated by the superior, unhypothetical science (see next section).

Lastly, Proclus holds that mathematics must be endowed with the dialectical methods in their entirety – i.e., in all its branches – since every mathematical disciplines adopt the methods of dialectic. Because of this, he introduces an overarching, general mathematical discipline that mediates between dialectic and the particular mathematical sciences, which he calls "common mathematics" (κοινὴ μαθηματική). Succinctly, Proclus characterizes this discipline through two main traits: (i) common mathematics comprises the most general theorems and methods shared by geometry and arithmetic and thus stands as the general science upon which all mathematical branches depend; (ii) common mathematics functions as the hinge between dialectic and mathematics: it serves as the bridge leading the learner from mathematical inquiry to dialectical speculation. Proclus illustrates this by noting that through the study of fundamental mathematical concepts, the practitioner is gradually drawn toward reflection on the fundamental principles underlying mathematical truth, thereby advancing toward the science of being *qua* being (τοῦ ὄντος, ᾗ ὄν ἐστιν) − the Aristotelian expression Proclus employs to denote dialectic in this section[15].

[12]. A further question arises as to whether Proclus truly attained the archetypal scientific rigour he envisioned for dialectic in his *Elementatio theologica*. By his own standards, a genuine *elementation* of the supreme discursive science would have to surpass Euclid's *Elementa* in precision, irrefutability, and freedom from hypotheses. Yet, whether Proclus actually achieved this remains open to legitimate doubt.

[13]. Lernould 1987.

[14]. For Proclus' understanding of definitions as suppositions of existence, see Martijn 2010, p. 84 ff. See also MacIsaac 2014.

[15]. Procl. *In Eucl.* 7.13−22: ἀλλ'ὅτι μὲν ἀρχαὶ καὶ τῶν μαθημάτων αὗται προεστήκασιν, αἳ καὶ τῶν ὄντων ἁπάντων, φανερόν. ὥσπερ δὲ τὰς κοινὰς ἀρχὰς αὐτῶν τεθεωρήκαμεν καὶ διὰ πάντων διηκούσας τῶν μαθηματικῶν γενῶν, οὕτω δὴ καὶ τὰ κοινὰ αὐτῶν θεωρήματα καὶ ἁπλᾶ καὶ τῆς μιᾶς ἐπιστήμης ἔγγονα τῆς πάσας ὁμοῦ τὰς μαθηματικὰς γνώσεις ἐν ἑνὶ ἐπεχούσης ἀναλογισώμεθα, καὶ ὅπως ἐφαρμόττει πάσαις καὶ δύναται καὶ ἐν ἀριθμοῖς καὶ ἐν μεγέθεσι καὶ ἐν κινήσεσι θεωρεῖσθαι σκοπήσωμεν. *In Eucl.* 9.11−19: οὐ γὰρ δήπου τῶν μὲν μεριστῶν εἰσὶν ἐπιστῆμαι καὶ γνώσεις, τῶν δὲ ἀΰλων καὶ τῆς νοερᾶς θεωρίας ἐγγυτέρω τεταγμένων οὐδὲ μίαν ἔχομεν ἐπιστήμην, ἀλλὰ πολλῷ πρότερον ἡ ἐκείνων γνῶσίς ἐστιν ἐπιστήμη καὶ ἀπ'ἐκείνης αἱ πολλαὶ τοὺς κοινοὺς ὑποδέχονται λόγους καὶ μέχρι τοσούτου γνώσεων ἄνοδος

The Interweaving of Discursive Sciences: Common Mathematics and Dialectic

So far, we may conclude that, according to Proclus, (a) the underlying logical structures of dialectic and mathematics stand in an exemplar-imitation relationship; (b) this structure is what may be termed "geometric-like" or, more properly, "elementary"; (c) the syntax of Intellect necessitates this structure within dialectic; and (d) the same structure is transmitted from dialectic to mathematics, which constitutes the quantitative image of dialectic.

From this, it follows that (1) the syntax of Intellect is manifested at each ontological level in a manner appropriate to that level. As noted above, one of the main differences between dialectic and mathematics lies in the hypothetical status of the latter. Another distinction may be found in the fact that whereas dialectic consists entirely of theorems, mathematics also includes problematic propositions – i.e., steps that require the mathematician to construct the objects about which theorems will be demonstrated. These constructive procedures reveal that mathematics stands closer to the realm of generation and change.

Second, it also follows that (2) if mathematics both ontologically and epistemically depends on dialectic and is perfected by it, then a logical bridge must exist between the logical syntaxes of the two sciences. Proclus locates this connection in what he calls common mathematics. We will now focus on the mediatory role of this science. In doing so, we will take propositions 1−4 of the *Elementatio theologica* as a token of dialectic and, as indicated by Proclus, attempt to reach them through an analysis of mathematical concepts.

Some details about the doctrinal transition from mathematics to dialectic are revealed in Proclus' *In Rem publicam* (I 291.28−292.15):

> (…) Intellect contemplates that which lies beyond dianoetic beings, not by proceeding toward a result, as in reasoning from hypotheses to derived conclusions while leaving the hypotheses unproven, but instead by ascending from principles to ever higher principles, until it reaches the indemonstrable and unhypothetical first principle that does not exist hypothetically but as the true principle of all things, beyond which nothing can be thought. It is not subordinate to anything else, but everything else is subordinated to it. Thus, while the point is the principle of all things in geometry, it derives from the common principle of all things and is subordinate to it. The unit (μονάς) is also the principle of certain things and not of all in the same way; it is the principle of

ἀπὸ τῶν μερικωτέρων ἐπὶ τὰς ὁλικωτέρας, ἕως ἂν ἐπ'αὐτὴν ἀναδράμωμεν τὴν τοῦ ὄντος, ᾗ ὄν ἐστιν, ἐπιστήμην. See also *In Eucl.* 7.26−27; 8.4−6. See Steel 2005.

> numbers and of everything in arithmetic, but it too is subordinated to the principle of all things. Such steps may be used by those ascending toward that principle of everything[16].

Proclus here illustrates the analytical ascent of the Intellect toward the supreme principle. He contrasts this movement with the synthetic progression from the hypothetical principles of mathematics to the proofs of propositions. Moreover, he declares that the first principles of arithmetic and geometry, as images of the One, can each be traced back to their metaphysical origin in dialectic, and that this is the path to be followed by the learner.

Similarly, in his *In Parmenidem*, Proclus observes that the beginner should be led to understand the divine principles of dialectic by reflecting upon arithmetical and geometrical units as images of divine undifferentiation (*In Prm*. IV 926.16–23):

> For whatever our discursive thinking is unable to contemplate in the gods, it can see in these [*scil*. mathematics] as in images (ἐν εἰκόσι); and having seen them, it will rest in the contemplation of those [*scil*. the gods] and believe also the things said about them; I mean, for example, that if it wonders how the many are in one and how everything is in the indivisible, it will reflect upon unity (τὴν μονάδα), and how within it all things are shown to be both even and odd, circle and sphere, and the other Forms of numbers[17].

From this it becomes clear that Proclus regards the notion of unity in arithmetic and geometry as the starting points of the soul's ascent toward dialectic. It is therefore reasonable to take as our own starting points the two authoritative Euclidean definitions (well-known to Proclus) of such principles: Book I, def.1 "A point is that which has no part(s)" (σημεῖόν ἐστιν, οὗ μέρος οὐθέν) and Book VII, def. 1 "A unit is that through which each of the beings is said one" (μονάς ἐστιν καθ'ἣν ἕκαστον τῶν ὄντων ἓν λέγεται)[18].

To attempt to grasp how the ascent to theology through common mathematics works according to Proclus, we must first clarify some aspects of his philosophy of mathematics. We shall focus on the notion of number, since Euclid's definition of unity seems philosophically promising.

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[16]. (...) ὃ δὴ τῶν διανοητῶν ὑπέρτερον <ὂν> νοῦς ἐπισκοπεῖ καὶ οὗτος οὐκ ἐπὶ τελευτὴν πορευόμενος, οἷον ἐξ ὑποθέσεών τινων ἐπὶ συμπεράσματα, τῶν ὑποθέσεων ἀναποδείκτων κειμένων, ἀλλ'ἀπὸ τῶν ἀρχῶν ἐπ'ἄλλας ἀνωτέρω κειμένας ἀρχάς, ἕως ἂν ἐπὶ τὴν ἀναπόδεικτον ἀναδράμῃ καὶ ἀνυπόθετον ἀρχήν, οὐ καθ'ὑπόθεσιν ταύτην οὖσαν, ἀλλὰ κατ'ἀλήθειαν τῶν πάντων ἀρχήν, ἧς οὐδὲν ἐπέκεινα νοεῖν θέμις, μηδενὶ μὲν ὑποκειμένης, αὐτῇ δὲ τῶν ἄλλων ὑποκειμένων. τὸ μὲν γὰρ σημεῖον, εἰ καὶ ἀρχὴ τῶν ἐν γεωμετρίᾳ πάντων ἐστίν, ἀλλὰ καὶ ἐκ τῆς κοινῆς ἀνῆπται πάντων ἀρχῆς καὶ ὑπόκειται ἐκείνῃ. τινῶν οὖν ἀρχὴ καὶ μονὰς καὶ οὐ πάντων ὡσαύτως, ἀριθμῶν μὲν ἀρχὴ καὶ τῶν ἐν ἀριθμητικῇ πάντων, ὑπόκειται δὲ τῇ πάντων ἀρχῇ. καὶ ἐπαναβασμοῖς τούτοις χρῆσθαι δυνατὸν τοῖς ἐπ'ἐκείνην ἀνιοῦσιν τὴν τοῦ παντὸς ἀρχήν.

[17]. ὅσα γὰρ ἡμῶν ἡ διάνοια θεωρεῖν ἐν τοῖς θεοῖς ἀδυνατεῖ, ταῦτα ἐν τούτοις ὡς ἐν εἰκόσι κατίδοι, καὶ κατιδὼν ἐπαναπαύσεται τῇ τούτων θεωρίᾳ, καὶ πιστεύσει καὶ τοῖς περὶ ἐκείνων λεγομένοις· λέγω δὲ οἷον, εἰ θαυμάζοι πῶς ἐν ἑνὶ τὰ πολλὰ καὶ ἐν ἀμερεῖ πάντα, τὴν μονάδα ἐννοήσει, καὶ ὅπως ἐν αὐτῇ δείκνυται ὄντα πάντα καὶ ἄρτια καὶ περιττὰ, κύκλος τε καὶ σφαῖρα, καὶ τἄλλα εἴδη τῶν ἀριθμῶν. See O'Meara (1989) chapter 10.

[18]. Accessing theology through meta-mathematical speculation does not exclude the simultaneous and necessary pursuit of other paths, such as theurgic practice and moral refinement.

Indeed, this definition – already in use and metaphysically interpreted in the pre-Euclidean age – extends beyond the disciplinary boundaries of mathematics, confronting the reader of the *Elementa* with a metaphysical proposition: unity, or *a* unity (the statement is ambiguous), is that by virtue of which each *being* is said to be "one". This definition is meant to pave the way for the following one, namely that of number (Euclid's VII, def. 2): "A number is a multitude composed of units" (ἀριθμὸς δὲ τὸ ἐκ μονάδων συγκείμενον πλῆθος). From these definitions, it follows that (i) the unit is not itself a number; (ii) numbers are not units (but collections of units); and (iii) each number is *one* number – that is, each multitude (πλῆθος) of units is, *qua* number, *one* multitude of units.

Here a philosophical puzzle arises: how can each number be one number if it is a multitude of units, yet not itself a unit? Proclus' philosophy of arithmetic, which he arguably inherited from Syrianus[19], resolves this difficulty by applying to number two general features of his metaphysics: (a) that all begins except the first principle are not one by themselves, and (b) that all beings except the first principle are unified (or said to be "one") by virtue of their subordination to some unifying principle. Accordingly, Proclus endorses the long-held distinction between monadic numbers (numbers composed of units) and eidetic or divine numbers (indivisible and unique numbers). In his view, the oneness of a monadic number derives from its participation in an eidetic number: five units are said to be *one* multitude – and thus a single monadic number – because of the Form of five, or five-ness itself. Hence, the eidetic number is conceived as a monad that bestows the character of "being one-five" upon a series of fitting, unified manifolds[20]. Ultimately, the source of the oneness imparted by the eidetic number is the One-itself, in which both eidetic and monadic numbers participate.

Considering all this, we now return to Euclid's arithmetical definitions. If a monadic number, *qua* that number, is said to be "one", then, according to Euclid's definition of the unit, a collection of units (say, five units) can only be said to be one number (one token of five-ness) through (a) unit. What does this mean precisely? If we interpret this statement through the lens of Proclus' philosophy of arithmetic, the function "Y is said to be X through a unit" must be reformulated as "Y participates in some way in X-ness". Consequently, a number is said to be one X by participating in and being unified through X-ness. If this principle applies not only to numbers but to all unified beings, as Proclus upholds and Euclid's VII def. 1 declares (ἕκαστον τῶν ὄντων),

[19]. See Syr. *In Metaph.* 133.8−14. See also Mueller 2002; Mouzala 2015; Opsomer 2014.

[20]. See, for instance, Procl. *Theol. Plat.* IV 34.101.8−11: ἐπεὶ κἀν τοῖσδε τοῖς μοναδικοῖς ἀριθμοῖς ἄλλα μὲν τὰ εἴδη τῶν ἀριθμῶν, οἷον ἡ τριάς, ἡ πεντάς, ἡ ἑπτάς, ἄλλαι δὲ αἱ τῶν εἰδῶν ἑνώσεις· καὶ γὰρ τούτων ἕκαστον τῶν εἰδῶν ἕν ἐστι καὶ πλῆθος. Note that, in this passage, Proclus emphasizes that every monadic number is simultaneously one and a multitude − a detail which will be crucial below. For Proclus' views on eidetic numbers, see *Theol. Plat.* IV 29.86.1−10; *In Ti.* II 173.13–174.3 Van Riel (= I 324.4−14 Diehl); IV 132.14–19 Van Riel (= III 102.17−23 Diehl). On monadic number, see *In Prm.* VII 1175.5−14; *In Eucl.* 95.26−96.3.

then a metaphysical claim follows: every composite being is said to be "one" by participating in oneness (or *a* oneness). From this, propositions 1−4 of the *Elementatio theologica* can be logically derived. Seeking to remain faithful to Proclus' own manner of reasoning, we may now propose the following tentative formalization:

Analysis from arithmetic

(0) Every (arithmetic) number is a (one) being (i.e., arithmetic numbers exist).

(1) Euclidean definitions:

(1.a, VII, def. 2) A number is a multitude composed of units.

(1.b, VII, def. 1) A unit is that by virtue of which each of the beings is said one.

Analysis within common mathematics (from arithmetic to dialectic)

(1.c, Proclean postulate) No number is one by itself.

(2) If every number (*qua* that number) is *one* multitude composed of units (from 0 and 1.a), then every number is said to be one (from 1.a and 1.b).

(3) Every multitude composed of units is a number (identity inversion of 1.a).

(4) Every being is said to be one either by itself or through another (from 1.b).

(4.a) Every being said to be one through another is not one by itself (from 4).

(5) If every multitude composed of units is a number (3), and every number is said one (2), and no number is one by itself (1.c), and every being is said to be one either by itself or through another (4), then every multitude composed of units is said to be one through another.

Analysis within dialectic

(6) If every multitude composed of units is said to be one through another (5), and a unit is that through which each of the beings is said to be one (1.b), then there is some unit through which every multitude composed of units is said to be one.

(7, re-definition as participation) If a being Y is said to be X through another (from 4), this means that Y participates in some way in X-ness, and X-ness is the unit through which any being is said to be X.

(8) If every multitude composed of units is said to be one through another (5), and every being is one either by itself or through another (2), and every being said to be X participates in some way in X-ness (from 7), then *every multitude* (composed of units) *participates in some way in one*(ness) (Proclus, prop.1).

(9) If every multitude composed of units is said to be one (5) and every being is said to be one either by itself or through another (2), and a being said to be X through another participates in some way in X-ness (from 7), and every being which is said to be one through another is not one by itself (4.a), then *every being that participates in one*(ness) *is both one* (through another) *and not one* (by itself) (Proclus, prop. 2).

(10) If there is some unit through which every multitude composed of units is said to be one (6), and a being said to be X through another participates in some way in X-ness, and X-ness is the unit through which any being is said X (7), then *every being that becomes one* (= it is said to be one through another) *does so by participation in one*(-ness) (Proclus, prop. 3).

(11) If *every being that participates in one*(ness) *is both one* (through another) *and not one* (by itself) (9, Proclus, prop. 2) and *every being that becomes one* (= it is said to be one through another) *does so by participation in one*(-ness) (10, Proclus, prop. 3), then *every being that is unified* (= it is said to be one through another) *is other than the One-itself* (*sive* oneness, *sive* one by itself) (Proclus, prop. 4).

Although this logical transition could be structured in alternative ways that may be more direct or more comprehensive, its overall features provide valuable insight into the logical continuity between mathematics and dialectic that Proclus postulates. Four aspects are particularly noteworthy:

First, it helps us to understand more clearly the preliminary role of common mathematics in relation to dialectic. As Proclus explains, it is through reflection on the first principles of mathematics that the learner is ultimately drawn into metaphysical speculation. Questions such as "What is a number?" lead the learner to consider the higher concepts upon which mathematical definitions depend. This constitutes one of the defining traits of common mathematics: it begins with propositions about mathematical concepts and concludes with metaphysical ones (see steps 1.c and 2 to 5).

Second, as is well known, Proclus' *Elementatio theologica* rests on certain implicit assumptions. Opsomer has convincingly argued that Proclus most likely avoided definitions and postulates in order to preserve the unhypothetical status of dialectic[21]. However, we should bear in mind that the Platonic hierarchy of the sciences is not merely a classification of disciplines according to their proximity to the Forms; it also outlines a structured curriculum for students progressing toward dialectic. Following Plato, Proclus regarded mathematics as preparatory to dialectic, situating common mathematics as the final pre-dialectical stage. Consequently, it is reasonable to assume that Proclus' *Elementatio theologica* was intended for readers who, following

[21]. Opsomer 2022, part 3.

the Platonic progression, were already familiar with the preliminary concepts of geometry and arithmetic. Thus, some of the notions developed within common mathematics would form part of the conceptual toolkit required for dialectical speculation, *while remaining part of common mathematics as a distinct discipline*. In this way, theology could be kept free from all hypotheses while nonetheless drawing its necessary background notions from common mathematics. Further evidence for this view lies in the fact that the key terms employed by Proclus in the opening propositions of the *Elementatio theologica* partially overlap with the lexicon of mathematics: unit (μονάς), multitude (πλῆθος), one (ἕν), as well as unlimited (ἄπειρον), and limit (πέρας). These are all terms found in Euclid − a Platonic philosopher himself, according to Proclus[22].

In pedagogical terms, therefore, when Proclus employs, for example, the abstract noun *πλῆθος* in the opening propositions of the *Elementatio theologica*, he most likely assumes that the learner, as part of his philosophical training, has already advanced from the study of physical multitudes to the contemplation of incorporeal, countable multitudes (i.e., monadic numbers) by studying Euclid's *Elementa.* Such a learner would thereby have acquired the necessary familiarity with immaterial objects and the conceptual background required to embark upon dialectical speculation. Hence, the reader would know from his previous training in common mathematics that a πλῆθος is "a composite being that is said to be one, composed either of units or of other composite beings that are also said to be one" − a definition implied in Proclus' first proposition and easily inferred from Euclid's arithmetical definitions.

Third, once proposition 4 of the *Elementatio theologica* has been analytically deduced from arithmetical principles, the reverse operation should, in principle, also be logically possible. In such a synthetic operation, arithmetical definitions – assumed as existential hypotheses within the domain of mathematics – are granted the status of demonstrable metaphysical theorems from the higher standpoint of dialectic. Thus, what Proclus' theory ultimately maintains is that arithmetical definitions, such as "A number is a multitude composed of units", should be read as existential claims when viewed from the perspective of dialectic: "There exists such a thing as a multitude composed of units, i.e., a number". This proposition would be established as a theorem derived from dialectical propositions: it follows from the One-itself that numbers – i.e., unitary multitudes – must factually exist. Therefore, every genuine mathematical principle ultimately stands as a metaphysical theorem synthetically demonstrated within the *interregnum* of common mathematics. This, in turn, shows how Proclus interpreted Plato's claim in the *Respublica* (510b) that mathematics becomes truly scientific through dialectic.

[22]. See *In Eucl.* 68ff.

Lastly, we are now in a position to better understand the nature of the all-pervading syntax of Intellect. According to Proclus, the logical structure of discursive sciences, taken as a whole, mirrors the underlying relations of priority and dependence among beings. Hence, the theorematic arrangements of discursive sciences cannot be regarded as discrete systems: the seamless continuity of powers structuring Proclus' cosmos allows for no gaps; ontological continuity in the procession and reversion of beings entails logical – and therefore doctrinal – continuity. Inferential bridges must thus exist between sciences whose objects of study occupy adjacent positions within the metaphysical hierarchy. With the sole exception of dialectic, the principles of every subordinate science must be derivable from certain elements belonging to the science, or sciences, that stand above it.

This leads us to a further question – one that, for the sake of simplicity, I have set aside in this chapter – namely, the place of Proclus' *Elementatio physica* within this framework and its relation to prior mathematical sciences. As things currently stand, we can draw two preliminary conclusions. (1) First, according to Proclus, the physical world also bears the imprints of Intellect, as it receives it from the Soul. This transmission endows the sensible realm with some degree of intelligibility and, ultimately, makes it possible to learn about it demonstratively. (2) Second, if Proclus is consistent in his thought, he must, *mutatis mutandis*, also assume the existence of a doctrinal transition between the principles of physics and those of mathematics.

Concluding Remarks: The Elementary Structure of Scientific Knowledge

In this chapter, we have explored how, for Proclus, the hallmark of scientific knowability lies in a being's capacity to be propositionally formalized and integrated into an inferential web governed by the operations of analysis, division, demonstration, and definition. When combined, these cognitive powers converge upon a precise structure in which propositions are deductively interwoven according to an order of explanatory priority and logical dependence: the elementary system. According to Proclus' philosophy of language, the ordering produced by these logical powers reflects the syntax of Intellect, which stands as the ultimate source of these powers and the compressed formal model of this logical arrangement.

Proclus also maintains that likenesses of the intellectual syntax cannot be achieved with equal efficacy across all sciences. The extent to which a science's logical structure can formally represent the noetic architecture depends on the ontological proximity of its subject matter to Intellect. Hence, the discursive science concerned with the loftiest object must be the one capable of

representing the most accomplished logical expression of the undifferentiated noetic syntax. Therefore, whereas dialectical elementations can, at their finest, be arranged as self-sustained elementary systems (imitating the self-sufficiency of divine being), mathematical or physical systems (dealing with derivative entities) are structurally constrained to rely on hypothetical principles that they cannot justify. Consequently, as illustrated above through the case of Proclus' common mathematics, formal transitions between disciplines must exist, allowing for the ascent from lower to higher sciences and enabling the hypothetical principles of the former to be demonstrated from the perspective of the latter. In this resulting hierarchy of sciences, unhypothetical dialectic stands as the pinnacle.

It must also be emphasized that, according to Proclus, it would be utterly mistaken to claim that the elementary arrangements of theology attempt to emulate mathematical methodologies: the elementary syntax inherently belongs to theology; the so-called geometrical method is only an imperfect reflection, dependent upon the dialectical one.

All of this provides insight into Proclus' conception of the *elementatio* as a literary genre. According to his view, the elementary structure is not merely one mode of philosophical exposition among others. Rather, when composed correctly, the logical structures of the elementations are closer or more distant iconic instantiations of paradigmatic intelligibility[23]. Therefore, for Proclus, composing *elementationes* was more than writing pedagogical textbooks aimed at clarity and thoroughness. In his own terms, by producing these elementary works, Proclus also sought to retrace the syntax of Intellect itself, envisioning it as iconically expressed across the three theoretical sciences.

[23]. However, not every elementation is equally thorough, elegant, or clear. As an accomplished mathematician, Proclus was aware that alternative elementary models of a given science could be constructed, differing in their completeness, propositional arrangement, and inferential pathways. For Proclus' account of the literary genre of elementation and its composition, see *In Eucl.* 71ff.

Bibliography

R. Chlup, *Proclus: An Introduction*, Cambridge, Cambridge University Press, 2012.

S. Gersh, *Κίνησις ἀκίνητος: A Study of Spiritual Motion in the Philosophy of Proclus*, Leiden / Boston, Brill, 1973.

S. Gersh, *Proclus' Theological Methods: The Programme of* Theol. Plat. *I. 4*, in Id., *Reading Plato, Tracing Plato: From Ancient Commentary to Medieval Reception*, London / New York, Routledge, 2005, pp. 15–27.

C. Helmig, *Forms and Concepts: Concept Formation in the Platonic Tradition*, Berlin, De Gruyter, 2012.

C. Helmig, *Proclus on Epistemology, Language, and Logic*, in P. d'Hoine – M. Martijn (eds.), *All From One: A Guide to Proclus*, Oxford / New York, Oxford University Press, 2016, pp. 183–296.

S. A. Kiosoglou, *Proclus' Theory of Seemingly 'Useless' Theorems*, in «Méthexis», 34 (2022), pp. 148–65.

A. Lernould, *La dialectique comme science première chez Proclus*, in «Revue des Sciences Philosophiques et Théologiques», 71 (1987), pp. 509−36.

A. Lernould, *La διάνοια chez Proclus*, in «Chôra», 14 (2016), pp. 139–54.

A. Lernould, *La ΣΤΟΙΧΕΙΩΣΙΣ ΘΕΟΛΟΓΙΚΗ de Proclus. Science première et dépassement de la méthode hypothético-déductive*, in G. Aubry - L. Brisson - P. Hoffmann - L. Lavaud (eds.), *Relire les* Éléments de théologie *de Proclus : Réceptions, interprétations antiques et modernes*, Paris, Hermann, 2021, pp. 70−91.

D. G. MacIsaac, *The Soul and Discursive Reason in the Philosophy of Proclus*, Doctoral Dissertation, University of Notre-Dame, 2001.

D. G. MacIsaac, *Projection and Time in Proclus*, in J. Inglis (ed.), *Medieval Philosophy and the Classical Tradition in Islam, Judaism and Christianity*, London / New York, Routledge, 2002, pp. 83–105.

D. G. MacIsaac, *Νόησις, dialectique et mathématique chez Proclus*, in A. Lernould (ed.), *Études sur le Commentaire de Proclus au premier livre des* Éléments *d'Euclide*, Villeneuve d'Ascq, Presses Universitaires du Septentrion, 2010, pp. 125–38.

D. G. MacIsaac, *Geometrical First Principles in Proclus' Commentary on the First Book of Euclid's* Elements, in «Phronesis», 59 (2014), pp. 44–98.

M. Martijn, *Proclus on Nature: Philosophy of Nature and Its Methods in Proclus' Commentary on Plato's* Timaeus, Leiden / Boston, Brill, 2010.

M. Martijn, *Proclus' Geometrical Method*, in P. Remes and S. Slaveva-Griffin (eds.), *The Routledge Handbook of Neoplatonism*, London / New York, Routledge, 2014, pp. 145–59.

G. R. Morrow (ed.), *Proclus. A Commentary on the First Book of Euclid's* Elements, Princeton, Princeton University Press, 1970.

M. G. Mouzala, *Syrianus on the Platonic Tradition of the Separate Existence of Numbers*, in «Peitho», 6 (2015), pp. 167–94.

I. Mueller, *Syrianus and the Concept of Mathematical Number*, in G. Bechtle – D. J. O'Meara (eds.). *La Philosophie des mathématiques de l'Antiquité tardive*, Fribourg, Éditions Universitaires, 2000, pp. 71–83.

D. Nikulin, *Physica more geometrico demonstrata: Natural Philosophy in Proclus and Aristotle*, in «Proceedings of the Boston Area Colloquium in Ancient Philosophy», 17 (2003), pp. 183–209.

D. J. O'Meara, *Pythagoras Revived: Mathematics and Philosophy in Late Antiquity*, Oxford / New York, Oxford University Press, 1989 (Oxford / New York, Oxford University Press, 2010).

D. J. O'Meara, *Mathematics and the Sciences*, in P. d'Hoine – M. Martijn (eds.), *All From One: A Guide to Proclus*, Oxford / New York, Oxford University Press, 2017, pp. 167–82.

J. Opsomer, *Mathematical Explanation and the Philosophy of Nature, Late Ancient Philosophy: Astronomy and the Theory of the Elements*", in «Documenti e studi sulla tradizione filosofica medievale», 23 (2012), pp. 65–106.

J. Opsomer, *Syrianus, Proclus, and Damascius*, in J. Warren – F. Sheffield (eds.), *The Routledge Companion to Ancient Philosophy*, London - New York, Routledge, 2014, pp. 626–42.

J. Opsomer, *Proclus' Elements of Physics and the Axiomatization of Kinematics*, in *F.* Baghdassarian - *I.* Papachristou - *S.* Toulouse (eds.), *Relectures néoplatoniciennes de la théologie d'Aristote*, Baden-Baden, Academia, 2020, pp. 83−102.

J. Opsomer, *Organiser la philosophie selon ses éléments. Structures argumentatives dans les* Éléments de Théologie, in G. Aubry - L. Brisson - P. Hoffmann - L. Lavaud (eds.), *Relire les* Éléments de Théologie *de Proclus : Réceptions, interprétations antiques et modernes*, Paris, Hermann, 2021, pp. 133−76.

J. Opsomer, *Proclus'* Elements of Theology *and Platonic dialectic*, in D. Calma (ed.), *Reading Proclus and the Book of Causes*, Vol. 3: *On causes and the noetic triad*, Leiden - Boston, Brill, 2022, pp. 17–36.

J. L. Solère, *L'ordre axiomatique comme modèle d'écriture philosophique dans l'Antiquité et au Moyen Âge*, in «Revue d'Histoire des Sciences», 56 (2003), pp. 323–45.

C. Steel, *Breathing Thought: Proclus on the Innate Knowledge of the Soul*, in J. J. Cleary (ed.), *The Perennial Tradition of Neoplatonism*, Leuven, Leuven University Press, 1997, pp. 293−309.

C. Steel, *Theology as First Philosophy. The Neoplatonic Concept of Metaphysics*, in «Quaestio», 5 (2005), pp. 2–21.

A. Vargas, *Proclus on Direct Knowledge of the Forms*, in «Methexis», 37 (2025), pp. 73–89.

ABSTRACT

Álvaro José Campillo Bo, *The Unfolded Syntax of Intellect: Proclus on the Metaphysical Foundations of Elementary Arrangements*

This chapter examines Proclus' epistemology and ontology of the discursive sciences, focusing on the foundations that ground the logical structures of dialectic and mathematics. It explores Proclus' account of demonstrative reasoning and elementary arrangements as the iconic reflection of the architecture of Intellect. The chapter considers aspects of Proclus' philosophy of language to show how the structures unfolded by discourse reproduce the relational patterns of the Intellect. It analyses how the dynamics of Intellect manifest in the logical operations of dialectics and, in a more attenuated way, in mathematical demonstration, establishing a hierarchical relation between the two sciences. This hierarchy clarifies the dependence of mathematics on dialectic and their mutual intertwining through the discipline of "common mathematics". This dependence is examined by exploring the continuous logical syntax linking both sciences. As a case study, the chapter explores how the opening propositions of the *Elementatio theologica* (1 to 4) can be analytically derived from Euclidean arithmetic definitions. The chapter concludes that, for Proclus, the elementary logical syntaxes in contemplative sciences are not mere methodological tools but discursive enactments of the noetic order.

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CAN THE WILL GROUND SCIENCE?

What is the ultimate ground of knowledge? Is it something like a self-evident proposition or a group of propositions that could be the basis for a *more geometrico* treatise? In this chapter, I show that Proclus answers that no, knowledge is not ultimately grounded in anything like a self-evident proposition. For the Neoplatonist, the science concerned with ultimate grounding is dialectic, and specifically the dialectic of Plato's *Parmenides*, and Proclus' *more geometrico* metaphysics, the *Elements of Theology*, cannot be considered an instance of this science. Some have speculated that it might be so considered because the *Elements* lacks an initial list of axioms and definitions, but I argue that this absence must be understood within the larger context of the *Elements'* many deficiencies vis-a-vis the geometric ideal. The absence of such a list of starting points is thus not especially significant. Furthermore, the *Elements* makes no use of what Proclus, in his *Parmenides Commentary*, identifies as our access to an unhypothetical first principle – namely, the so-called "one of the soul" (*hen tes psyches*), which on the basis of the *De Providentia* can also be identified with the will[1]. I therefore complement the denial of the *Elements'* status as dialectic with a positive argument on how the will in Proclus can serve as a ground for knowledge. This will is a ground for analyzing our simple intuition of Being (which Proclus thinks Parmenides' Way of Truth describes) into Being itself and the Unity of Being. It thus institutes a tension between, on the one hand, the unity of Being and Unity, which is intuited by *nous* and, on the other hand, the transcendence of Unity itself, which is indicated by the will. Such a tension then forms the basis for the derivation of the orders of the gods in Proclus' interpretation of the second hypothesis of Plato's *Parmenides*.

[1]. *Voluntas* in Moerbeke's Latin, *bouleusis* in Strobel's 2014 retroversion. I favor the term "the will" in this context to bring out its non-cognitive character and thus to highlight the strangeness of Proclus' position. I also find it fitting given its connection to (i) *to agathon*, which I translate here, following convention as "the Good", though it clearly carries something more sinister in Proclus (see Vargas 2020) – a nuance that justifies a more neutral reading such as "the will" rather than, say, "love"; and (ii) its connection to the self (see section IV below). I would like to thank Anne Sheppard for pushing me on this terminological issue.

On the Hypothetical Character of the Elements of Theology

One reason why readers have thought that the *Elements* might embody dialectic is that it does not begin with a list of axioms and definitions – a feature that would seem to correspond to an unhypothetical science[2]. But this is clearly insufficient. The puzzle regarding the lack of a list of starting points in the *Elements* is, in fact, an illusion created by the spell of its mathematical form: the reader sees how relatively didactic the text is when compared to other ancient metaphysical treatises and correctly attributes this, in part, to its organization into propositions and demonstrations[3]. They then reason that clarity has been achieved – that the work is complete – and thus begin to wonder why it does not conform more closely to Euclid. Indeed, if everything in this text were formally perfect, and the lack of axioms and definitions at the beginning were the texts' only blemish, I would concede that there might be some significance to this fact. But once we snap out of this enchantment cast by its mathematical form, we realize that Proclus' text is incomplete in many respects and that the absence of a list is merely one among many such deficiencies[4]:

1. There are passages written in a more elaborate or baroque style than is typical for a text modeled on mathematics. See propositions 52, 71, 75, 86, 93, 98, 129, 145, and 209.
2. Some propositions fail to draw upon earlier propositions for their arguments when it would obviously be appropriate to do so, as in prop. 115.
3. Certain propositions contain incomplete arguments, such as prop. 91[5], or require extensive supplementation to be valid, as in prop. 6.
4. Some propositions seem misplaced, especially propositions 104–107 on perpetuity and immortality, which would fit better alongside propositions 50–55 in the subsection on time and *aion*.
5. There is some inconsistency as to when a proposition is enunciated as a compound statement (as in prop. 21) or when two propositions are proved separately (as in props. 84 and 85).
6. Some corollaries are treated as propositions (prop. 109, prop. 111). In other cases, enunciations are missing (e.g. props. 71 and 209, where what is taken as the enunciation is far too complex a statement) and many demonstrations lack explicit conclusions.

[2]. As in Opsomer 2021, p. 156 and Opsomer 2022, p. 32, but proposed earlier in e.g. O'Meara 1989, p. 197. Opsomer's suggestion that the lack of a list of starting points is due to reliance on common notions perplexes me: if these common notions were readily available to Proclus and his readers, he could just as well have written them down.
[3]. The logical organization of Proclus' concepts is certainly the primary source of the text's clarity.
[4]. I expand here slightly on the list of deficiencies given in my translation of the *Elements*, Vargas 2024, pp. 29–30, which can be consulted for the arguments concerning individual propositions.
[5]. I suspect that prop. 91 and 92 were originally intended to form a single proposition, since the enunciation of prop. 92 appears to be expand upon the second, undemonstrated half of 91.

7. The text ends abruptly at prop. 211, with an attack on Plotinus' theory of the soul – an odd place to conclude an introduction to theology[6].

8. And, last but not least, it appears that a final section on the natures of bodies is missing.

Several features of the text suggest this:

8.1 In the corollaries of prop. 21 and prop. 62, and also in prop. 20, the three orders – henads, *noes*, and souls – are presented alongside the order of natures. Thus, if the *Elements of Theology*, in its second part, aims to present a metaphysics of all orders of being, it would also need a section dedicated to natures.

8.2 If, on the other hand, we emphasize the theological character of the investigation in the *Elements of Theology*, we find that Proclus repeatedly highlights the presence of gods in the world (props. 139, 140, 144, 145, 165).

8.3 In sections of the book devoted to henads, *noes*, and souls, each one addresses not only its respective order but also the principle of the order that follows. Thus, the section on the gods concerns not only the gods but also real being (see prop. 138); the section on *noes* concerns not only *noes* but also the forms known by souls; and, finally, the section on souls concerns not only souls but also their "vehicles," the primary bodies in the world.

These are all reasons indicating that the *Elements* is incomplete, and therefore that we should not read too much into its lack of a list of axioms. Indeed, it is plausible that elaborating such a list would be the very last thing the author of a work entitled *Elements* would do: only after establishing all the claims he seeks to ground, and exhausting attempts to derive those claims from simpler ones, would an "elementarizer" finally settle on a group of basic claims and terms. Thus, when faced with a text that is deficient in many respects, we should even expect it to lack the crowning glory of any such endeavor: a minute list of basic definitions and assumptions from which, astonishingly, the principles of an entire science can be derived.

On the Starting Point of Dialectic

An even more important point to be made here is that dialectic in Proclus could never take the form of an *Elements* – not even that of a completed *Elements* with perfectly valid proofs and self-evident definitions and axioms. For there can be no self-evident grasp of the unhypothetical first principle, Unity or *to hen*, which is "more ineffable than all silence and more unknowable than

[6]. We probably tend to exaggerate the distance between Plotinus and Proclus on this issue, given that Proclus held that Plotinus (and others like him) possessed something that remained "up there", namely their divine *daimon* (see Procl. *In Alc.* 72.16–73.8).

all individual existence" (*Theol. Plat.* II 11.65.13–14). As I will discuss below, human beings "know" unity, if at all, not by *nous*, not in the clarity of *noesis*, but through our constitutive will for the Good, which Proclus also calls the unity or one of the soul, *hen tes psyches*. It is only this immediately present unity within us that can serve as an unhypothetical starting point. By definition, any unity discovered through an argument for a first principle is hypothetical, insofar as it depends on the premises and starting points of that argument. It is true that Proclus, in propositions 5, 8, 11, 12, 13, and 20 of the *Elements*, argues about the first principle, demonstrating its existence (5, 8, 11), the identity of its notions (12 and 13), and its superiority to the principles of change (20). Yet in these arguments he presupposes certain premises as conceded or self-evident, relies on definitions, and builds on earlier propositions. Thus, although the first principle is discussed and indeed present in the *Elements*, it does not appear *qua* an unhypothetical first principle. Some have been tempted to see in the first few propositions (1–5 or 1–13) an ascent to the One, which would then be followed by a descent[7]. But although there may be some thematic truth to this interpretation, it is certainly not logically true: later propositions often introduce premises and concepts that simply cannot be derived from 1–13 alone (for instance, the necessary existence of change presupposed by prop. 14). There is therefore no logical sense in which prop. 1–13 brings us to the principles for the rest of the book.

There are certainly affinities between the *Elements of Theology* and dialectic. In the first place, it is unmistakably a theological text – not only because of its title, but because it treats all things as dependent on the first principle[8]. All readers of the text seem to agree that it falls into two parts: a first, more general part on the first principle and causality in general, and a second part dealing with the various specific orders of existence. It thus seems clear that the work is intended, at least ideally, to encompass all kinds of being[9]. Dialectic too is supposed to show how all things depend upon the unhypothetical first principle, ascending to it and then descending again, proving those hypotheses that other sciences simply take for granted.

Furthermore, Proclus tells us that dialectic makes ample use of the methods of division, definition, demonstration, and analysis – and indeed, it is in dialectic that these methods properly have their home (*In Prm.* 1003.6–7 Steel, *In Eucl.* 42.13–43.1). Starting from Proclus' claims to this effect, many researchers have shown how these methods are present in dialectic and employed in the *Elements*[10]. However, this is not sufficient to show that the *Elements* is dialectic, for the key feature that distinguishes dialectic from all other sciences is its relation to the unhypothetical first

[7]. E.g. Opsomer 2021, p. 157, and Opsomer 2022, pp. 28–30.
[8]. See *In Prm.* I 640.13–643.4, and also Steel 2005.
[9].On the division of the text, see Ruz 2024, pp. 113–5. As noted in Section I, a concluding set of theorems on natures and bodies appears to be missing.
[10]. See Opsomer 2022, Lernould 1987, Bonelli 2016, and Kiosoglou 2023.

principle *qua* unhypothetical. The analyses in propositions 5, 8, 11, and 20, for instance, which argue for the existence of a first principle, all depend on some definition or hypothesis; thus, they do not provide the absolute starting point of dialectic, but something that depends on other starting points.

Opsomer (2021) and Kiosoglou (2023) have observed in this regard that certain propositions which are accompanied by proofs in the *Elements* are mentioned elsewhere in Proclus' works as axioms to be accepted without proof. This raises the interesting possibility that Proclus regarded the whole list of propositions in the *Elements* as axiomatic[11]: each might, in principle, be susceptible to being known as self-evident through *noesis*. In his interpretation of the *Timaeus*, Proclus often remarks that Timaeus first announces a conclusion and then provides an argument for it, taking this to indicate that the character directly intuits the truth of the conclusion though *nous* and then offers an argument, by way of *dianoia*, for the benefit of his listeners[12]. Likewise, Proclus construes the dialectic exercise of the *Parmenides* as a laborious "task" for the Eleatic philosopher, since he is descending from his habitual state of intuitive apprehension to the use of discursive reasoning for the sake of Socrates' training[13]. Thus, perhaps the *Elements* as a whole is knowable through *noesis* and there is no list of axioms because it itself constitutes such a list, merely accompanied by proofs for the benefit of learners[14].

By introducing the theme of *noesis* and self-evidence, this reconstruction of the *Elements'* internal logic brings us to what truly separates it from dialectic. For even if we were to imagine that the *Elements* constitutes a list of self-evident truths, self-evidence alone is not sufficient to bestow "unhypothetical" status upon a proposition. Indeed, Proclus is quite clear that the One-Being – the hypothesis from which Parmenides sets out in the dialectical exercise – is Being itself, existing separately from all particular beings, and is thus known only through *noesis* (Procl. *In Prm*. I 704.7–18):

> All beings actually come from a single unit (*monas*) which is called primarily Being, by which they are and are named beings according to their respective orders; and from this unit all beings share a predicate with one another (συμπαθῆ ἐστιν ἀλλήλοις) and are in a sense the same, as coming from the One-Being. It is to this union of all beings that Parmenides was looking when he demanded that we call the All one, primarily and

[11]. To be clear, neither author makes the leap I am making here from their starting points.
[12]. See *In Ti.* II 114.5–115.5 Van Riel (= I 282.27–283.18 Diehl); II 182.18–183.3 Van Riel (= I 330.12–20 Diehl); II 225.17–226.10 Van Riel (= I 360.5–17 Diehl); and see also *In Prm.* VI 1125.14–16.
[13]. *In Prm.* V 1020.9–12.
[14]. This would correspond to the claim in *Resp*. VI 511b–e that dialectic knows through *nous*, a topic which Proclus discusses at *In Eucl*. 42.9–44.24.

> most strictly the All which is united with the One, but also the All simply (ἁπλῶς); for all things, in so far as they participate in the One-Being are in a sense the same as one another and one[15].

However, Being itself is not the unhypothetical first principle, because it has a further cause, namely the One or Unity itself (Procl. *In Prm* V 1033.23–33):

> Parmenides begins from the One Being (…) and ascends from the One Being to the One, thus indicating clearly that the One in the strict sense wishes only this, to be One, and "snatches itself away" from Being; and that the One Being is second to this by reason of its descent, going forth towards being, whereas the One itself is superior even to the "is", and the hypothesis "if it is"; for together with the "is", the One in the strictest sense no longer remains[16].

That there is no *noesis*, no self-evident grasp of Unity, is clear for three reasons familiar to scholars of Neoplatonism:

(i) because Unity is the Good beyond being, and only beings can be known by *nous* (*El. Theol.* 8, 13, 123);

(ii) because the very duality of *nous* – which characterizes self-evidence, in which there is an activity (knowing, "being evident") and a being (the *noeton*, the "self" that is evident) with identical content – makes it clear that it is not simple Unity and is below the One (*El. Theol.* 20);

(iii) and finally, because all beings and all *noeta* are a mixture of limit and the infinite, and thus fall short of Unity itself (*El. Theol.* 89).

A Short Digression on Plat. Theol. II 12: What is the First Thought of Dialectic?[17]

The above presentation of an *eidos*, the One-Being, as Parmenides' starting point in the hypotheses of the eponymous dialogue may come as a surprise. Although Proclus clearly refers to Parmenides as beginning from the One-Being in his commentary on the dialogue (e.g. *In Prm.* V 1033.17–1034.29, VI 1071.4–7, 1079.4–20), a passage in his *Platonic Theology* has given many readers the impression that his starting point is actually a proposition (Procl. *Theol. Plat*. II 12.66.2-20):

[15]. Tr. Morrow and Dillon 1987, modified. See *In Prm* V 986.21–27 on how the primary *eide* are not definable and are apprehensible solely by *nous*.
[16]. Tr. Morrow – Dillon 1987, modified.
[17]. I would like to thank Carlos Steel for his insistence that I revisit this passage.

> What, then, would be the very first thought (*noema*)[18] of the science that goes forth from *nous* and shows itself? What else will we say but that it is the simplest and most knowable of all thoughts within that science? For this is also especially most like the contemplation of *nous*. What is it then? "Unity", says Parmenides; "if there is Unity, it would not be many". For multitudes participate in Unity necessarily, but Unity does not participate in Unity but is Unity itself. But neither is primary Unity participated in: for pure Unity would not be mixed with any multitude, nor would what it is to be one receive any addition of what is worse than it [...]. This, therefore, Parmenides did not deem worthy of demonstration; rather, as most evident to all, he announced it first, given the (as it were) 'opposition' between multiplicity and Unity.

To reconcile this passage with those in which Proclus clearly states that the One–Being is the starting point of the first hypothesis, Lernould (2021, pp. 100–1) goes so far as to identify the One-Being with the proposition "Unity is not many". I see no need, however, for such a radical reinterpretation. Indeed, it seems clear that what is called here "the first thought" of this science is the first negation of the first hypothesis (*In Prm.* VI 1089). Yet this is clearly presented as a consequence of the claim "Unity is", which expresses the existence of the One-Being. As Proclus explains: "Parmenides begins from the One-Being for "if there is Unity", refers to this order of realities, since it contains besides "Unity" also "is" (*In Prm.* V 1033.18–20).

However, if the One Being is known through *nous* (as *In Parm* V 986.21–27 suggests), then a science that begins by stating its existence is indeed one that starts *apo nou*, "from *nous*". The very first predication it makes of the One-Being, "Unity is not many", may thus be legitimately called a *noema* – not simply because it is a thought, but because it is a product of *nous*[19]. It is true that no demonstration or argument is offered for this inference, nor is there any appeal made to a principle such as "something is not its opposite"; rather, the inference is presented as immediately evident. But it still rests on a more basic insight into the One-Being.

Proclus is eager to show that Parmenides' premises are self-evident and that his arguments are valid. However, neither the *Platonic Theology* nor the parallel passage *In Prm.* VI 1092.13–1093.13 describes his starting points as "the unhypothetical first principle", in such a way that Platonic dialectic could be reduced to a model of deriving all knowledge from a set of self-evident truths. But if science is not grounded in self-evidence, then in what is it grounded? The answer, as we will see, is "the one of the soul" or "the will". But how can the will ground science?

[18]. On *noema* in Proclus, see his discussion of Socrates' suggestion at *Prm.* 132b3–6 that the *eide* are thoughts (*In Prm.* IV 891.4–898.1). The term appears to denote both an activity of thinking and an innate concept belonging to the soul's essence.
[19]. See *In Prm* IV 897.2–5.

The Discovery of the Will as Ground in the First Hypothesis

In *De Providentia* §57.60, Proclus defines the will (*voluntas*) in contrast to the power of choice: whereas the latter is capable of pursuing both real and apparent goods, the will is a unitary power directed solely toward the good. The man who lives according to the will "becomes god and governs the whole world" as he says. This agrees with *In Ti.* II 241.15–243.10 Van Riel (= I 371.9–372.18 Diehl), where Proclus presents the will (*bouleusis*) as mediating between the goodness of the gods and their exercise of *pronoia*, understood as their characteristic act of bestowing goodness upon things, prior to any divine plan or *nous* (*El. Theol*. 120). The potential of the will to deify us, together with its role in the gods' exercise of their most characteristic activity, would already be sufficient to identify it with the one of the soul, the soul's participation in Unity itself (see *De dec. dub*. §63–65). But there is no need to rest my argument solely on this. Indeed, in the *Commentary on the* Parmenides we learn that the soul's innate striving for the Good itself is actually identified with the one of the soul (*In Prm.* VII 509.10–24). Proclus explains this as the faculty through which the soul unites itself as an "I", and says "I am" and "I do such and such" (*In Prm.* IV 957.22–958.10)[20].

Proclus turns to this faculty when resolving an aporia concerning the first hypothesis: how is it possible that the soul should not "lose itself in the void of non-being" once everything has been denied of Unity itself, and imagination and thought have nothing to hold on to? (Procl. *In Prm*. VI 1072.1–11)

> So then, we need demonstrative power in our preliminary assumptions, whereas we need intellectual activity in our investigations of being (for the orders of being are denied of Unity), and we need inspired impulse in our consciousness of that which transcends all beings, in order that we may not slip unawares from our negations into Not-Being and its invisibility by reason of our indefinite imagination[21], but rousing up the unity within us and, through this, warming the soul we may connect ourselves to Unity itself and, as it were find mooring, taking our stand above everything intelligible within ourselves and dispensing with every other one of our activities, in order that we may consort with it alone and perform a dance around it, leaving behind all the intellections of the soul which are directed to secondary things[22].

Proclus here claims that it is our own striving for unity – which points to Unity itself as what is ultimately desired, even in the lack of any concept or image of unity – that allows us to refer to

[20]. On this passage, see Costa Lima 2012, pp. 119–21 and Vargas (forthcoming). In this context, it is significant that Proclus' explanation of the one of the soul as the source of selfhood arises precisely as he seeks an analogy to divine *pronoia*.
[21].Cf. *In Prm.* VI 1105.32–42.
[22]. Tr. Morrow – Dillon 1987, modified.

Unity itself after total negation. This teaches us something about how the first hypothesis of the *Parmenides* and its negations of all predicates of Unity is supposed to work. It shows us that Proclus interprets the final denial – "Therefore it is not named or spoken of, nor is it the object of opinion or knowledge, nor does anything that is perceive it" (*Prm.* 142a) – not as a denial of every kind of knowledge of unity, but only of the kinds of knowledge that are listed: the will and its reference to unity remain untouched. The first hypothesis thus begins with the One-Being, known through *noesis*, and ends by showing that our reference to Unity itself, as the cause of the Unity of the One-Being, is independent of that *noesis*. Phrased differently, the series of negations shows that when we, together with Parmenides, claim that "All is one" (see *supra* 000 p. 88), the unity we predicate and intend is prior and independent of any insight into being. The argument therefore is not a process of acquiring new knowledge, but of bringing the reasoner to realize that they already possess an innate reference to Unity itself.

Proclus stresses that our striving toward unity is not, as it were, a presence or activity of Unity itself within us (Procl. *In Prm.* VII Finis Graece Redditus 511.3–8):

> But our notion and apprehension of Unity is in our nature itself and does not come about through perception or cognition. The other notions, being cognitions, coexist with their objects and can name them, for their objects can be grasped by them. This notion, however, is not cognitive and does not grasp Unity, but is essentially an operation of nature and a natural desire for unity[23].

The denial Unity's presence in the will is crucial, for it is precisely because Unity itself is not present in the will except as its ever-absent goal that our own unity can serve as a truly absolute and unhypothetical starting point for our thinking, one that contains no duality at all. In this respect, it is significant that Proclus also discusses the unity within us as a *synthema* of Unity[24], as opposed to an *eikon*: for a *synthema* need not resemble what it refers to, nor mirror its content in any way (*In R.* I 198.1–25)[25].

We can compare this to Iamblichus' reply to Porphyry's remark that he "concedes the existence of the gods" (*Myst.* I 3.1–11):

> That is not at all the way to put it. For an innate knowledge about the gods is coexistent with our nature, and is superior to all judgement and choice, reasoning and proof. This knowledge is united from the outset with its own cause and exists in tandem with the essential striving of the soul towards the Good. Indeed, to tell the truth, the contact we have with the divine is not even knowledge. Knowledge, after all, is separated (from its

[23]. Tr. Morrow and Dillon 1987, modified.
[24]. See *Theol. Plat.* II 56.16–57.3, *In Ti.* II 8.9–14 Van Riel (= I 210.3–8 Diehl), *In Or. Chald.* IV 63–66.
[25]. The innate concepts (*logoi*) of the soul, however, are clearly called *eikones* of the forms in *El. Theol.* 195.

object) by some degree of otherness. But prior to what knows another as being itself other, there is the unitary connection with the gods that is natural <and indivisible>[26].

It is important to bear in mind that Iamblichus and Proclus disagreed on the interpretation of the first hypothesis: Iamblichus thought that it concerned Unity and the gods, whereas Proclus believed it concerned only Unity, with the orders of the gods discussed exclusively in the second hypothesis (*In Prm.* VI 1066.13–1070.12). Iamblichus can nonetheless help us to understand more clearly the character of the one of the soul as an absolute starting point. Yet when it comes to the knowledge of the gods in Proclus, we must turn to how our striving for unity functions in the second hypothesis, where positive conclusions are derived concerning the One. How, indeed, can something that is not a judgement – indeed, not even a concept – allow us to draw any positive conclusions at all?

The Use of the Will as Ground in the Second Hypothesis

We should recall that Proclus does not think that the unity of the soul alone suffices for the interpretation of the *Parmenides*. As *In Prm*. VI 1072.1–11 (quoted above, 000 p. 91) states, *nous* and *dianoia* are also required. *Nous* allows us to grasp the definitions of the orders of beings and to intuit the truth of the initial hypothesis, "Unity is", which Proclus interprets to mean "there is the One-Being". What the one of the soul appears to ground is the claim at the beginning of the second hypothesis, that "unity" has a meaning that is entirely independent of beings (Pl. *Prm*. 142b–c):

"Consider from the beginning: if unity is, can it be, but not partake of being?" —"It cannot."
So there would also be the being of unity, and that is not the same as unity. For if it were, it couldn't be the being *of* unity, nor could unity partake of it. On the contrary, saying that 'unity is' would be like saying that unity is unity. But now that is not the hypothesis – namely, what the consequences must be, if unity is unity – but rather if unity is. Isn't that so?"—"Of course."
Is that because 'is' signifies something other than 'unity'?"—"Necessarily."
"So whenever someone, being brief, says 'unity is,' would this simply mean that unity partakes of being?"—"Certainly."[27].

[26].Tr. Clarke *et al.* 2003, modified.

[27]. Tr. Gill – Ryan 1997, modified.

For reasons that would take us too far afield, Proclus interprets this initial passage in what appears to be a topsy-turvy manner. He takes it to claim not that unity participates in being, but that being – or, rather, the One-Being – participates in a unity (a henad). Thus, according to Proclus, Plato is offering an analysis of the One-Being into a triad that comprises a henad, its power, and the One-Being that is generated by the unity through its power (Procl. *Theol. Plat*. III 24.84.20–26):

> This triad is thus the summit of the intelligibles: the unity, the power, the being – i.e., the producer, the product, and the power dependent on the unity, which is brought together with the being. It is this triad, then, that Parmenides immediately presents at the beginning of the second hypothesis, attaching to the unity the simplest participation of *ousia* (…).

The one of the soul preserves this analysis of the One-Being from a possible objection of the following kind: "The 'unity' of Being is a mere abstraction and nothing real – as the first hypothesis' attempt to think a pure unity shows." Against this, one can reply: "I know Unity is something beyond being from my own case, for I desire unity in the necessary absence of any kind of knowledge (and where there is no knowledge, there is no being)".

Furthermore, our unity not only allows us to distinguish Being from Unity; it also allows us to distinguish between Unity, which is entirely unparticipated, and the unity of Being, which must be participated by Being. That is, it allows us to recognize the henad or divine unity that must ground Being's unity. Thus, Proclus can perhaps accept Iamblichus' claim that we know the gods to the extent that we desire the Good – for, in cooperation with *nous*, the one of the soul also allows souls to have an innate knowledge not only of the first principle itself, but also of the gods.

The key to understanding how the cooperation of unity and *nous* allows one to know the gods lies in Proclus' repeated emphasis that each negation in the first hypothesis corresponds to an affirmation in the second (e.g. *In Prm.* VI 1061.17–1063.4). Thus, for instance, the affirmation that "Unity is" at the beginning of the second hypothesis contradicts the negation "Unity is not" at the end of the first hypothesis at *Prm.* 141e9–10. Furthermore, for Proclus, the only predications that can be affirmed – and thus the only ones that can be denied of Unity – are those asserting that Unity itself belongs to a certain "order of being" (*In Prm.* VI 1085.11–1086.7). Examples of such claims would be that unity depends on being in time, or that unity depends on being a self-identical substance. According to Proclus, the former is denied in the conclusions that "Unity is not in time" (*Prm.* 140e–142e, *In Prm.* VII 1212.26–1213.1), and the latter in the denial that Unity is the same as and different from itself and the things other than it (*Prm.* 139b–e, *In Prm.* VII 1176.22–27). According to Proclus, by denying that Unity depends on these specific kinds of unification, the argument also shows that Unity is distinct from the specific kind of unities or henads that are

responsible for producing it, what he calls an "order of the gods". Thus, the negations of the first hypothesis reveal Unity's transcendence with respect to all these divine orders.

The affirmative conclusions of the second hypothesis, however, all demonstrate – by contrast – the existence of these divine orders. As we saw above, the very first conclusion allows Proclus to distinguish within Parmenides' One-Being the divine unity or god responsible for unifying it. To see how the one of the soul continues to function as a productive ground, let us also consider the following conclusion, that Unity is a whole and has parts (Pl. *Prm*. 142c7–d8):

> "Let's again say what the consequences will be, if unity is. Consider whether this hypothesis must not signify that unity is such as to have parts."—"How so?"
> "In this way: if we state the 'is' of the One-Being (τοῦ ἑνὸς ὄντος), and the 'unity' of the Unity that is (τοῦ ὄντος ἑνός), and if being and oneness are not the same, but both belong to that same thing that we hypothesized, namely, the One-Being, must it not itself, since it is one, be a whole, and the parts of this whole be Unity and being?"—"Necessarily."
> "Shall we call each of these two parts a part only, or must the part be called part of the whole?"—"Of the whole."
> "Therefore whatever is one both is a whole and has a part."—"Certainly" [28].

"The One-Being" appears here alongside the inverse expression, "the Unity that is". If Proclus explicitly interprets the former as Being itself in its transcendent unity beyond all beings (see *supra* 000 p. 88), then "the One that is" can plausibly be understood as the henad that unifies the One-Being and whose existence is, for Proclus, demonstrated in the previous argument of the second hypothesis. From a Procline point of view, then, this argument now shows that once the distinction between the One-Being and its henad has been drawn, we are confronted with a new kind of unity: the unity of a whole composed of parts. This whole exists because of the causal relationship between them, whereby the henad produces the One-Being through its power. As Proclus puts it "from the power [of the henad] comes the second kind of wholeness [i.e. being a whole of parts: see *El. Theol*. 65]" (*Theol. Plat.* III 25.88.18). Thus, once we have analyzed the One-Being into Being and its specific kind of unity, this analysis requires the existence of another kind of unity: the specific unity of wholeness. The one-of-the-soul is necessary in maintaining the separation of one of the two parts of this wholeness, the henad or separately existing unity, and also in ensuring a further conclusion that according to Proclus is demonstrated in the passage quoted above: namely, that there is a henad responsible for this particular kind of unity, i.e., wholeness, in the same way that the first argument showed that there is a henad responsible for the peculiar kind

[28]. Tr. Gill – Ryan 1997, modified.

of unity that is being. In the argument of the second hypothesis, then, the one of the soul always serves two functions: on the one hand, our innate knowledge of unity beyond being grounds the analyses in all the prior arguments, thereby enabling the separation of the henads from the beings they cause; on the other, in each new argument, a new kind of unity of being is established (wholeness, totality, number, and so on). For each of these unities, the one of the soul enables the analysis of that new unity of being into a kind or order of being on the one hand, and a transcendent unity or henad on the other, which unifies this kind or order of being. This constant and renewed analysis drives the argument toward its conclusion that there are ever-new and increasingly complex forms of unity, ultimately producing, according to Proclus, an exhaustive list of all the orders of the gods and the kinds of being they are responsible for (*In Prm.* VI 1083.1–1088.2).

Conclusions

In conclusion, our will, the unity of the soul, grounds the science of dialectic (and thereby, all sciences) by requiring that *dianoia*, our discursive thought, at every step break up and analyze the simple unity of being grasped in *noesis* into modes of being and orders of gods. Thus, it is not surprising that the deduction of the essence of the transmigrating soul and its thought in time appears in the third hypothesis, according to Proclus, motivated by the need to both deny and affirm all predicates of unity[29]. *Dianoia* arises precisely because of the interplay of Unity, represented by the first hypothesis, and *nous*, represented by the second hypothesis.

In any case, it should be clear that this absolute ground – this willing of our own selfhood, which has no further cause – does not function as a self-evident proposition from which all science is derived. Rather, it provides science with its foothold in the absolute, but science itself still requires *nous* (the insight into self-evident unities) and *dianoia* (discursive thought). In this regard, there is an analogy between Proclus' theory of causes and his account of dialectic. In his view of causes, a cause essentially remains what it is, and its effect manifests itself around the cause[30]. Correspondingly, in his account of dialectic, the knowledge of unity remains what it is, and *nous* and *dianoia* generate science by circling around the absolute certainty of our will.

Finally, it should be obvious to all that none of this is present in the *Elements of Theology.* The *Elements of Theology* is an attempt to render the theorems of theology in a mathematical and

[29]. Pl. *Prm.* 1554–6; see Trouillard 1972, pp. 133–54.

[30]. Procl. *El. Theol.* 30.6–7: τὸ γὰρ πάντῃ προϊὸν οὐδὲν ἂν ἔχοι ταὐτὸν πρὸς τὸ μένον, ἀλλ'ἔστι πάντῃ διακεκριμένον. What goes forth entirely has "nothing" in common with its cause, which implies that the cause, qua cause, has no activity" or "effort" emanating from it, because it is entirely immutable. Therefore, the effect must possess an aspect of its own that remains in the cause, and from which it separates itself in its production.

thus more easily learnable form, but it does not do so dialectically. It does not start from an unhypothetical first principle; rather, it seeks to discover a set of basic hypotheses from which all theological truths can be derived. It is an incomplete attempt, and that is why there is no list of axioms in the text as we have it. Nevertheless, this is the direction it tends toward – this is the project, insofar as we can recognize one in the text. But even if this project were completed, it would never start from an unhypothetical first principle, because the mode of reasoning here is fundamentally different from that in Plato's *Parmenides* (in light of the *Parmenides* commentary).

Bibliography

M. Bonelli, *Proclus et la dialectique scientifique*, in J.-B. Gourinat – J. Lemaire (eds.), *Logique et dialectique dans l'Antiquité*, Paris, Vrin, 2016, pp. 267–84.

E.C. Clarke – J. M. Dillon – J. P. Hershbell (eds.), *Iamblichus*, De Mysteriis, Atlanta, Society of Bibl. Literature, 2003.

D. Costa Lima, *Sementes para uma leitura transcendental da filosofia do neoplatonismo tardio*, in «Revista Ética e Política», 24.2 (2021), pp. 34–6.

M.L. Gill – P. Ryan, Plato, *Parmenides*, in J. M. Cooper – D. S. Hutchinson, *Plato, Complete Works*, Indianapolis and Cambridge, Hackett, 1997, pp. 359–97.

S.-A. Kiosoglou, *Variations of reception. Proclus' axiomatic method of presentation and some milestones of its legacy*, Doctoral Dissertation, KU Leuven, 2023.

A. Lernould, *La dialectique comme science première chez Proclus*, in «Revue des Sciences Philosophiques et Théologiques», 71.4 (1987), pp. 509−36.

A. Lernould, *La ΣΤΟΙΧΕΙΩΣΙΣ ΘΕΟΛΟΓΙΚΗ de Proclus. Science première et dépassement de la méthode hypothético-déductive*, in G. Aubry – L. Brisson – Ph. Hoffmann – L. Lavaud (eds.), *Relire les* Éléments de théologie *de Proclus : Réceptions, interprétations antiques et modernes*, Paris, Hermann, 2021, pp. 91–131.

G. R. Morrow – J. M. Dillon (eds.), *Proclus' Commentary on Plato's* Parmenides, Princeton, Princeton University Press, 1987 (Princeton, Princeton University Press, 1992).

D.J. O'Meara, *Pythagoras Revived. Mathematics and Philosophy in Late Antiquity*, Oxford, Clarendon, 1989.

J. Opsomer, *Organiser la philosophie selon ses éléments. Structures argumentatives dans les* Éléments de Théologie, in G. Aubry – L. Brisson – Ph. Hoffmann – L. Lavaud (eds.), *Relire les* Éléments de théologie *de Proclus : Réceptions, interprétations antiques et modernes*, Paris, Hermann, 2021, pp. 133–76.

J. Opsomer, *Proclus'* Elements of Theology *and Platonic dialectic*, in D. Calma (ed.), *Reading Proclus and the Book of Causes*, Vol. 3: *On causes and the noetic triad*, Leiden / Boston, Brill, 2022, pp. 17–36.

G. J. Ruz Troncoso, *The Argumentative Structure in Proclus' Elements of Theology*, Doctoral Dissertation, KU Leuven, 2024.

C. Steel, *Theology as First Philosophy. The Neoplatonic Concept of Metaphysics*, in «Quaestio», 5 (2005), pp. 2–21.

B. Strobel, *Proklos: Tria Opuscula. Textkritisch kommentierte Retroversion der Übersetzung Wilhelms von Moerbeke*, Berlin, De Gruyter, 2014.

J. Trouillard, *L'Un et l'âme selon Proclos*, Paris, Les Belles Lettres, 1972.

A. Vargas, *Os Elementos da Teologia*, São Paulo, Odysseus, 2024

A. Vargas, *The Conspiracy of the Good: Proclus' Theodicy* qua *Political Theological Paradigm*, in «Political Theology», 21.8 (2020), pp. 723–37.

ABSTRACT

Antonio Vargas, *Can the Will Ground Science?*

This chapter has two aims: to determine the ultimate ground of knowledge in Proclus and to argue that his answer to the problem of an unhypothetical first principle is not a self-evident proposition. The author contends that the science grounding all knowledge on an unhypothetical first principle is the Parmenidean dialectic, and Proclus's Elements of Theology, despite its *more geometrico* style, is not an instance of this ultimate science. The speculation that it might be so considered because the Elements lacks an initial list of axioms is challenged; the author argues this absence is insignificant given the text's overall deficiencies vis-à-vis the geometric ideal. Furthermore, the Elements fails to utilize what Proclus, in the Parmenides Commentary, identifies as our true access to an unhypothetical first principle: the so-called "one of the soul." Based on the De Providentia, this "one of the soul" can be partially identified with the will. The essay thus complements its negative argument regarding the Elements with a positive argument demonstrating how the will in Proclus can serve as a viable ground for knowledge.

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II

MATHEMATICS: A BRIDGE TO THEOLOGY

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PROCLUS ON THE USEFULNESS OF MATHEMATICS FOR THEOLOGY

How Can Geometry Prepare for Theology?

In the first prologue of his *Commentary on Euclid*, Proclus discusses at length why it is useful for philosophers to study mathematics[1]. He draws upon a long-standing tradition in defence of mathematics, beginning with Plato, who emphasised the importance of mathematics, particularly geometry, in the advanced education of philosophers, the future leaders of the state. However, not all philosophical schools accepted this Platonic prejudice in favour of mathematics. Proclus refers to Zeno of Sidon, an Epicurean who attacked the scientific nature of geometric proofs[2]. Neither the Epicureans nor the Stoics considered the study of mathematics to be a valuable pursuit for a philosophical life. They observed that advanced mathematics with demonstrations of general theorems offered little practical benefit. Applied sciences that dealt with specific problems, such as mensuration, were considered to be more useful in everyday life. Even within the Platonic school, as Proclus mentions, some people questioned the value of mathematics as a science. They referred to Plato's criticism of mathematics in comparison to dialectic in the *Republic*. However, Proclus offers an extensive response to these arguments, defending the essential role of mathematics in intellectual formation alongside Plato, though he acknowledges that it is not first philosophy[3]. He refers to mathematics with an expression coined by Plato in the *Timaeus*: "the paths of education" (τῶν κατὰ παίδευσιν ὁδῶν). The phrase is used at a pivotal moment in the exposition. Before explaining how the four elementary bodies are formed from a combination of triangles, Timaeus warns his audience that his account will be

1. For some arguments, this contribution draws upon the introduction I composed together with Alain Lernould for our new edition with a French annotated translation of Proclus' commentary on Euclid. For various reasons, the publication of this edition by Vrin has been delayed and is now scheduled for early 2026. To avoid complications with other contributions in this volume, references will not be to the new edition, but to Friedlein 1873. Fortunately, the old edition is in most cases reliable. When I diverge from Friedlein's edition, a variant reading will be noted. For quotations and paraphrases in English I rely on Morrow's 1992 excellent translation, adapting it freely.
2. On the Epicurean criticism of geometry, and in particular of Zenon of Sidon, see Benatouïl 2010.
3. See on this discussion within the Platonic school: *In Eucl*. 29.14–32.20.

"unfamiliar": "but since you have been schooled in the 'paths of education' through which I must demonstrate what I am saying, you will follow me"[4]. The context makes it clear that Timaeus assumes that his audience is well trained in geometry, as all his arguments start from geometrical figures (triangles) and their properties. As Proclus explains, Timaeus uses the phrase κατὰ παίδευσιν ὁδοί because in his view mathematics is related to philosophy in the same way that education leads to a virtuous life (*In Eucl.* 20.14–17):

> As education prepares the soul (προευτρεπίζει) for a perfect life through infallible dispositions, mathematics makes ready our understanding and the eye of our soul for turning towards the upper world[5].

As he explains later, one should practise mathematics for its own sake and not for some *external* utility (*In Eucl*. 28.1–7):

> However, if one must relate its usefulness to something outside of itself, it is to intellectual insight that it must be said to contribute. For to that it leads the way and prepares us (ποδηγεῖ καὶ προευτρεπίζει) by purifying the eye of the soul and removing the hindrances from the senses for the knowledge of the whole of being.

Just as the purifying virtues are not useful for the necessities of life but prepare the soul for the contemplative life, in the same way one must relate the study of mathematics to the achievement of ultimate wisdom (*In Eucl*. 49.9–12).

> We say that geometry releases us from the sensible world and leads us to the incorporeal being and that it habituates us to the view of the intelligibles and prepares us for activity according to the intellect[6].

Proclus adopts the term "habituation" (συνεθισμόν) from Plotinus, whom he quoted earlier. It is not enough to have a soul that is "by nature" philosophical; it must be educated (*In Eucl.* 21.19–24):

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4. Pl. *Ti*. 53c1–4: ἀήθει λόγῳ πρὸς ὑμᾶς δηλοῦν, ἀλλὰ γὰρ ἐπεὶ μετέχετε τῶν κατὰ παίδευσιν ὁδῶν δι' ὧν ἐνδείκνυσθαι τὰ λεγόμενα ἀνάγκη, συνέψεσθε.
5. [15 ἤθεσιν correxi (cf. 23.24 and *In R.* I 161.28): ἔθεσιν Friedlein with M].
6. τὴν γεωμετρίαν τῶν αἰσθητῶν ἡμᾶς ἀπολύειν φήσομεν καὶ περιάγειν εἰς τὴν ἀσώματον ὑπόστασιν καὶ συνεθισμὸν εἶναι πρὸς τὴν θέαν τῶν νοητῶν καὶ προευτρεπίζειν εἰς τὴν κατὰ νοῦν ἐνέργειαν.

> Certainly, someone of this nature has already from himself awakened and taken flight towards being, but one must give him, says Plotinus, mathematics to accustom him with the incorporeal nature, and, while he uses mathematics as steps, it is necessary to lead him to the reasonings of dialectic and, in a general way, to the study of beings[7].

Proclus also uses the phrase "paths of education" in his introduction to the *Platonic Theology*, when he discusses the necessary preparations and dispositions for studying theology. First, the student of theology must be purified by all moral virtues. Next, he should be trained in logic. Thirdly, he should have studied physics in order to investigate "the nature of the separate and primordial substances" and become acquainted with "the truth in appearances". Finally, he should not be ignorant neither of the "paths of education": "for through them we know the divine being more immaterial"[8]. Proclus is himself an excellent example of such an education, which he received from Heron in Alexandria. His teacher is said to have had a perfect training in the mathematical "paths of education"[9].

We may, therefore, conclude with Proclus that the utility of mathematics should not be measured by considering human needs. As with all theoretical activities "it is worthy of pursuit for its own sake and not merely because it satisfies human needs" (27.28–28.1). If we are to continue discussing its usefulness, we must consider its essential role in the formation of a philosophical mind and its ascent to higher immaterial reality. Proclus also makes this point in his introduction to the commentary. He wants:

> to emulate the Pythagoreans whose byword (σύμβολον) was "a figure and a stepping-stone" (βᾶμα), not "a figure and three obols". By this they meant that we must cultivate that (ἐκείνην) sort of geometry that, with each theorem, lays the basis for a step upward and elevates the soul to the higher world[10].

"Prefer the figure and step instead of the figure and three obols" is listed as the 36th Pythagorean *symbolon* in Iamblichus' *Protrepticus*. Iamblichus explains it as an exhortation to study mathematics with the goal of contemplating the incorporeal beings

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7. Cf. Plot. *Enn*. I 3 [20], 3.5–6 and 9–10.
8. Μήτ' οὖν ταύτης, ὅπερ εἴπομεν, τῆς ἐν τοῖς φαινομένοις ἀληθείας, μήτε αὖ τῶν κατὰ παίδευσιν ὁδῶν καὶ τῶν ἐν αὐταῖς μαθήσεων ἀπολελείφθω· διὰ γὰρ τούτων αὐλότερον τὴν θείαν οὐσίαν γινώσκομεν.
9. Marin. *V. Pr.* § 9.15: ἐπὶ δὲ μαθήμασιν Ἥρωνι ἐπέτρεψεν ἑαυτόν, ἀνδρὶ θεοσεβεῖ καὶ τελείαν παρασκευὴν ἐσχηκότι τῶν κατὰ παίδευσιν ὁδῶν.
10. *In Eucl*. 84.15–20: ζηλοῦντες τοὺς Πυθαγορείους, οἷς πρόχειρον ἦν καὶ τοῦτο σύμβολον „σχᾶμα καὶ βᾶμα, ἀλλ' οὐ σχᾶμα καὶ τριώβολον" ἐνδεικνυμένων, ὡς ἄρα δεῖ τὴν γεωμετρίαν ἐκείνην μεταδιώκειν, ἣ καθ' ἕκαστον θεώρημα βῆμα τίθησιν εἰς ἄνοδον καὶ ἀπαίρει τὴν ψυχὴν εἰς ὕψος,

(p. 150). A later commentator, Elias, interprets this goal as a speculation about the divine: "let us make on each figure a step up to theology[11]. Three obols (a day pay given to members of the assembly or tribunal in Athens) stands for the ordinary applied mathematics, for calculation involved in mensuration and commerce. In the *Vita Pythagorica* (§ 5.21–24), Iamblichus recounts how Pythagoras, having observed a young man in Samos wasting his time on physical training, convinced him to study mathematics with him, promising to pay for his training. Pythagoras paid him three obols for each figure he learnt. After some time, Pythagoras said he had no more money to pay him. However, by that time, the young man had become so highly motivated that he continued his study of mathematics without any financial reward. A similar anecdote is told about a disciple of Euclid who once asked him, after having studied the first proposition, what profit he will obtain from having studied it. Euclid said, "give him a threeobol" for he must have some profit from what he learned[12]. Proclus shares the Pythagorean view. According to him, the only benefit of studying mathematics is "elevating the soul to the higher world, instead of letting it descend among sensible things to satisfy the common needs of mortals" (84.20–23).

However, this emphasis on mathematics as a study worthy of choice for its own sake should not make us forget that it also has a utility that extends to all sciences and all technical arts. Drawing inspiration from Iamblichus' arguments in *On common mathematics* Proclus discusses how mathematics contributes to the study of theology, physics, ethics, and politics, and to the different applied arts[13]. I will focus in this contribution on the utility of mathematics for theology, which Proclus formulates as follows (*In Eucl*. 22.1–16):

> Mathematics prepares for theology intellectual insights (προευτρεπίζει τὰς νοερὰς ἐπιβολάς). Mathematical arguments reveal through images (εἰκόνων) truths concerning the gods that are difficult for imperfect (minds) to discover and (too) elevated to discern, making them trustworthy, evident, and irrefutable. For they show, reflected in numbers, the properties of that which transcends being and reveal in the objects of discursive thought the powers of intellectual figures. That is why Plato teaches us many wonderful doctrines about the gods by means of mathematical forms and the philosophy of the Pythagoreans also uses such veils to cover its

[11.] El. *In Porph.* 28.21–23: οὕτω γάρ φασι καὶ οἱ Πυθαγόρειοι "σχᾶμα καὶ βᾶμα" ἀντὶ τοῦ 'καθ' ἕκαστον σχῆμα βαθμὸν ἄνιμεν ἐπὶ θεολογίαν.
[12.] Stob. *Anth.* II 31.114 Wachsmuth: Παρ' Εὐκλείδῃ τις ἀρξάμενος γεωμετρεῖν, ὡς τὸ πρῶτον θεώρημα ἔμαθεν, ἤρετο τὸν Εὐκλείδην· "τί δέ μοι πλέον ἔσται ταῦτα μαθόντι;", καὶ ὁ Εὐκλείδης τὸν παῖδα καλέσας "δός", ἔφη, „αὐτῷ τριώβολον, ἐπειδὴ δεῖ αὐτῷ ἐξ ὧν μανθάνει κερδαίνειν".
[13.] See on the contribution of mathematics to the different sciences O'Meara 1989, pp. 160–4.

> mystical initiation into the divine doctrines. So is the entire *Sacred Discourse*[14], so does Philolaos in the *Bacchae*[15] and so is the whole way in which Pythagoras teaches (τρόπος τῆς ὑφηγήσεως) about the gods[16].

The way of teaching of Pythagoras. As is well known, Proclus distinguishes four *tropoi* of theology, one through oracular language (Chaldaean way), one through myths (Hesiod, Orphic hymns), one through (mathematical) images (Pythagorean way), one through dialectic[17]. The privileged Platonic way of theology is the rational scientific approach, as displayed in the dialectic of the *Parmenides*. Yet Plato occasionally uses the three other modes; he inserts in the dialogues myths, he writes an almost oracular speech on Love in the *Phaedrus*, and uses arithmetic and geometry to explain the structure of the divine world soul. However, the Pythagorean theology, as exemplified by Philolaos and the Sacred Discourse, is entirely in the mathematical mode (Procl. *Theol. Plat*. I 4.20.8–11):

> The mode of exposition that makes use of images is Pythagorean, since the discovery of mathematics was made with a view to the recollection (ἀνάμνησιν) of divine principles, and by means of these entities as images they endeavoured to gain access to the principles there; hence they consecrated numbers and figures to the gods.

When discussing the utility of mathematics to theology Proclus draws upon Iamblichus, who even used the formula προευτρεπίζει παρασκευὴν τῇ θεολογίᾳ[18]. Among the different ways to study mathematics Iamblichus puts the theological first (*Comm. Math*. 63.24–30):

> The first way to study any mathematical object (wether generic or specific) is the theological approach, which harmonizes (συναρμοζόμενον) it according to its rank and acts with the being and power of the gods through a suitable analogy. This is indeed what deserves special attention

14. On the ἱερὸς λόγος attributed to Pythagoras, see Thesleff 1965, pp. 164–8, and the recent update Huffman 2024.
15. On (pseudo-)Philolaos see Huffman 1993, on Proclus' use of Philolaos in his commentary see Steel 2007.
16. ὅτι θεολογίᾳ μὲν προευτρεπίζει τὰς νοερὰς ἐπιβολάς. ὅσα γὰρ τοῖς ἀτελέσι δυσθήρατα καὶ ἀνάντη φαίνεται τῆς περὶ τῶν θεῶν ἀληθείας εἰς διάγνωσιν, ταῦτα οἱ τῆς μαθηματικῆς λόγοι πιστὰ καὶ καταφανῆ καὶ ἀνέλεγκτα διὰ τῶν εἰκόνων ἀποφαίνουσι. τῶν μὲν γὰρ ὑπερουσίων ἰδιοτήτων ἐν τοῖς ἀριθμοῖς τὰς ἐμφάσεις δεικνύουσι, τῶν δὲ νοερῶν σχημάτων ἐν τοῖς διανοητοῖς τὰς δυνάμεις ἐκφαίνουσιν. διὸ καὶ ὁ Πλάτων πολλὰ καὶ θαυμαστὰ δόγματα περὶ θεῶν διὰ τῶν μαθηματικῶν εἰδῶν ἡμᾶς ἀναδιδάσκει καὶ ἡ τῶν Πυθαγορείων φιλοσοφία παραπετάσμασι τούτοις χρωμένη τὴν μυσταγωγίαν κατακρύπτει τῶν θείων δογμάτων. τοιοῦτος γὰρ καὶ ὁ ἱερὸς σύμπας λόγος καὶ ὁ Φιλόλαος ἐν ταῖς Βάκχαις καὶ ὅλος ὁ τρόπος τῆς Πυθαγόρου περὶ θεῶν ὑφηγήσεως.
17. See on the four modes of theology, Steel 2005, Steel 2007, Hoffmann and Gavray 2024.
18. Iambl. *Comm. Math.* 55.8–9 (ch. 15). On Iamblichus' scientific theology, see Lecerf – Taormina 2024.

> from the Pythagoreans [τοῖς ἀνδράσιν]; for instance, in the case of numbers, which sorts of numbers are of like nature and akin to which sorts of gods, and in the case of other branches of mathematics they are used to think the same[19].

Theological mathematics consists in finding similitudes, analogies between mathematical objects such as numbers or figures with different classes of gods and their respective powers and acts. Like Iamblichus, Proclus recognises two modes of images in the Pythagorean theological tradition: either numbers or geometrical forms. The former is found in the tradition of theological arithmetic, best known by the *Theologoumena arithmeticae*[20]. The second method involves the use of geometric figures such as lines, triangles and circles. Philolaos, quoted by Proclus, provides an interesting example of this method. According to Proclus, geometrical arguments may reveal "the powers of intellectual figures in the objects of discursive thought" (τῶν δὲ νοερῶν σχημάτων ἐν τοῖς διανοητοῖς τὰς δυνάμεις). Τὰ διανοητά is the realm of being that corresponds to discursive thought (διάνοια) and is distinguished both from the intelligible and the sensible. As Proclus shows in the opening paragraph of the prologue (following Plato's analogy of the divided line in *Resp*. VI), the mathematical objects and the knowledge related to them occupy that intermediate region. In this realm one may study triangles and other figures in their essence and powers (and the effects they have in the construction and demonstration of theorems). However, when considered from a theological perspective, these geometrical figures can reveal properties and powers of gods. For certain classes of gods, as we shall see, are characterised by "intellectual figures" that transcendently anticipate the powers of geometrical figures[21]. Thus, the study of geometry can offer, opportunities to grasp through images the divine paradigms of its objects.

In fact, as Proclus says, mathematics is a preparation for theology because it offers through images intuitive insights (νοεραὶ ἐπιβολαί) that transcend the multiplicity of discursive argumentation[22]. These insights are necessary in order to understand divine beings, which are themselves beyond all multiplicity[23]. We can only grasp their nature by an act of the intellect. Mathematics prepares us for theology, helps us to reach this intellectual intuition of divine orders not directly by its definitions and demonstrations

19. Cf. Dillon – Urmson 2020, p. 73.
20. See on this work the recent study by Salerno 2024.
21. See below, 11 and Steel 2007.
22. Proclus uses a similar expression for Cratylus' attempt at intuitive knowledge of the gods through images of "names": τὴν ἐκεῖ διὰ τῶν εἰκόνων ἐπιβολήν (*Theol. Plat*. VI 12.60.3–4).
23. See *Theol. Plat*. II 12.73.9–11).

but because some of its basic notions (not only figures, but also propositions) can be used as images (*eikones* or *sumbola*) to "indicate" a truth that is hard to reach by demonstrations. As Proclus says, a certain argument "may offer an indication": ἴσως ἔνδειξιν ἂν φέροι (102.4; 110.4) ἔχοι ἂν ἔνδειξιν (291.15) εἰς ἔνδειξιν (91.11)[24]. Some examples of geometrical symbols are obvious, and they belong to a long tradition of Pythagorizing reading of geometry. Points, lines, surfaces, triangles, cubes, circles, spheres can be used as symbols to reveal fundamental metaphysical principles. Take 108.10–15: the straight line is a symbol of the inflexible providence, and the circumference a symbol of the activity that returns to itself; or 115,10: "if we take this [i.e. definition VI of lines as limits of a surface] as image (ἀπὸ τούτων ὡς εἰκόνων ληπτέον) we can understand that every simpler being supplies a limit to what follows"; 164.8: the straight line is symbol of procession and movement and infinity; 290.17: the line at right angles is symbol of life lifting itself to the upper world, the perpendicular is an image of life following the path downwards, the right angle is symbol of undeviating activity.

However, in some cases, the supposed images do not actually help us to understand the superior divine beings. In fact, one must be well versed in a theological system, such as Chaldean, Orphic or Proclus' own Platonic theology, to understand how a certain element of a gometrical argument may function as an image of a paradigmatic reality. Take, for instance, the commentary on the first Proposition (the construction of an equilateral triangle on a straight line 213.14f.). Proclus noticed that Euclid in his demonstration constructs two circles that enclose this triangle (each of them in part). "This seems to indicate in a likeness (ὡς ἐν εἰκόσιν)", he notices, "how the things that proceed from first principles receive perfection, identity and equality from these principles" and "how that things that move in straight line are carried about in a circle through the eternal world-process, and souls are likeness of the immovable activity of Nous because of their periodic return" (213.21-26). So far, we may understand the analogies. But what follows is a more difficult step: "It is said also that the live-giving source of souls is bounded by a twofold Intellect" (214.2–3). Specialists in Proclus' theology may identify the "source of souls" with Rhea, who occupies the second place in the first triad of intellectual gods between two intellects, Kronos and Zeus. Proclus

[24] Simplicius (*In Cael*. 641.23–28) notices that Timaeus uses the verb ἐνδείκνυσθαι in his remarks on the "unusual (geometrical) account he will give to explain the elementary bodies". In Simplicius' view, this is an indication that we should not take this hypothesis literally: "Plato makes clear that these things are like the hypotheses used by the astronomers with which it is possible to save the phenomena".

integrated this Orphic-Chaldean doctrine in his Platonic theology[25]. Only in that context one may understand how Euclid's demonstration of the first proposition can function as "image" of a theological doctrine. But how could Proclus still claim that Euclid's proof of Proposition 1 reveals truths about the gods that are difficult for imperfect minds to discover, and makes them "trustworthy, evident, and irrefutable" through images? In this case, the geometrical demonstration is not an image helping us to understand a divine truth, it is the inverse: unless one is familiar with a particular theological system it is not possible to recognize images in the argument. For the non–initiated it is "obscura per obscuriora". And yet Proclus concludes this digression in a Pythagorean manner: "But all this reminds us, as through images, of the nature of things" (Ἀλλὰ ταῦτα μὲν ὡς ἐξ εἰκόνων ἡμᾶς ἀναμιμνησκέτω τῆς τῶν πραγμάτων φύσεως 214.12–13). *Anamnesis* of the divine was, as we have seen, the main motivation for the Pythagoreans to study mathematic.

Rather than being images that evoke divine beings, the geometrical definitions and arguments serve as triggers for theological digressions on topics dear to Proclus. Take 91.1–3: "if I may give my own view (εἴ με δεῖ τοὐμὸν εἰπεῖν), the centres of all the spheres and poles are symbols of the wry-necked gods (τῶν ἰυγγικῶν)". The epitheton ἰυγγικός used to designate a particular class of gods originates from the Chaldean oracles (see fr. 77). In his *Platonic Theology* (IV 1.6.10–18) Proclus makes this class corresponds to what is called in the *Phaedrus* the "supercelestial place" (247c), which represents the first triad of intelligible-intellectual gods. When introducing "his own view" in the Euclid commentary, Proclus gives a summary of his doctrine of the three triads of the intelligible-intellectual gods showing how they are represented in this world. As is evident again in this case, Proclus uses the argument to find a mathematical confirmation of a doctrine from his Platonic theology. Another example of this "exegetical" practise can be seen in his speculations about a "triadic god who proceeds from 'the hidden world' (= *Or. Chald.* fr. 198) and comprehends in himself the primary cause of the rectilinear figures" (155.19–21). Glenn Morrow suggested that that this name 'triadic' "given by the most mystic theology" might be a reference to Hermes Trismegistos, what in this context is implausible[26]. It is again a reference to the first triad of the intelligible-intellectual gods.

In sum, one gets the impression that Proclus wants to confirm his complex Platonic theology, and above all the doctrine on the triads of intelligible-intellectual

25. See *Theol. Plat*. V 11.36.12–38.9.
26. See Morrow 1970, p. 123 note 120, see also 38, n. 88 and Goulding 2022, p. 382 note 35.

gods, with references to Euclid's geometry. In that way, the Platonic theology, which finds its ultimate scientific foundation in the first and second hypothesis of the *Parmenides*, not only integrates the oracular wisdom of the Chaldean tradition and of the Orphic mythology but also allows us to interpret symbolic elements of geometry. The idea that we have intuitive access (ἐπιβολή) to this complicated theology through geometrical figures serving as images is wishful thinking. In fact, we should first study Proclus' Platonic theology in order to understand which gods the images represent.

Text and Theoria

Proclus' commentary on Euclid's work is clearly different from those of his predecessors, Heron and Pappus (which, unfortunately, we only know partially). Although he does not neglect a technical-mathematical approach to Euclid's text, he is not primarily interested in it. His main interest lies in *theoria*: a speculative approach that investigates the reality (τὰ πράγματα) corresponding to mathematical objects, as well as the first causes and principles. This approach is evident in the way he presents his commentary after the prologues. He urges his readers not to expect a technical analysis from him, the kind usually found in commentaries on Euclid. As he says, there are already enough explanations of the technical aspects of Euclid's *Elements*. "We are surfeited with those topics and shall touch on them only sparingly (...). But whatever involves a speculation more about reality (πραγματειωδεστέραν θεωρίαν) and contributes to philosophy as a whole, we shall make it our chief concern to mention" (84.11–15). Not only in this introduction, but also in the conclusion of his commentary Proclus expresses the same view, as an advice to those who might consider continuing his commentary beyond book One (432.15–19):

> For the commentaries (ὑπομνήματα) which are now in circulation present much confusion of every kind: they do not expose the causes, and we see in them neither dialectical judgment (κρίσιν διαλεκτικήν) nor philosophical speculation (θεωρίαν φιλόσοφον).

Throughout the commentary we find remarks with the same emphasis (200.15–16 and 294.13–14):

> we should give greater attention to the examination of reality (τῇ τῶν πραγμάτων ἐπεξεργασίᾳ) than to the variety of cases.

so much can we derive from this theorem for understanding the whole of things (εἰς τὴν τῶν ὅλων φύσιν).

From all these passages the same interest emerges: one must approach Euclid's text as a philosopher, try to understand how the definitions and propositions relate to reality (τὰ πράγματα), seek to establish the causes in the demonstrations. By proceeding in this way, says Proclus, geometry comes close to the "first science" (243.18), i.e. metaphysics, and even theology, since the first principles of beings are divine.

Proclus often indulges in speculative considerations of first principles. For example, after his explanation of the definition of a line (I 2), he provides an extensive exposition of the Pythagorean doctrine on points, lines, surfaces and solids, and their respective correspondences with the monad, dyad, triad and tetrad, and finds confirmation of this doctrine in the *Parmenides*. Proclus concludes (100.4–5): "So much can be said about the line according to more speculative views" (κατὰ τὰς θεωρητικωτέρας ἐπιβολάς)". Another example is the *theoria*, which comes after the explanation of propositions 11 and 12. "If we must now add a speculative consideration (θεωρίαν) to these last two problems, it seems that the straight line raised at right angles imitates Life lifting itself up to the upper world (...) whereas the perpendicular is a likeness of life following the path downwards (...) For the right angle is a symbol of undeviating energy (…)" (290.15–22) Or after having discussed what the circle are, and its circumference are (definition 15/16): "let us once more ascend from these to the consideration of their paradigms (εἰς τὴν τῶν παραδειγμάτων ἀναδράμωμεν θεωρίαν)" (153.12–13).

In many cases the theory offers theological speculations about different classes of gods. As I mentioned earlier, the commentary on Proposition I,1 demonstrates that the equilateral triangle and the two circles used to construct it are images of the first triad of intellectual gods. However, the conclusion does not make any explicit theological claims: "Let these remarks bring us to reminiscence, as from images, of the nature of things" (214.14–15).

Definition 9 offers Proclus the occasion for a long speculation on how the angles are symbols of divine connective powers, with reference to Philolaos. Proclus concludes: "Thus these [features of the angles] bring us around to the consideration of beings (τὴν τῶν ὄντων θεωρίαν)" (130.22). The commentary on the definition of trilateral figures (24–29) leads to a long speculation (θεωρεῖν 168.3) on how and why

the Pythagoreans, and in particular Philolaos, dedicated the angles of triangles to particular gods, Kronos, Hades, Ares and Dionysos[27]. Similarly, when commenting on the definition of quadrangular figures (30–34), Proclus draws upon Philolaos to develop the analogies between the angles of these figures and certain divinities. He concludes (174.17–21):

> This is all we could say about quadrilaterals to explain the thought of the author of Elements and to give those who wish to know the intelligible and invisible beings (τῶν νοητῶν καὶ ἀφανῶν) starting points (ἀφορμάς) towards more speculative intuitions (θεωρητικωτέρας ἐπιβολάς).

This conclusion highlights two objectives of the previous commentary: (1) to explain Euclides' argument and (2) to provide a starting point for "a speculative contemplation (intuition)" of invisible (divine) beings. As we have seen, Proclus used the term ἐπιβολή when examining the contribution of mathematics to theology. Mathematics can prepare for theology by providing intuitions.

Proclus clearly likes to use the term θεωρία for these speculative considerations, often connecting it with πράγματα or πραγματειώδης, once with φιλόσοφος but never with θεολογικός. The πραγματειώδης θεωρία[28] is distinguished from, and opposed to, the λέξις which offers a technical exegesis of the argument, as is the practice in Proclus' commentaries on Plato[29]. As we have seen, most of these speculative considerations deal with metaphysical questions that are beyond the scope of mathematics. Some of them are explicitly theological, making connections with deities. This raises the question of this commentary's overall purpose, in addition to its immediate purpose of explaining Euclid's text. Do why find this overall purpose in de speculative *theoriai*? As is well known, Proclus conceived of his Platonic theology as an endeavour to unify all interpretations of Plato's texts (τὴν τῶν λέξεων ἔφοδον), as set out in his commentaries, into a comprehensive and holistic theory (πραγματειώδη θεωρίαν) applicable to all

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27. See on this interpretation Steel 2007.

28. Cf. Procl. *In Ti*. III 260.25 Van Riel (= II 193.9 Diehl): πραγματειώδης ἐξήγησις. On the meaning of πραγματειώδης see Festugière 1963, pp. 94–100, and in this volume, see Sheppard 000.

29. See, for instance, *In Prm*. III 784.12: Ἀλλὰ περὶ μὲν τῶν λέξεων τοσαῦτα εἰρήσθω· μεταβατέον δὲ ἡμῖν ἐπὶ τὴν τῶν πραγμάτων θεωρίαν. VI 1092.14: αὐτὴν δὲ τὴν λέξιν λοιπὸν ἐπισκεπτέον, ἵνα καὶ ταύτην ἐπὶ τὴν τῶν πραγμάτων θεωρίαν ἐπαναγάγωμεν. VII 1146.23: Ταῦτα μὲν οὖν εἰρήσθω περὶ τῆς πραγματειώδους τῶν προκειμένων θεωρίας. περὶ δὲ τὴν λέξιν ἐπισημαντέον ὅτι. *In R*. I 164.9: Εἰ δὲ τὴν περὶ τὴν λέξιν πολυπραγμοσύνην ἄλλοις ἀφέντες ἐπὶ τὴν τῆς θεωρίας ἀναδράμοιμεν. *In Alc*. 30.5–6: Ταῦτα μὲν οὖν τῆς περὶ τὴν λέξιν θεωρίας ἀντεχόμενα μέχρι τούτων ἐξητάσθω, τὸ δὲ ἐντεῦθεν ἐπ' αὐτὴν τὴν τῶν πραγμάτων τραπόμενοι θεωρίαν. *In Ti*. II 138.10–12 Van Riel (= I 299.20 Diehl): ἡμᾶς δὲ πρῶτον χρὴ τὴν λέξιν αὐτὴν καθ' αὑτὴν ἐξετάσαντας ἔπειτα οὕτω πρὸς τὴν ὅλην θεωρίαν ἀναδραμεῖν. On the distinction between λέξις and θεωρία in Proclus' commentaries see Festugière 1963 and Saffrey and Westerink 1987, p. 83 note 3 (note compl. 142).

classes of gods[30]. Should we also include in this overall project his commentary on Euclid with its distinction between *lexis* (the technical-mathematical) and *theoria* (philosophical/theological)?

Listening to Anne Sheppard's presentation I was struck by a passage in Proclus' *Commentary on the* Timaeus (*In Ti*. III 236.20–237.6 Van Riel = II 174.15–22 Diehl).

> Mathematical theory ought neither to be entirely scorned, nor sought after for its own sake. The first option means that Plato will not be able to indicate to us, as he wishes to, the things in their images (τὰ πράγματα ἐν ταῖς εἰκόσι), while the second option makes the whole exegesis of the text as unstable as a ship without ballast, for it is necessary to be firmly anchored, as it were, when one is heading off after the essence of those realities with which the dialogue is concerned (τῆς τῶν πραγμάτων περὶ ὧν ὁ λόγος ἐστὶν οὐσίας). We, however, will steer a middle course, as we said before, having first set out the text mathematically[31].

"Mathematics is not sought after for its own sake". Although this advice may seem relevant to interpreting the Euclid commentary, it does not apply. In fact, the interpretation of the *Timaeus* is a quite different case. After all, the *Timaeus* is about physical matters, not about mathematics, which in this dialogue is subservient to the goal of explaining nature. Proclus rightly insists that the mathematical sections in the *Timaeus* should be studied seriously. For they are essential for understanding the text's main purpose: to explain the physical world and its causes. However, mathematical constructions only offer "things in images"[32], as they are not themselves the things they represent. Therefore, although the mathematical arguments must be examined seriously, they should not be considered the ultimate explanation of the *Timaeus*. That is why the interpreter must be steering "a middle course" between neglecting mathematics and making it the main purpose.

But what if we turn to the commentary on Euclid? Is there a "middle course" to follow? Should we study geometry seriously, but not make it our ultimate purpose? What, then, could be the overall purpose of studying geometry with Euclid? Proclus discusses the σκοπός of the *Elements* in his prologue (70.20–71.24). He distinguishes between the treatise's purpose, as determined by the things (πράγματα) investigated, and its purpose with regard to the student. With regard to the student the *Elements* offer a

30. See Procl. *Theol. Plat*. III 23.83.7–10: Δεῖ δὲ ἡμᾶς τὴν πραγματειώδη καὶ συνοπτικὴν περὶ ἑκάστης τάξεως συνάγειν εἰς ἓν θεωρίαν, ἐπειδὴ καὶ τὴν τῶν λέξεων ἔφοδον ἐν τοῖς ὑπομνήμασι ποιησαμένους τὰ αὐτὰ καὶ νῦν ἀνακυκλεῖν οὐκ ἂν ἔχοι λόγον.
31. Quoted and discussed by Anne Sheppard in this volume, 000.
32. "Things in images" to be compared with what Proclus says in *Theol. Plat*. I 2.11.5 of physics: the study of "truth in appearances" (τῆς ἐν τοῖς φαινομένοις ἀληθείας).

training in the elements of geometry that makes the student capable to apply its reasoning to the whole of geometry. With regard to the matters it investigates it helps us understand the geometrical structure of the four elementary bodies and of the world as a whole, as set out by Plato in the *Timaeus*[33]. Some interpreters, Proclus notes, thought that each of the books had a purpose related to the understanding of the cosmos, and that its "usefulness" (χρεία) lay in its "contribution to a theory of the universe" (πρὸς τὴν τοῦ παντὸς θεωρίαν). This corresponds to what Proclus said about the importance of mathematics for understanding the natural world in the above-quoted text from the *Commentary on the* Timaeus.

So far for Euclid's *Elements*. But what is the purpose of Proclus' commentary, and what place did it have in the school curriculum? For the students, it is undoubtedly an advanced training in geometry, which will help them considerably when studying dialogues in which Plato indulges in geometrical arguments. But what about the "things examined"? should it have a more overarching, sublime goal? Dominic O'Meara writes: "The purpose of Proclus' *Commentary on Euclid* is (…) to use mathematical science as a bridge, as a fore-shadowing of and initiation to metaphysics. The overall purpose of the Commentary (...) is Pythagorean"[34]. The Pythagorean–Platonic emphasis on the essential role of geometry in the education of philosophers may well have been the main justification for including this treatise in the curriculum. However, despite the theological nature of much of its speculative theory, the *Commentary on Euclid* is certainly not an example of theological geometry, a kind of *theologoumena geometriae*[35]. After all, we should not exagerate the theological scope of this commentary. The Pythagorean approach is most evident in the section on definitions, as speculations on points, lines, circles and angles fit well here. There is already a long tradition of symbolic interpretation of these definitions. However, we should not forget that the most important section of the commentary, which constitutes half of the volume, concerns the propositions and contains very few speculations that go beyond the text. The commentaries on the propositions are often very technical for non-specialists. Furthermore, some of the longer speculations focus on "logical and methodological issues closely related to Euclid's text rather than on theological or metaphysical

33. The same purpose is mentioned in *In Eucl*. 82.13–83.2 (συντελεῖ πρὸς ὅλην τὴν τῶν κοσμικῶν στοιχείων θεωρίαν); 384.3–4; 423.13–18.
34. O'Meara 1989, p. 174.
35. See Mueller 1987, p. 309: "In the Euclid commentary Proclus blends these two approaches to mathematics (that is the Pythagorean approach and the technical) in an interesting and unique way". Even 'blending' may be too much said, as both approaches are clearly distinguished.

questions[36]. Above all, the philosophical-theological approach in the *theoria* does not cause Proclus to neglect the technical exegesis of the text. He carefully analyses the Euclidean text, addresses the objections raised against some of his proofs, and presents alternative proofs, including his own[37]. His explanation of the distinction between problems and theorems, and his discussion on the fifth postulate (also known as the parallel postulate), are remarkable[38]. Clearly, Proclus was interested in discussing technical geometrical problems, despite his stated intention not to devote too much time to them.

In conclusion, the Pythagorean speculative *theoriai* do not provide a perspective on an overarching purpose of the commentary different from the textual and technical explanations it offers. Rather, they are digressions triggered by certain arguments, in which the author indulges his theological interests. Fortunately, they do not interfere in the textual explanation. On the contrary, as Iamblichus already observed, it is only when knowing the "common elements of mathematics" that one may also understand "the symbolic and outlandish use of mathematical expressions" in theological speculations[39].

Mathematics and Theurgy

The most significant theological digression is found in the commentary on the definition of figure (σχῆμα) (136.20 ff.). This should come as no surprise, given that we have seen that «figure» is the central concept for making symbolic analogies to divine powers. Proclus concludes this lengthy section "we have drawn out at great length these matters of Pythagorean doctrine"[40].

To explain how figures could be symbols of gods, Proclus first gives a survey of the various levels where figures may be found: figures produced by art, figures in nature, in the soul, in the intellect, and "transcending all these figures, the perfect, uniform, unknowable and unutterable figures of the gods" (138.6–7). Theurgy, Proclus claims, may represent the properties of these divine figures by creating statues of the gods and assigning different forms to each deity. It symbolizes some of the divine figures by

36. Harari 2022, p. 64.
37. On Proclus' own mathematical contributions see Vitrac 2004 and Maronne – Rabouin 2022, ch. 1, pp. 9–12.
38. Cf. Salerno 2021.
39. Iambl. *Comm. Math.* 67.16–17 (chapter 22): ἀκοῦσαι τὴν συμβολικὴν καὶ ἀπεξενωμένην χρῆσιν τῶν μαθηματικῶν λέξεων.
40. *In Eucl.* 142.8: Ἀλλὰ ταῦτα μὲν κατὰ τὸ Πυθαγόρειον ἀρέσκον ἐμηκύναμεν.

means of magical "characters" upon the statues; it "imitates other through forms and shapes making some [statues] standing, some sitting; and some spherical, some heart-shaped and some fashioned still otherwise, etc." (138.12–18).

That the gods can be characterized by figures can also be deduced "dialectically" from the second hypothesis of the *Parmenides*. "If the One is of this sort, it would partake of some figure, as it seems, either straight or round, or some figure mixed from both" (145b2–6). Proclus argued that in this deduction Parmenides refers to the third intelligible-intellectual triad, which corresponds to the Teletarchs (τελετάρχαι) in the Chaldean Oracles. These divinities play an important role in the theurgical rituals, and it is plausible that certain geometrical forms as triangles and circles were used[41].

The association between divine figures and theurgy has been examined in a recent study by Robert Goulding[42]. Building upon the studies of Gregory Shaw[43], the author distinguishes between *external* theurgy, which uses statues and other magical objects in rituals to purify the soul and draw it towards the divine, and a superior *inner* theurgy, which is "conducted within the contemplative space of the mind to raise the soul to the highest reaches of the intelligible". Goulding (2022, p. 383) radicalizes this view arguing that "geometry, practiced in a certain way, is not just a preparation but constitutes for Proclus the core of the higher theurgy".

> Geometrical objects are tokens of the gods in the *phantasia*, which are employed in an internal ritual. The efficacy of this inner theurgy depends on the practitioner being aware of the objects just as symbols and recognizing the inner rituals just as rituals (p. 376). The propositions themselves of the *Elements* constitute the theurgic rituals—that is, the whole of each proposition, including the instructions for making the construction and the demonstration, as well as the geometrical truth stated in the proposition, and the finished diagram that accompanies it. The process of inscribing the diagram in the *phantasia*, line by line and circle by circle, is a ritual act, as is the movement of the *dianoia* through the steps of the demonstration; theurgy differs from contemplation precisely in the fact that something is done (p. 383).

In short, Proclus' commentary does not merely contain theological digressions (*theoriai*) and analogies with theurgical practises; the Euclidean demonstrations themselves become "rituals of inner theurgy". Goulding has two main arguments (1) The role of *phantasia* in geometry: Proclus held that the same organ that was the vehicle

41. For a study of Proclus' doctrine on divine figures and the connection with the *Parmenides* see Steel 2007.
42. Goulding 2022.
43. Shaw 1995.

and receptacle of theurgy was also the location of the mathematical objects that are the subject of Euclid's geometry. (2) Proclus observed "connections between geometrical objects and the gods" and structured them "into σειραί, chains of correspondences, which formed the basis of theurgy" (380). I will not discuss the first argument, on the status of *phantasia*, which is as such not convincing[44]. Besides, *phantasia* is certainly not an "organ" that is the "location of mathematical objects" Mathematical objects belong essentially to the realm of dianoetic knowledge though they can be projected within the sphere of imagination. Concerning the second argument it is correct that Proclus (and his Pythagorean predecessors) found connections between geometrical objects and divine principles. As Proclus says: "The figures first appear in the successive classes of the (intellectual) gods" (156.2–5). Because of these connections with the gods it is possible to use a geometrical notion as an image to grasp indirectly the divinity that is beyond understanding. However, to develop those symbolic relations is in no way an act of theurgy, but is, as we have seen, one of the *tropoi* of theology. Moreover, not all notions or elements of a geometrical demonstration can be seen and experienced as images of the divine. Geometrical arguments have a technical aspect that cannot be reduced to theological speculation. Claiming that practising Euclidean demonstrations is an act of theurgy removes the intrinsic rational finality of those arguments.

However, one might not focus on the technicalities of the demonstrations, but rather defend the theurgic claim from Proclus's perspective on mathematical objects as projections originating from the *dianoia* in the imagination[45]. Although the soul may find in its own being the geometrical reasons (*logoi*), it is too weak to consider them in a unified way within itself, it has to project them in its imagination. This projection gives extension and plurality to figures that have in their ideal state neither extension nor place or plurality: the ideal circle, the ideal triangle. In the compositions and divisions of the figures as well as in the related arguments discursive thought makes use of imagination as an intelligible matter. With his projection doctrine, Proclus attempts to explain the tension within geometry between, on the one hand, the understanding of its ideal content, and, on the other hand, the discursivity of its argumentation. This is the type of geometrical knowledge, as we practice it in explaining Euclid's *Elements*. However, as Proclus claims, this discursive geometrical reasoning is not an end in itself, it should be "a path" to go back up, to rediscover the richness of the soul's thought (*In Eucl*. 55.13–18).

44. One could also use this argument to make the role of *phantasia* in sense-perception theurgical.
45. On this well-studied doctrine see *inter multos* Nikulin 2008, p. 2.

> If [the soul] should be able to roll up the extension and view the multitude of the figures in a non-figurative and unitary way, then in turning back to itself it would see in a superior way the undivided, unextended, and essential geometrical ideas (λόγους) of which it is the fullness.

Such an activity would be "the supreme end of studies in geometry". "It is to this exercise", says Proclus, "that he who is truly a geometer must devote himself, and this should be the goal he must set for himself: to awaken and turn away from the imagination towards thought alone in itself"[46].

Goulding (2022) comments on this passage: "Thus, simple visualization of geometrical objects and reversion to their dianoetic models must have been part of the practice of Proclus's inner theurgy" (381). However, what has this reversion from imagined figures to their *logoi* in the soul to do with theurgy? The projection of its content in imagination and rolling it up when returning to itself is a relation within the soul. The dianoetic *logoi* are not the ideal paradigms in the intellect, and a fortiori not the secret figures of the intelligible-intellective gods: they are *logoi* in the soul. Proclus is right in saying that no true geometer would like to remain in endless argumentation and demonstration; rather, he would want to return to the state of knowledge that the soul has always possessed in its essence. However, Proclus never considered the projection from the essential *logoi* to the realm of imagination, and their subsequent return to an ideal status as an internal ritual of theurgy. One might argue that the geometrical approach is analogous to the aims of theurgic practice. However, even this more modest assertion is not put forward by Proclus.

Rather than relating geometry to an esoteric practice, Proclus' doctrine of projection, which situates the practice of Euclidean geometry within the movement of the soul – outgoing in imagination and returning to its innate content – provides geometry with its ultimate philosophical foundation. Moreover, even his Pythagorean-Platonic approach does not remove geometry frow its natural location, discursive reasoning (διάνοια). To be sure, Proclus likes to show how geometry is capable of extending its reach to all levels of reality, including the divine. However, in this range to all beings, it remains within its own level and its own possibilities. Even when revealing through images sublime insights about the gods, the soul is not transported to a higher sphere but remains on its own level. This is evident in a beautiful text wherein Proclus demonstrates the extension of geometry to all things.

46. *In Eucl*. 55.20–23.

> Geometry is coextensive with all things; it applies its reasoning to all things and contains in itself the forms of all things (*In Eucl*. 62.1–22).

> At its highest intellectual level, it thoroughly inspects real beings and, through images, teaches the properties of the divine orders and the powers of the intellectual forms. For it also contains within its own speculations the *logoi* of these entities and shows which figures are proper to the gods, which to primary substances, and which to the natures of souls.
> In the middle range of knowledge, it develops and unfolds its dianoetic *logoi*, considering the diversity within them and revealing their essences, properties, commonalities, and differences. Starting from these *logoi*, it encompasses within defined limits the figures shaped in imagination referring them to ther essential *logoi*.
> At the *third* stage in the development of its reasoning (διανοήσεως) it examines nature and the species of the sensible elements and their powers and explains how they are contained in a causal manner in its own *logoi*[47].

The three levels correspond to 1) the use of geometry in theological speculations, 2) the level of ordinary mathematics as Proclus understands it, 3) the use of mathematics in explaining the physical world.

What Proclus says about the superior dimension of geometry ("it teaches through images the properties of the divine orders and the powers of the intellectual forms"), corresponds to what he said earlier about its contribution to theology : mathematical *logoi* show through images the properties of that which transcends being and reveal the powers of intellectual figures"[48]. Proclus insists that geometry finds these *logoi* of the divine beings, "in its own speculations", that is in discursive thought.

The second level, which lies between the intelligible and the sensible, is characteristic of the rational soul when it is engaged in discursive thinking (*dianoia*). This is the level of Euclidean geometry, as explained by Proclus in his commentary. It develops the notions (*logoi*) corresponding to the different figures, studies their properties, shows what they have in common and in what they are different. Starting from those *logoi*, which belong to the very essence of the human soul, geometry shapes the figures in imagination but keeps them within their defining limits by connecting them with the essential *logoi*.

The third level is the extension of geometry in its application to the physical world. It examines in particular, as in the *Timaeus*, the geometrical structure of the elements.

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47. Mueller 1987, pp. 317–8 quotes this text to show how Proclus used his doctrine of projectionism as a foundation of ordinary geometry. See on this text also also O'Meara 1989, p. 173.
48. See text quoted above note 17.

Proclus concludes (*In Eucl*. 62.22–26):

> Geometry has images of the total intelligible kinds and paradigms of sensible ones *but it has its essence in the dianoetic forms* and through this middle region it ranges upwards and downwards to everything that is or comes to be.

The conclusion recalls an important principle of Proclus' psychology. The soul is everything, in its own manner: what is above the soul, it knows as image, what is inferior, it knows in a paradigmatic way. This principle figures as proposition 195 in the *Elements of Theology*: "Every soul is all things, in a paradigmatic way (παραδειγματικῶς) the sensible, in an iconic way (εἰκονικῶς) the intelligible". Unlike Plotinus and Porphyry, Proclus maintains that, in its attempts to ascend or descend, the soul always remains at its own intermediate ontological level, characterised by discursive thinking (*dianoia*). Even in its theological speculations the soul keeps a mode of vision proper to it (ἐν τοῖς οἰκείοις θεάμασι). Rather than a theurgic esoteric transformation, there is a reasoning that enables us to understand, through images, that which we cannot directly grasp. This is also what Proclus had said earlier in the prologue (20.4–5):

> Geometry reveals in its own arguments (ἐν τοῖς οἰκείοις λογισμοῖς) the truth about the gods and a theory of being (καὶ τὴν περὶ θεῶν ἀλήθειαν καὶ τὴν περὶ τῶν ὄντων θεωρίαν).

Bibliography

T. Benatouïl, *Les critiques épicuriennes de la géométrie*, in P.-E. Bour – M. Rebuschi – L. Rollet (eds.), *Construction. Festschrift for Gerhard Heinzmann*, London, College Publications, 2010, pp. 151–62.

J. M. Dillon – J. O. Urmson (eds.), *Iamblichus: On the General Science of Mathematics*, London, Bloomsbury, 2020.

A.-J. Festugière, *Modes de composition des Commentaires de Proclus*, in «Museum Helveticum» 20.2 (1963), pp. 77–100.

R. Goulding, *Geometry and the Gods: Theurgy in Proclus's Commentary on the First Book of Euclid's* Elements, in «Perspectives on Science», 30.3 (2022), pp. 358–406.

O. Harari, *Philosophical Commentaries on Mathematical Texts: The Case of Proclus' Commentary on the First Book of Euclid's Elements*, in K. Chemla – G. W. Most (eds.), *Mathematical Commentaries in the Ancient World*, Cambridge, Cambridge University Press, 2022, pp. 53–95.

P. Hoffmann – M.-A. Gavray, *Modes d'exposition de la théologie dans le néoplatonisme athénien (Proclus, Damascius, Simplicius)*, in O. Boulnois – Ph. Hoffmann – C. Lafleur – J.-M. Narbonne (eds.), *L'Esprit critique dans l'Antiquité*, II. *La Naissance de la théologie comme science*, Paris, Les Belles Lettres, 2024, pp. 219–58.

C. Huffman, *Philolaus of Croton: Pythagorean and Presocratic*, Cambridge, CUP, 1993.

C. Huffman, *Pythagoras*, in *The Stanford Encyclopedia of Philosophy*, https://plato.stanford.edu/archives/spr2024/entries/pythagoras/. (Seen July 30 2025), 2024.

A. Lecerf – D. P. Taormina, *Jamblique et la théologie scientifique*, in O. Boulnois – Ph. Hoffmann – C. Lafleur – J.-M. Narbonne (eds.), *L'Esprit critique dans l'Antiquité*, II. *La Naissance de la théologie comme science*, Paris, Les Belles Lettres, 2024, pp. 167–217.

S. Maronne – D. Rabouin, *La philosophie des mathématiques dans son histoire de l'Antiquité à la période moderne*, in A. Arana – M. Panza (eds.), *Précis de philosophie de la logique et des mathématiques*, Vol. 2 : *Philosophie des mathématiques*, Paris, Ed. Sorbonne, 2022, pp. 4–47.

G. R. Morrow (ed.), *Proclus. A Commentary on the First Book of Euclid's* Elements, Princeton, Princeton University Press, 1970.

I. Mueller, *Mathematics and Philosophy in Proclus' Commentary on Book I of Euclid's Elements*, in J. Pépin – H. D.Saffrey (eds.), *Proclus lecteur et interprète des anciens*, Paris, Ed. CNRS, 1987, pp. 305–18.

D. Nikulin, *Imagination and Mathematics in Proclus*, in «Ancient Philosophy», 28 (2008), pp. 153–72.

D. J. O'Meara, *Pythagoras Revived. Mathematics and Philosophy in Late Antiquity*, Oxford, Clarendon, 1989.

H. D. Saffrey – L. G. Westerink (eds.), *Proclus.* Théologie Platonicienne. *Livre III*, Paris, Les Belles Lettres, 1987.

L. Salerno, *Il quinto postulato di Euclide nel Commento di Proclo al primo libro degli* Elementi, in «Studia graeco-arabica», 11.1 (2021), pp. 19–81.

L. Salerno, Theologumena Arithmeticae *dello ps.-Giamblico: alcune note testuali al capitolo sulla Tetrade*, in «Studia graeco-arabica», 14 (2024), pp. 39–57.

G. Shaw, *Theurgy and the Soul: The Neoplatonism of Iamblichus. Hermeneutics, Studies in the History of Religions*, University Park, Penn. State University Press, 1995.

C. Steel, *Le jugement de Proclus sur le style du* Parménide, in J. M. Dillon – M. Dixsaut (eds.), *Agonistes. Essays in Honour of Denis O'Brien*, London, Routledge, 2005, pp. 209–25.

C. Steel, *Divine figures. An essay in Platonic-Pythagorean Theology*, in M. Bonazzi – C. Lewy – C. Steel (eds.), *A Platonic Pythagoras. Platonism and Pythagoreanism in the Imperial Age*, Turnhout, Brepols, 2007, pp. 215–42.

H. Thesleff, *The Pythagorean Texts of the Hellenistic Period*, Åbo, Åbo Akademi, 1965.

B. Vitrac, *À propos des Démonstrations Alternatives et autres Substitutions de Preuves dans les* Éléments *d'Euclide*, in «*Archive for History of Exact Sciences*», 59 (2004) pp. 1–44.

ABSTRACT

Carlos Steel, *Proclus on the Usefulness of Mathematics for Theology*

Despite Proclus's programmatic declarations in the introduction of his *Commentary on Euclid* that mathematics "prepares intellectual insight for theology" and should be used as a "stepping-stone" to a higher world, this chapter cautions against overemphasizing the theological purpose of his geometrical arguments. Such declarations have led to a neglect of the real import of Proclus's technical geometrical discussions, with some arguing that geometry for him constituted the core of higher theurgy. This chapter argues that scholars should not be misled by Proclus's Neoplatonic claims about mathematics as a bridge to theology. The theological speculations present in his work are best understood as digressions that illustrate how it is possible, from our discursive perspective (the proper domain of mathematics), to gain an intellectual insight into the divine causes. Specifically, the chapter examines the crucial difference between the technical 'textual' analysis and the speculative approach (*theoria*) to reality.

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LA MATHEMATIQUE GENERALE CHEZ PROCLUS

Mise au point terminologique

« Mathématique générale » traduit le grec ἡ ὅλη μαθηματική. Maintenant l'expression ἡ ὅλη μαθηματική peut avoir deux sens : (i) la « mathématique tout entière » (la « mathématique *en général* », *i.e*. l'ensemble des sciences mathématiques particulières, en premier lieu : arithmétique, géométrie, musique, astronomie), ou (ii) la « mathématique générale » ou « universelle », par opposition aux sciences mathématiques particulières[1].

La notion de « mathématique universelle » apparaît pour la première fois chez Aristote, fugitivement, en *Métaphysique* E 1.1026a25–27 et K 7.1064b8s. Dans ces deux passages Aristote distingue les sciences mathématiques particulières, qui traitent d'un genre déterminé (comme la géométrie ou l'astronomie) d'une « mathématique universelle » qui est « commune » : ἡ δὲ καθόλου πασῶν κοινή, 1026a27)[2].

Jamblique, dans son *De communi mathematica scientia*, du IV^e^ s., glisse parfois du sens (i) au sens (ii)[3].

Le genre transcendant

Dans le français « mathématique générale » il y a « genre ». Proclus oppose très fortement le genre « logique », le genre « abstrait », élaboré *a posteriori* par la raison à partir d'une multiplicité et le genre transcendant, qui a le statut de principe, de cause productrice, antérieure à et séparée de

[1]. Comme le dit très justement Rabouin 2009, p. 143 : « l'exposé (*sc*. dans le Prologue I, chapitre sept de l'*In Eucl*.) semble valoir aussi bien des mathématiques 'en général' que d'une possible 'mathématique générale', deux manières très différentes de traduire et de comprendre la ὅλη μαθηματική, qui se retrouveront chez les commentateurs ».

[2]. Voir Rabouin 2009, p. 37ss.

[3]. Sur la « mathématique commune » chez Jamblique, voir pour commencer Bechtle 2010, pp. 31–44, qui aborde la question importante de savoir si par « mathématique commune » (κοινὴ μαθηματική) Jamblique dans le *De communi mathematica scientia* entend une science antérieure aux mathématiques particulières (une « mathématique générale ») ou l'ensemble des sciences mathématiques (une mathématique « en général »). Dans le chapitre cinq du *Comm. Math.* il est clairement question d'une « mathématique générale » (comme dans le chapitre trois du Prologue I de l'*In Euclidem* de Proclus). Sur la mathématique générale chez Proclus, voir aussi O'Meara 1989, pp. 157–66.

son dérivé[4]. C'est surtout dans le commentaire sur le *Parménide* que l'on trouve des développements sur ce point. Mais le thème du genre transcendant apparaît clairement aussi dans un passage de l'*In Euclidem*. C'est dans le cadre du commentaire sur la définition de la figure dans les *Éléments* (Déf. I, 14) : *Une figure est ce qui est contenu par quelque ou quelques frontières*. Cette définition a été critiqué d'un point de vue logique en ce qu'elle définit le genre par l'espèce. À cela Proclus répond (*In Eucl.* 143.18–144.9) :

> Si un logicien subtil reproche à la définition d'Euclide de définir le genre à partir des espèces — car ce qui est contenu par une frontière et ce qui est contenu par plusieurs frontières sont des espèces de la figure, il faudra lui répondre que les genres ont pré-embrassé en eux-mêmes les espèces potentielles. Et les Anciens, chaque fois qu'ils veulent mettre en lumière les genres à partir des espèces potentielles qui sont dans les genres, semblent procéder à partir des espèces, mais, en réalité, c'est à partir des genres eux-mêmes c'est-à-dire à partir des espèces potentielles en eux qu'ils nous enseignent ce qu'ils sont. La raison de la figure, tout en étant une, embrasse donc, en vertu de la Limite et de l'Illimitation qui est en elle, les différences entre les multiples figures, et pour celui qui définit cette raison il n'est pas absurde d'inclure dans la définition les différentes puissances qui sont dans cette raison.

Le genre transcendant pré-contient les espèces dont il est cause. Il les *contient en acte*, mais de manière concentrée, non différenciée, si bien que ces mêmes espèces peuvent être appelées « puissances » en tant qu'elles ne sont pas encore réalisées, *i.e.* pas encore formellement différenciées. C'est là une doctrine fondamentale dans le Néoplatonisme tardif. On la trouve exposée chez Proclus dans le *Commentaire du Parménide* et aussi chez Simplicius, dans son commentaire aux *Catégories* d'Aristote[5]. On ne peut comprendre ce qu'est la mathématique générale chez Proclus si l'on n'a pas en tête cette doctrine néoplatonicienne du genre transcendant[6].

Le rang de la mathématique générale dans la hiérarchie des connaissances

Dans le chapitre 14 du Prologue I du commentaire de Proclus sur le premier Livre des *Éléments* d'Euclide il est très explicitement question en certains passages de la « mathématique

[4]. Sur la notion de genre transcendant cf. l'ouvrage important de Helmig 2012 et la recension de celui-ci dans Lernould 2016. Voir aussi Luna 2001, citée infra note 5.

[5]. Voir *e.g.* Simpl. *In Cat.* 69.24–32 ; 77.12–78.3. Sur cette doctrine qui s'inspire de la métaphysique néoplatonicienne tardive voir les pages excellentes que lui consacre Luna 2001, pp. 794–804. Voir aussi Procl. *In Prm.* V 981.5–21 Steel : la raison substantielle en nous de l'homme contient en acte sous un mode unifié toutes les différences constitutives de l'individualité d'un Socrate par exemple, tandis que le genre « homme » coordonné aux multiples hommes particuliers *ne contient qu'en puissance* ces mêmes différences, « si bien qu'il est comme une matière pour les différences spécifiques qui s'ajoutent à lui » (981.14s. ; souligné par moi).

[6]. C'est ce qui manque dans les pages que D. Rabouin consacre dans son livre *Mathesis universalis* à la mathématique générale chez Proclus.

générale ». On peut en tirer une idée très claire de la place qu'il faut attribuer à la mathématique générale dans une hiérarchie des connaissances. Proclus traite dans ce chapitre de la question de savoir en quel sens Platon, dans la *République* (534e), dit que la dialectique est le « couronnement » des mathématiques. Puis il aborde le problème du sens qu'il faut donner au mot « lien » que Platon emploie dans l'*Epinomis* (991e) en parlant de « lien des mathématiques ». Il commence par rejeter l'opinion d'Ératosthène, le contemporain d'Archimède, pour qui le lien est la proportion. Proclus nous donne la raison pour laquelle Ératosthène avait pensé cela. C'est parce que « la proportion est dite être, et de fait est, une des choses qui sont communes aux mathématiques » (*In Eucl*. 43.20–21)[7]. Mais, rétorque Proclus, la proportion n'est pas la seule chose qui soit commune à tous les êtres mathématiques[8]. Lui-même donne trois autres interprétations du mot « lien », en fonction d'une hiérarchie de liens qu'il parcourt de bas en haut. Le « lien », c'est d'abord (i) la mathématique générale :

> Nous dirons au contraire ceci : leur lien immédiat est la mathématique une et générale qui contient en elle-même sous un mode plus simple les principes de toutes les sciences mathématiques particulières, qui examine ce qu'elles ont de commun et ce qui les différencie, qui enseigne ce qui est identique en toutes ces sciences, ce qui appartient à une majorité d'entre elles et ce qui appartient à une minorité d'entre elles. Et, pour ceux qui apprennent [les mathématiques] comme il faut le faire, c'est jusqu'à elle que s'effectue la remontée à partir des sciences mathématiques (*In Eucl*. 44.1–8).

Proclus parle ici de « lien immédiat » (*proseches sundesmos*) en ce sens que sont aussi « lien », d'abord et avant la mathématique générale, la science première (la dialectique) et plus fondamentalement encore l'Intellect (voir la suite immédiate du texte sur laquelle je passe ici).

Les objets de la mathématique générale : les théorèmes communs

Quels sont les objets de cette mathématique générale ? Pour répondre à cette question je prendrai d'abord un passage tiré du chapitre 2 du Prologue II de l'*In Euclidem* (*In Eucl.* 60.6–23 ; souligné par moi) :

> [La proposition] : 'tout rapport est exprimable' convient à la seule arithmétique et pas du tout à la géométrie car en elle il y a aussi des rapports inexprimables. Et le principe qu'il y a une limite inférieure aux gnomons des carrés[9] est propre à l'arithmétique car en géométrie il n'existe absolument pas de minimum. Appartiennent

[7]. On peut se demander si Ératosthène avait en tête le *Timée* de Platon, où il est dit (31b–32a), à propos du lien qui doit unir les deux extrêmes que sont le Feu et la Terre, que la proportion (ἀναλογία) est le plus beau des liens.
[8]. Cf. *In Eucl*. Prologue I, chap. 3.
[9]. Le nombre gnomique minimal pour les carrés est 3. Le « gnomon » en géométrie est la figure qui, ajoutée au carré, accroît sa taille sans altérer sa forme (cf. Arist. *Cat*. chap. 14, 15a30). Sur le « gnomon » et l'usage qu'en ont fait les Pythagoriciens dans leurs spéculations sur le pair et l'impair, cf. Vitrac 1990, pp. 318–20 et 326s.

exclusivement à la géométrie les théorèmes sur les positions – car les nombres n'ont pas de position ; ceux sur les contacts – car c'est dans les continus qu'il y a contact[10] ; ceux sur les irrationnelles – car c'est là où il y a division à l'infini qu'il y a aussi l'irrationnel. Mais sont communs à l'arithmétique et à la géométrie les théorèmes relatifs aux sections, comme ceux qu'Euclide présente dans son deuxième Livre[11], exceptée la section d'une droite en extrême et moyenne raison[12]. De ces théorèmes communs, les uns passent de la géométrie à l'arithmétique, d'autres, inversement, passent de l'arithmétique à la géométrie, *d'autres, qui leur viennent de la mathématique générale, sont également propres aux deux ensemble*. Car la permutation et les conversions des rapports ainsi que les compositions et les divisions sont de cette manière choses communes à toutes les deux, mais c'est l'arithmétique en premier qui étudie ce qui concerne la commensurabilité, puis en second la géométrie, qui imite celle-là.

Un théorème peut être commun à l'arithmétique et à la géométrie parce ce que l'une le reçoit de l'autre. Il peut aussi leur être commun parce que toutes deux le reçoivent en même temps d'une autre science qu'elles, antérieure à elles, à savoir la mathématique générale. C'est ce que veut dire « sont également (ὁμοίως) propres aux deux ensemble »[13].

Proclus, dans le Prologue I, chapitre 4, parle des théorèmes communs en donnant les mêmes exemples. Mais il donne en plus une précision cruciale quant au statut de ces théorèmes communs (*In Eucl.* 8.18–9.10) :

Il ne faut donc pas penser que ces théorèmes communs[14] existent d'abord dans les espèces mathématiques multiples et divisées, et qu'ils sont produits postérieurement, c'est-à-dire à partir des espèces multiples ; il faut au contraire dire qu'ils existent avant ces multiples espèces et qu'ils l'emportent sur elles en simplicité et en précision. C'est en effet pour cela que la connaissance de ces théorèmes communs a la préséance sur les diverses connaissances mathématiques, qu'elle donne à ces dernières leurs principes, et que les diverses sciences mathématiques existent *en référence à*[15] elle et remontent à elle[16]. Que le géomètre dise en effet que si

[10]. Arist. *Ph*. E 3.227a21s.
[11]. Eucl. *El.* II 1-10. Sur l' « algèbre géométrique » et l'interprétation algébrique du Livre II des *Éléments*, cf. Vitrac 1990 p. 366ss. (en part. 368s. sur les démonstrations arithmétiques des Propositions du Livre II des *Éléments* d'Euclide dans les scholies à ce livre et par le moine Barlam). Voir aussi le commentaire de an–Nayrîzî dans la version arabe.
[12]. Cf. *El.* II 11.
[13]. Rabouin 2009, pp. 147–8, pense que les théorèmes communs dans le Prologue II (sur la géométrie) relèvent d'une mathématique générale « au sens étroit » du terme par opposition aux théorèmes communs dont il est question dans le Prologue I (sur la mathématique), chapitre 3.
[14]. Ce sont les théorèmes qui portent sur les proportions, les rapports, l'égal et l'inégal considérés d'une manière générale et non pas en tant qu'ils sont dans les figures, les nombres, etc. cf. *In Eucl.* 7.13–8.3.
[15]. 9.1s. Sur l'emploi de la préposition περὶ + acc. pour signifier dans un contexte métaphysique ce « relativement à quoi » ou « en référence à quoi » quelque chose existe (*i.e*. la cause première ou la cause antécédente) cf. *Théol. plat*. II 9.60.26 et la note 4 Saffrey-Westerink (*Notes Complémentaires*, p. 117). Proclus développe cette interprétation de περὶ + acc. (qui s'inscrit dans le cadre plus général d'une métaphysique des prépositions) à partir d'un passage de la *Lettre* II, où il est dit que « tous les êtres existent relativement au Roi de l'Univers » (312e1s.), cf. *Theol. Plat.* II 9.61.2–5 : « Le "relativement à quoi" (τὸ περὶ ὅ) convient au sommet des intelligibles, puisque tous les mondes divins procèdent relativement à lui qui leur est supérieur d'une manière cachée » ; *Theol. Plat.* II 9.59.10–13 : « Dire que tous les êtres n'ont d'existence qu'*en référence à* lui (τὸ γὰρ πάντα περὶ αὐτὸν ὑφεστηκέναι) révèle l'existence des êtres inférieurs tout en laissant ce qui est au-delà de tout l'univers sans aucune liaison avec les êtres qui le suivent » ; voir aussi *Theol. Plat.* I 3.17.6ss. ; II 8.56.18 ; II 9.57.15 ; *In Prm.* I 708.37. Appliquée à l'Un l'expression περὶ ὅ marque la

> quatre grandeurs sont proportionnelles, elles le sont encore par permutation, et qu'il le démontre selon les principes qui lui sont propres et dont jamais un arithméticien ne ferait usage ; quant à l'arithméticien, qu'il démontre à son tour que quatre nombres étant en proportion, ils le seront encore par permutation, et ceci à partir des principes de la science qui lui est propre. Qui est alors celui à qui il revient de connaître la permutation aussi bien dans les grandeurs que dans les nombres[17], ainsi que la division aussi bien des grandeurs que des nombres composés, et pareillement la composition aussi bien des grandeurs que des nombres divisés ? [souligné par moi][18].

Proclus ne donne pas la réponse à la question : qui est alors celui à qui il revient de connaître la permutation aussi bien dans les grandeurs que dans les nombres ?, mais on peut raisonnablement penser qu'il s'agit du philosophe.

Mais ce que je voudrais souligner est la chose suivante : les théorèmes communs ne sont pas des généralisations produites postérieurement à partir des multiples et divers théorèmes qu'on trouve dans les diverses sciences mathématiques. Ils ont un statut analogue à l'universel *ante rem*, par opposition à l'universel postérieur dans l'ordre de l'être. Loin d'être « abstraits » des multiples théorèmes divers ils sont antérieurs à ceux-ci. Loin d'être des formes « logiques » vides, ils ont le statut de principes producteurs. Et la science de ces théorèmes communs elle aussi est antérieure aux diverses connaissances mathématiques. L'antériorité se traduit par l'unité, une plus grande simplicité, une plus grande rigueur, par le statut de source, d'origine, *i.e*. de cause productrice. C'est la « connaissance des théorèmes communs » qui *donne* aux diverses sciences mathématiques leurs principes spécifiques, plus déterminés, plus particuliers. Cette « connaissance » n'est pas la totalité composée (qui résulte) des parties (ceci est la « mathématique tout entière », la mathématique « en général »), encore moins la totalité dans la partie. Elle est totalité avant les parties[19]. Par « connaissance des théorèmes communs » Proclus veut dire ici : « mathématique générale ».

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transcendance absolue du premier principe en même temps qu'elle exprime l'idée que l'Un est cause de la procession et de la conversion de toutes choses. Ficin se fera l'écho de cette interprétation, cf. *Commentaire sur le Banquet de Platon. De l'amour* II 4.32 Laurens : « *Autour du roi* signifie non pas "dans le roi", mais "en dehors" de lui : car en Dieu il n'y a pas point de composition. Quant au sens de ce terme, *autour*, Platon l'explique en ajoutant : *Tout existe grâce à lui, il est cause de tout ce qui est beau*. Comme s'il disait : tout gravite autour du roi parce que tout, par nature, revient à lui comme à sa fin, de même que tout a été produit par lui en tant que principe ». Ici c'est la science première, la science de l'être en tant qu'être, qui, par rapport aux diverses sciences mathématiques, se voit attribuer le rôle de cause antécédente, analogue donc à ce qu'est le τὸ περὶ ὅ en métaphysique. Pour d'autres occurrences dans l'*In Eucl*. de περὶ + acc. au sens de « en référence à », cf. par exemple 56.18 ; voir aussi 154.25 (le centre du cercle est le « ce en référence à quoi » les rayons et la circonférence existent).

[16] Cf. Iambl. *Comm. Math.* ch. 5, 20.1–5.

[17] Cf. Arist. *APo*. I 5.

[18] Cf. Eucl. *El.* V 16, 17 et 18 : si A : B = C : D alors, par permutation, A : C = B : D ; si A : B = C : D alors par séparation (A-B) : B = (C-D) : D ; et si A : B = C : D alors par composition (A+ B) : B = (C+D) : D. Voir aussi Iambl. *Comm. Math.* ch. 5, 19.2–4.

[19]. Cf. *El. Theol*. 67–69.

La mathématique générale est un genre transcendant

La mathématique générale est-elle une cinquième science mathématique ? La réponse à cette question est bien sûr : non. La mathématique générale n'est pas une science à côté et sur le même plan que les autres sciences mathématiques. Par « sur le même plan » je veux dire que même si les diverses sciences mathématiques peuvent faire l'objet d'une hiérarchisation en fonction du degré d'unité plus ou moins grand de leur objet propre (l'unité en arithmétique, n'a pas de position est un objet plus simple que le point, qui est en géométrie unité ayant en plus une position), néanmoins elles se situent toutes à un même niveau de discursivité. La mathématique générale déploie, elle, une science plus simple. Mais la mathématique générale n'est pas non plus une méta-mathématique. Elle ne serait, si c'était le cas, qu'un savoir logiquement « abstrait », un genre « postérieur dans l'ordre de l'être » comme l'est le genre logique « vivant » par opposition aux espèces (vivant ailé, terrestre, etc.)[20] ; et le théorème commun ne s'appliquerait à aucun objet qui lui serait propre, un universel « substantiel ». Un théorème commun, sur l'égal en tant que tel par exemple, a le statut d'un universel *ante rem*. L'égal dans les nombres et l'égal dans les grandeurs sont, eux, des universaux *in re*. C'est précisément parce qu'elle est connaissance du commun comme genre transcendant que la mathématique générale n'est plus une connaissance du même type que celui des sciences particulières qu'elle unifie. En ce sens la mathématique générale n'est plus une science mathématique. La mathématique générale est bien au-dessus (séparée) des mathématiques particulières. Elle a le statut d'une monade d'où procède une multiplicité qui lui est homogène (*scil*. les diverses sciences mathématiques), de la même manière que la Nature Universelle (ou Totale) est suivie des natures multiples. Puisqu'elle est « lien », principe d'unité, elle transcende ce qu'elle unifie. Elle est donc bien une « connaissance » (γνῶσις), ce terme marquant un type de savoir spécifiquement caractérisé par son degré de proximité ou d'éloignement par rapport à la connaissance ultime qu'est l'union intellective avec les Intelligibles (sensation, opinion, science, intellection). Le fait que cette connaissance est encore de la « mathématique » trahit, si l'on peut dire, sa proximité d'avec les sciences mathématiques. Mais ce n'est plus une science mathématique en ce sens qu'elle ne porte pas sur un genre mathématique particulier (comme toute entité médiatrice elle communique avec ce qui la précède, la dialectique, et avec ce qui la suit, les mathématiques particulières).

[20]. Proclus parle parfois des sciences mathématiques comme des espèces d'un genre commun, cf. *In Eucl.* 35.17ss. : « Car après avoir considéré le genre total et complet de la mathématique il faut aussi sans doute voir les différences spécifiques des sciences mathématiques particulières ». Le « genre commun » est ici la mathématique tout entière. Sur l'idée que la mathématique générale n'est pas un genre par opposition à des espèces cf. *e.g.* Dillon – O'Meara 2008, p. 6.

Ajoutons que les théorèmes communs (les axiomes) sont les principes de toute démonstration. Ils s'appliquent à toutes choses en tant que ces choses sont, au sens fort du terme, *i.e*. en tant que toutes choses ont en commun l'être. C'est pourquoi Aristote peut dire que leur connaissance relève de la connaissance de l'être en tant qu'être (*Metaph*. Γ 3.1005a19–30).

Mathématique générale ou science de l'être en tant qu'être

Reprenons notre lecture du Prologue I, chapitre 4. Proclus a souligné l'antériorité des théorèmes communs par rapport aux objets propres à chaque science mathématique et il a donné quelques exemples de théorèmes communs. Il poursuit en disant (*In Eucl*. 9.10–10.13) :

> Il ne se peut en effet qu'il y ait des connaissances scientifiques des choses particulières sans que nous ayons une science *une* des choses qui sont universelles et plus proches de la contemplation intellective. Au contraire, c'est bien plutôt la connaissance de ces choses-là qui est science; c'est d'elle que les diverses sciences mathématiques reçoivent leurs principes, et <la> remontée des connaissances, depuis les plus partielles vers les plus universelles, se fait jusqu'à ce que nous remontions à la science elle-même de l'être en tant qu'être[21]. Celle-ci ne daigne pas examiner les attributs essentiels des nombres, ni non plus à ce qui est commun à toutes les quantités ; elle étudie l'une et unique essence et existence[22] de tous les êtres, et c'est pour cela qu'elle est la plus enveloppante de toutes les sciences et que toutes les sciences reçoivent d'elle leurs principes. Ce sont en effet toujours les sciences supérieures qui fournissent à celles qui leur sont inférieures les hypothèses premières de leurs démonstrations ; quant à la plus parfaite des sciences, elle donne à toutes les sciences leurs principes, qu'elle tire d'elle-même : aux unes, elle procure des principes plus universels, aux autres, des principes plus particuliers – c'est pourquoi, dans le *Théétète* aussi, Socrate mêlant la plaisanterie au sérieux compare à des *colombes* les sciences qui sont en nous. Il dit qu'elles *volent, celles-ci en troupe, celles-là à part des autres*. En effet, d'un côté, les sciences qui sont plus communes et plus universelles englobent en elles de multiples sciences plus partielles, et de l'autre celles qui touchent aux choses connues divisées en espèces s'écartent les unes des autres et sont sans contacts les unes avec les autres, parce qu'elles partent de principes premiers différents. Au-dessus des multiples sciences mathématiques on doit donc mettre une science unique, qui connaît les choses qui sont communes et qui s'étendent à tous les genres et qui fournit à toutes les sciences mathématiques leurs principes. Que s'arrête ici notre enseignement consacré à cette science.

[21]. Dans le chapitre sept la science une et supérieure aux diverses sciences mathématiques est la mathématique générale (cf. *In Eucl.* 18.5–11). Il est à nouveau fait mention de la mathématique générale plus loin dans le chapitre quatorze où il est dit que la mathématique générale est le lien immédiat, le premier lien des mathématiques ; au–dessus de la mathématique générale le lien des mathématiques est la dialectique ou science première, et, en dernière analyse, le lien est l'intellect (*In Eucl*. 43.19–44.21).

[22]. L'essence, c'est-à-dire l'existence, est première par rapport aux déterminations qui viennent s'ajouter à elle et dont elle est comme le substrat. Pour le couple οὐσία καὶ ὕπαρξις cf. *e.g. In Ti*. III 170.6–7 Van Riel (= II 123.16 Diehl) : « l'essence et l'existence (οὐσία καὶ ὕπαρξις) dérivent du Principe tout premier ».

Dans ce chapitre quatre Proclus, dans sa remontée jusqu'à la science une au-dessus des diverses sciences mathématiques, s'élève directement jusqu'à la science de l'être en tant qu'être en laissant derrière lui, si l'on peut dire, la mathématique générale (et aussi la dialectique[23]). Comme le fait remarquer Rabouin 2009, p. 142 : c'est là une des raisons pour laquelle nombre de commentateurs renaissants considéreront que la « mathématique universelle » n'est en fait rien d'autre pour Proclus que la philosophie (science) première elle-même. Il ajoute (ibid. 142ss.) que la possibilité d'une mathématique générale n'est pas fermée et il voit, semble-t-il, dans la science de « ce qui est commun à toutes les quantités » (*In Eucl.* 9.21) quelque chose comme une allusion à la mathématique générale.

Ce qui peut conduire à faire dépendre les sciences mathématiques directement de la philosophie première c'est bien sûr l'enseignement de Platon, qui attribue à la dialectique (que les Néoplatoniciens identifiaient avec la science de l'être en tant qu'être chez Aristote) la fonction de « couronnement » des mathématiques (cf. *Resp.* VII 534e2s. et *In Eucl*. 42.8–43.18). C'est aussi celui d'Aristote, cf. *Metaph*. Γ 3.1005a19–30 :[24] c'est une même science, à savoir la science de l'être en tant qu'être, qui porte à la fois sur les principes à la base de toute démonstration et sur les causes premières de l'être (voir aussi E 1, en part. 1025b3–16). Syrianus se fera l'écho de cette doctrine.[25]

Mais je ne pense pas qu'ici dans le chapitre quatre le silence de Proclus sur la « mathématique générale » comme science intermédiaire entre la dialectique (la science première) et les mathématiques implique une identification chez lui de la mathématique générale avec la science première. La « mathématique générale », ou « mathématique universelle », n'est pas la « science universelle ». Et si l'on suit Rabouin, il n'y a pas véritablement une omission de la mathématique générale, mais bien une allusion à celle-ci. Et même si on dit qu'il y a omission, l'omission n'est pas une négation. Comme souvent chez Proclus un cadre doctrinal peut prendre des formes diverses selon le contexte et l'idée directrice du développement. Je pense que l'étagement à

[23]. Sur la dialectique comme « couronnement » des mathématiques, cf. *In Eucl*. 42.8ss., en part. 42.16ss. : « La dialectique (…) donne aux sciences mathématiques leurs diverses puissances, qu'elle tire d'elle-même (…) je veux dire : l'analyse, la division, la définition et la démonstration ». Là-dessus voir Lernould 1987.

[24]. « Il faut dire s'il relève d'une seule science ou de deux sciences différentes de traiter de ce qu'on appelle axiomes en mathématiques et de la substance. Certes, il est manifeste que l'examen de ces axiomes relève d'une seule science et que c'est celle du philosophe, car ils appartiennent à tous les êtres et non à un genre particulier séparément des autres. Tous s'en servent parce que les axiomes relèvent de l'être en tant qu'être, et que chaque genre est un être, mais ils ne s'en servent qu'autant qu'il leur suffit, c'est-à-dire autant que s'étend le genre sur lequel ils font porter leurs démonstrations. En conséquence, puisque, à l'évidence, les axiomes appartiennent à tous les genres en tant qu'êtres (c'est en effet ce qui leur est commun), leur étude relève aussi de celui qui acquiert la connaissance de l'être, en tant qu'être. C'est pourquoi, aucun de ceux qui mènent une recherche partielle n'entreprend de dire quoi que ce soit sur la vérité ou la fausseté de ces axiomes, qu'il soit géomètre ou arithméticien » (trad. Duminil-Jaulin).

[25]. Cf. *In Metaph*. 20.27–34. Sur cette science première des principes de toute démonstration comme « sagesse » – comme « monade » des sciences – identifiée par Syrianus à la philosophie première, et sur cette « sagesse » comme image de l'Intellect divin, voir O'Meara 2009.

quatre termes dans le chapitre 14 : 1. sciences mathématiques multiples, 2. mathématique générale, 3. dialectique, 4. Intellect, est réduit ici dans le chapitre 4 à deux termes : 1. sciences mathématiques multiples, 2. science de l'être en tant qu'être (= dialectique), sans mention de la mathématique générale et sans mention non plus de l'Intellect.

En outre la science première s'élève au-delà des *objets* propres à la mathématique générale. Certes les principes que connaît la mathématique générale sont les axiomes ou principes de toute démonstration. Mais les théorèmes qui lui sont propres portent sur les rapports généraux, comme on peut le lire chez Marinus, dans son Commentaire aux *Data*, 254.5–13 Menge : la mathématique générale est la science qui a pour objet les multiplicités, les grandeurs, les temps, les vitesses, les rapports, proportions et médiétés de toutes sortes. Marinus ajoute : Euclide dans le livre V a adapté aux grandeurs, en géomètre qu'il était, les rapports généraux (ibid. 254.223–227). La mathématique générale a ainsi pour objet une essence mathématique, qui est intermédiaire entre l'Intelligible et le Sensible (voir *In Eucl.* Prologue I, chapitre 1). Seule la science première (la théologie ou métaphysique) s'élève jusqu'aux Intelligibles. La science première déploie toute la hiérarchie divine en montrant comment tout dépend de l'Un. Elle est tout entière contenue dans le *Parménide*.

Maintenant, pour en revenir à notre chapitre sept, le mouvement de remontée vers la science unique au-dessus des sciences multiples ne peut s'arrêter avant le terme ; c'est lui qui prime et une étape intermédiaire peut très bien être omise quand il s'agit d'aller rapidement à l'essentiel, au terme culminant. Et le terme ultime c'est ici la science de l'être en tant qu'être. Pour le dire un peu familièrement : certes il existe ce qu'on appelle la mathématique générale, qui intéresse le mathématicien ; mais le philosophe, lui, ce qui l'intéresse c'est la science première ; alors on peut bien passer (quand il faut être bref) par-dessus la mathématique générale et aller directement à la science première. Et Proclus s'arrête là. Il ne remonte pas jusqu'à l'Intellect parce que le chapitre porte sur le mode d'être des théorèmes communs en tant qu'ils font l'objet d'une science *discursive*, en tant qu'ils font l'objet d'une démonstration. La thèse est, comme on l'a vu, que ces théorèmes communs ne sont pas tirés par abstraction des divers théorèmes spécifiques aux diverses sciences mathématiques mais qu'ils préexistent à ces derniers et en sont la source, ce qui fait qu'ils sont plus simples et plus précis.

Le caractère fantomatique de la mathématique générale dans l'In Euclidem

Le premier passage où semble bien apparaître le thème de la mathématique générale est dans le chapitre 3 du Prologue I. Dans le chapitre précédent Proclus a exposé comment le Limitant et

l'Illimité parcourent l'essence mathématique tout entière. Après l'exposé des principes communs (Prologue I, chap. 2) vient celui sur les théorèmes communs (Prologue I, chap. 3 : *In Eucl*. 7.13–8.4) :

> Maintenant**,** de même que nous avons considéré les principes qui sont communs aux objets mathématiques et qui s'étendent à travers tous les genres mathématiques, de la même manière considérons les théorèmes qui leur sont communs, qui sont simples et issus de la science une qui maintient ensemble dans l'unité toutes les connaissances mathématiques, et examinons comment ces théorèmes s'adaptent à elles toutes et peuvent être étudiés et dans les nombres et dans les grandeurs et dans les mouvements. Tels sont les théorèmes qui portent sur les proportions, les compositions et divisions, les conversions et inversions, ou encore ceux qui portent sur tous les rapports comme par exemple les rapports multiples, épimores, épimères, et leurs inverses, et, plus simplement, ceux qui portent sur l'égal et l'inégal considérés d'une manière générale et commune, non pas en tant qu'ils sont dans les figures, ou les nombres, ou les mouvements, mais en tant que l'égal et l'inégal, pris en tant que tels, possèdent une nature commune et se font connaître d'une manière plus simple[26].

Là encore il semble bien qu'on puisse mettre le doigt sur la mathématique générale, à ceci près que l'expression n'y est pas. Mais le rapprochement avec le passage en 60.6–23 cité plus haut peut nous permettre de reconnaître, voilée dans la brume d'une périphrase, comme la silhouette de la mathématique générale. Passons au dernier texte qui peut ici nous intéresser. Il s'agit du début du chapitre 7 dans le Prologue I. Ce chapitre a pour objet l'acte propre et les puissances (*scil*. les méthodes) de la science une au-dessus des multiples sciences mathématiques :

> Mais après l'essence des espèces mathématiques, remontons jusqu'à la science une des celles-ci, que nous avons montré être au-dessus des multiples [sciences mathématiques], et voyons (i) quel est son acte propre, (ii) quelles sont ses puissances [cognitives] et (iii) jusqu'où ses puissances s'étendent dans leurs opérations. Il faut poser, comme nous l'avons dit auparavant, que l'acte propre de la mathématique générale (τῆς ὅλης μαθηματικῆς) est dianoétique et qu'il n'est ni semblable à l'acte de l'intellect, qui est établi de manière stable en lui-même, parfait et autosuffisant de par lui-même, et tourné vers lui-même, ni semblable à celui de l'opinion ou celui de la sensation (*In Eucl*. 18.5–13).

Je traduis ici ὅλη μαθηματική par « mathématique générale » car la « science unique au-dessus des multiples sciences mathématiques » semble bien être la « mathématique générale ». Mais en même temps le contexte ne corrobore pas, semble-t-il, une telle interprétation. Quand Proclus dit : « la science unique que nous avons montré être au-dessus des multiples sciences mathématiques », il renvoie manifestement aux chapitres trois et quatre qui précèdent. Or dans le chapitre quatre c'est la science de l'être en tant qu'être qui est mentionnée comme étant la science

[26]. Par exemple l'axiome : si, à partir de choses égales, des choses égales sont retranchées, les restes sont égaux.

une au-dessus des multiples sciences mathématiques (cf. 8.18–10.13), avec le problème que pose l'apparente mise entre parenthèses de la mathématique générale ; et dans le chapitre trois on ne trouve pas l'expression « mathématique générale ». Et surtout la suite de son propos tout au long de ce chapitre sept porte à l'évidence sur les sciences mathématiques en général, la mathématique tout entière, et certainement pas sur ce qui serait propre à la mathématique générale. Comme le dit très justement Rabouin 2009, p. 143 : « l'exposé (*scil*. dans le chapitre sept) semble valoir aussi bien des mathématiques 'en général' que d'une possible 'mathématique générale', deux manières très différentes de traduire et de comprendre la ὅλη μαθηματική, qui se retrouveront chez les commentateurs ».

Il reste que nous avons deux passages où la notion de « mathématique générale » apparaît distinctement dans l'*In Euclidem* (Prologue II, *In Eucl*. 60.19, et Prologue I, chap. 14, 44.1–8). C'est, il est vrai, un peu court pour une notion si importante, qu'on ne peut manquer de rapprocher de la *mathesis universalis* comme ambitieux programme du rationalisme classique[27].

Bibliographie

G. Bechtle, *Sur la mathématique générale comme σκοπός (sujet) du* De communi mathematica scientia *de Jamblique et sur la composition de ce traité*, in A. Lernould (ed.), *Études sur le Commentaire de Proclus au premier livre des* Éléments *d'Euclide*, Lille, Presses universitaires du Septentrion, 2010, pp. 31–44.

J. M. Dillon – D. J. O'Meara (eds.), *Syrianus On Aristotle* Metaphysics *3-4*, London, Duckworth, 2008.

C. Helmig, *Forms and Concepts. Concept Formation in the Platonic Tradition*, Berlin, De Gruyter, 2012.

A. Lernould, *La dialectique comme science première chez Proclus*, in «Revue des Sciences Philosophiques et Théologiques», 71.4 (1987), pp. 509–36.

A. Lernould, *Recension du livre de Chistoph Helmig*, *Forms and Concepts. Concept Formation in the Platonic Tradition*, in «Revue de philosophie ancienne», 34.2 (2016), 207–53.

C. Luna, *Simplicius, Commentaire sur les* Catégories *d'Aristote, ch. 2–4*, Paris, Les Belles Lettres, 2001.

[27]. Sur le thème du rapport entre la mathématique générale néoplatonicienne et la *mathesis universalis* classique, cf. Rabouin 2009.

D. J. O'Meara, *Pythagoras Revived. Mathematics and Philosophy in Late Antiquity*, Oxford, Clarendon Press, 1989.

D. J. O'Meara, *Le fondement du principe de non-contradiction chez Syrianus*, in A. Longo (ed.), *Syrianus et la Métaphysique de l'Antiquité Tardive*, Napoli, Bibliopolis, 2009, pp. 299–309.

D. Rabouin, Mathesis universalis. *L'idée de « mathématique universelle » d'Aristote à Descartes*, Paris, PUF, 2009.

B. Vitrac (ed.), *Euclide*, Les Éléments, vol. I, Paris, PUF, 1990.

ABSTRACT

Alain Lernould, *The General Mathematics according to Proclus*

Above the particular mathematical sciences—namely, arithmetic, geometry, music, and the spherical sciences—Proclus introduces General Mathematics in the first Prologue of the *In Euclidem*. This concept is distinct from mathematics in general and represents a transcendent genre that surpasses the specific mathematical sciences. This chapter aims to demonstrate how General Mathematics constitutes an ultimate degree within the discursive sciences in the ascent towards the Intellect and the divine. By occupying this position, it offers the soul the highest level of rigorous, discursive knowledge before the non-discursive apprehension of the divine. The analysis will show how this General Mathematics serves a crucial role in the Neoplatonic hierarchy, linking the demonstrable principles of specialized mathematics to the ultimate, unified truths of the Intellect.

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RELATIONS THAT ARE GIVEN AND ESTABLISHED. PROCLUS' USE OF ANALOGICAL REASONING IN THEOLOGY

Introduction

The first form of reasoning human beings use, is arguably analogical reasoning[1]. Apparently from the moment humans start perceiving, they start drawing conclusions on the basis of similarities. Besides this all-pervasive use, analogical reasoning, most famously the *analogia entis*, also has a more specific role in philosophy, as a tool for understanding the divine.

As Aubenque shows, although the tradition often identifies Aristotle as the source of medieval and later ideas of the *analogia entis*, this is actually the result of Platonizing Aristotle − and indeed, Plato is the primary source for the most important influence in that tradition: mathematically based analogical reasoning[2]. One of the thinkers that play a crucial role in that tradition is Proclus. This chapter will discuss his views of analogy, to find a unified theory bringing together a colourful palet of analogies. Although it is not one of the dialectical methods, Proclus does value analogical reasoning as a crucial scientific and philosophical method, because of its contribution to reversion or *epistrophe*. It is therefore *a fortiori* a crucial theological method as well. At the same time, however, analogy comes in different forms, each with its limitations and challenges, that were also matters of debate in Proclus' writings, as in antiquity more generally. The aim of this chapter is to discuss the different types of analogy relevant to theology, and show that even the most problematic ones, according to Proclus either involve or are related to a proportional relation, with an ontological foundation.

In section 2, I will discuss contemporary and ancient definitions of analogy, and the main limitation of this form of reasoning. The most pressing limitation, one

[1]. Holyoak *et al*. 2001, p. 2.

[2]. Pierre Aubenque published various articles on the history of what he calls the "pseudo-Aristotelian" notion of *analogia entis*. See e.g., Aubenque 1989. Fronterotta 2016, following Aubenque, analyzes the use of *analogia* and related terms in Plato's dialogues. For another history of the *analogia entis* that includes the Neoplatonic tradition, see Pannenberg 2007. I have not been able to consult this work yet.

could say, is the problem of justification, or the problem of the unknown universal. I will also discuss the mathematical foundation of proportionality, and the justification of analogy found in the system of causality. In the subsequent sections, we will look at some passages from Proclus' writings, such as the *Platonic Theology*, the *Commentary on the* Republic and the *Commentary on the* Parmenides, to highlight the kinds of analogy he distinguishes and their uses in theology. In this context, we will also address his discussions of the limitations of analogy, and his proposed solutions.

Analogy Defined and Justified

Let me start out by distinguishing between analogy as a form of reasoning and analogy as a stylistic tool. This chapter is about both, but for the purposes of this discussion, the most relevant *contemporary* discussions on analogy are those looking at it as a form of reasoning[3]. Analogical reasoning is a method that does not require any prior knowledge, just a capacity for observation and subsequent identification patterns. That is its core: "to identify recurrences of these patterns despite variation in the elements that compose them"[4]. Or in more technical terms, analogy involves two domains, or sets of objects and statements about them: a familiar *source domain*, and a less familiar *target domain*; and accepted similarities between these two domains. On the basis of these similarities and our further knowledge of the source domain, we infer additional similarities between the domains, that allow us to conclude certain things about the target domain[5]. For example: if we know that (lit) candles and stars are similar in that they are bright, and in addition we know about candles that they are hot, we may infer that stars are hot as well. The first problem immediately presents itself: this is a form of induction, and therefore the certainty of the conclusion is not guaranteed[6]. In part, this problem is due to a certain vagueness in the English use of the term. This vagueness diminishes once we look at the Neoplatonic tradition[7]. The definitions given by Holyoak et al. and Bartha are similar to the description we find in G. E. R. Lloyd's famous *Polarity and analogy*, in which he analyses the use of two

[3] This is because, as we will see, stylistic analogy, or *simile*, for Proclus is nothing other than a specific use of analogical reasoning.
[4] Holyoak *et al*. 2001, p. 2.
[5] Bartha 2013, § 2.2.
[6] Cf. Bartha 2013.
[7] Cf. Gersh 1973, p. 87.

modes of reasoning in early Greek (pre-)philosophy. As emphasized by the title of his book, the two modes start from polar opposites and from analogies respectively. Lloyd mentions that there has not been "general agreement on the definition of analogy"[8]. In a broader sense, when we speak of analogy as a part or a method of *reasoning*, we may say that the hallmark of any analogy (like inductive reasoning) is *similarity*. This brings Lloyd to take on board the following definition, which maps onto Bartha's:

> Any mode of reasoning in which one object or complex of objects is likened or assimilated to another of the two particular instances between which a resemblance is apprehended or suggested, one is generally unknown or incompletely known (Bartha's target domain, MM), while the other is, or is assumed to be, better known (Bartha's source domain, MM)[9].

In the subsequent elaboration of pre-philosophical examples, Lloyd shows its pervasive use: he discusses homoeopathy, omen, simile, comparison, metaphor, imagery, and, in Greek cosmology, the anthropocentric understanding of the cosmos in political, biological, and technological images (the cosmos as a *politeia*, as a living being, or as a product of craft).

Lloyd's definition, as that of Bartha, ultimately derive from an uncontroversial and very concrete ancient sense of the word analogy (*analogia*): the relation between two mathematical proportions, especially what is called "geometrical proportion" − a relation which is itself also proportional: e.g., 1 : 2 :: 2 : 4, or 1 : 2 :: 4 : 8. In both cases, the two proportions stand in a proportional relation to one another (e.g., 1:2 and 2:4 − which is just another form of 1:2 −, in turn stand in a 1:2 relation). This primary sense is also generally identified as a crucial part of the justification for the success of analogical reasoning: in ancient philosophical sources discussing the nature of analogy, we find that, by extension, just as analogy holds as a proportional relation between different kinds of mathematical entities (numbers, magnitudes)[10], so too it holds as a proportional relation between non- mathematical properties or entities: as a is to b, b is to c, or as a is to be, c is to d. Excellent examples of this geometrical

8. Lloyd 1966, p. 172.
9. Lloyd 1966, p. 175.
10. See e.g., *In Eucl.* 9.2−8. As Morrow 1970 points out, in that passage Proclus also refers to the geometrical proof of alternate proportionality in Euclid's *Elements* theorem V.16 ("if four magnitudes are proportional they will also be proportional alternately": ἐὰν τέσσαρα μεγέθη ἀνάλογον ᾖ, καὶ ἐναλλὰξ ἀνάλογον ἔσται).

formula, as we may call it, are found in Plato's *Republic* and *Timaeus*, e.g., "as what is believed is to what is known, so the likeness is to the thing it is like"[11].

In the following, analogy will stand more narrowly for such applications of geometrical proportions to non-mathematical domains. Analogical reasoning will stand for argumentation relying on the two relations that are similar and themselves related.

There is a problem with the above-mentioned justification through the underlying mathematical example: that justification itself relies on an analogy. Other domains are considered similar to the mathematical domain, and because the proportional relations exist and work among mathematical entities, they are taken to exist and work among non-mathematical entities. This is one part of what I will call the ontological problem. For the other part see below.

In addition, analogical reasoning is essentially different from proportional relations in mathematics: whereas in mathematics the ratio of comparison is known, outside it analogical reasoning relies on an unknown universal (or an unknown ratio). This problem was addressed by Aristotle in his discussion of "arguments from similarity (*homoiotes*)". That these "arguments from similarity" are what we call analogy is clear from Aristotle's example, which echoes the geometrical formula[12]: "as knowledge and ignorance of contraries is the same thing, so is the perception of contraries the same thing, or, conversely, since the perception of them is the same, so also is the knowledge". The problem is that such arguments compare things through an implicit universal, which makes them useful in dialectic (in the Aristotelian sense), but also unreliable. This is what we may call the epistemological problem of analogical reasoning.

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[11] *Resp*. VI 510a9−10: ὡς τὸ δοξαστὸν πρὸς τὸ γνωστόν, οὕτω τὸ ὁμοιωθὲν πρὸς τὸ ᾧ ὡμοιώθη. And further *Resp*. VII 534a3−5: ὅτι οὐσία πρὸς γένεσιν, νόησιν πρὸς δόξαν, καὶ ὅτι νόησις πρὸς δόξαν, ἐπιστήμην πρὸς πίστιν καὶ διάνοιαν πρὸς εἰκασίαν; *Ti*. 29c3: ὅτιπερ πρὸς γένεσιν οὐσία, τοῦτο πρὸς πίστιν ἀλήθεια. Another important passage for later ancient ideas about analogy is *Ti*. 31b4−32a7, where Plato describes the proportional bond between the elements.

[12] Arist. *Top.* Θ 1.156b10−18. See also Fronterotta 2016, p. 50 and Bartha 2013, § 3.2: "Although Aristotle employs the term analogy (*analogia*) and discusses analogical predication, he never talks about analogical reasoning or analogical arguments *per se*. He does, however, identify two argument forms, the *argument from example* (*paradeigma*) and the *argument from likeness* (*homoiotes*), both closely related to what would we now recognize as an analogical argument". I would say, however, that the *argument from likeness* or similarity comes much closer to analogy in our sense. Paraphrasing his illustration of the argument from example: if we wish to prove that to fight with the Thebans is an evil for the Athenians, and we know that the fight against the Phocians was evil for the Thebans, we may assume an underlying universal that applies to both − namely that to fight against neighbours is an evil in general. The difference, of course, is that we here have an argument concerning two particulars falling under the same implicit universal, not (as in analogies) two relations between pairs of similar particulars. Cf. Aristotle's own observation at *Top.* Θ 1.156b14−18. Aristotle criticizes arguments from example, not as relying on an implicit universal, but an incomplete induction from a particular to a general principle: "it does not draw its proof from all the particular cases" (*APr*. 68b38−69a19).

Let us first sum up the well-known Neoplatonic reply to the ontological problem. One of the advantages of Neoplatonic, or in our case specifically Proclus' thought, is that it offers a clearcut universal justification for the analogy between mathematics and other domains: while the mathematical analogy itself as many other cases of analogy apparently function in a horizontal plane, i.e., between entities that are ontologically of the same level or category, for Proclus and most Neoplatonic, as well as Christian thinkers like the Areopagite and Thomas Aquinas, such analogies are dependent on primary analogies functioning in a vertical plane, i.e. between prior and posterior, or cause and effect[13]. It is salient in this respect that Proclus in the *Commentary on Euclid's* Elements, despite the fact that proportions are not introduced until book 5 of Euclid's *Elements* (as he points out at *In Eucl.* 427.13–4), does bring in the method of analogy as part of the geometrical method in the commentary on book 1: it is one of the methods grounded in the superior science of dialectic[14]. In addition, and even more saliently, he often refers to analogical relations in that same commentary, but of all instances most concern the vertical relations between the prior and posterior, and especially between causes and effects[15].

The likening or assimilation of what Bartha calls the "target domain" to the "source domain" relies on and is therefore justified by a metaphysical assumption regarding causal relations. Or to be more precise, according to Neoplatonists, in proper analogies the objects are not *likened* or *assimilated,* they are instead *alike* and *similar* or even in part *the same* by nature due to causation − the resemblance therefore is not suggested (as in Lloyd's definition), but real. Moreover, for any thinker with a Neoplatonic background, the mathematical proportions remain in the background also in the vertical application of analogy: for them, the mathematical number 1 is ontologically prior to and the cause of 2 etc., and therefore even a

13. On horizontal vs. vertical in the context of analogy see also Fronterotta 2016. On analogy as part of Proclus' geometrical method see also Martijn 2014.

14. For Proclus' discussions of mathematical proportions and the "common theorems" related to them, see *In Eucl*.
7.15−24. For the grounding of common principles, including those governing analogy, in dialectic, see In Eucl. 8.21−10.1.

15. For the use of mathematical proportions see 67.5−6, and especially in the exegesis of the first propositions, e.g., In Eucl. 205.24−206.2, 213.2−7, 220.17−221.6, cfr. 405−406. For the analogical relations between prior and posterior, or causes and effects (including the relation between superior and subordinate sciences), see 39.20−22, 66.5−8, 77.12−15, 97.18−100.2, 104.8−16, 117.2−17, 147.3−5, 168.13−15. An additional role of analogy is the Timaeus' "bond" of the cosmos, see e.g., *In Eucl*. 22.17−23.11. Note that at first sight, *In Eucl*. 43.22−44.24 seems to contradict my claims, because Proclus there says that proportion is not the bond between the mathematical sciences. His point there, however, is that although proportional relations are present in all mathematical sciences, they are not what relate them to one another − that bond is instead found on higher levels (universal mathematics, dialectic, and ultimately Nous).

mathematical analogy is in a sense already an expression of the connection between the horizontal with the vertical plane[16].

The general underlying causal principle can be found in Proclus' *Elements of Theology*. As has been discussed elsewhere, two elements are key to causality for Proclus as elaborated there: productive *motion* and degrees of sameness[17]. Productive "motion" is the activity of the prior that of necessity produces the posterior; and the degrees of sameness point to the fact that the posterior either has the same essence but in a lower degree, or a different essence. It is the former kind of sameness which is especially relevant for us, as in this case the posterior consists of a *transverse* chain of products or effects with an *analogically* or proportionally increasing distinction from their cause[18]. For example, lower kinds of souls that are still essentially souls, but, to differing degrees, inferior to, less perfect, and more divided, etc. than their source, the monad of Soul. In other words, one of the fundamental structures of reality consists in analogical relations. This is clear, for example, in *El. Theol.* 100:

> Again, all the unparticipated monads are referable to the One, because all are analogous to the One: in so far as they too are affected by a common character, namely their analogy to the One, so far we can refer them to the One[19].

So the primary causal background of the ontological analogy between a principle and its chain is, of course, emanation, which combines a degree of sameness (remaining) with difference through proportional declension (*hyphesis*/*hypobasis*), i.e., gradual disappearance or inversion of attributes[20]. Together, and on the assumption that the declension takes the same steps in every chain (i.e., follows at least partially same sequence of inverting attributes to their opposite or to privation), these two give us "analogous" members within two adjacent series: the monads to the One, the souls to different gods[21] − there are members of the one series that are in

16. Moreover, as Beierwaltes 1979, p. 65; pp. 153–8 in his discussion of analogy as horizontal ontological "bond of the world" ("Band von Welt", p. 158) points out, a horizontal analogy as a structure of sameness is a kind of unity and therefore also relates lower ontological levels to higher ones with greater unity.
17. See the discussion in Martijn – Gerson 2016, but also Beierwaltes before us (see the reff. in Martijn and Gerson). On the difference between similarity, sameness, and identity, see Martijn –Gerson 2016, p. 53 note 43.
18. In the latter case, the posterior is an image of the former, which means that the essence is not the same, but only similar to the essence of its cause. One may here think of the soul being similar to the intellect (*El. Theol*. 195).
19. πᾶσαι δὲ αὖ αἱ ἀμέθεκτοι μονάδες εἰς τὸ ἓν ἀνάγονται, διότι πᾶσαι τῷ ἑνὶ ἀνάλογον· ἦ οὖν ταὐτόν τι καὶ αὗται πεπόνθασι, τὴν πρὸς τὸ ἓν ἀναλογίαν, ταύτῃ εἰς τὸ ἓν αὐταῖς ἡ ἀναγωγὴ γίνεται.
20. In addition, there is a secondary sameness that is the result of reversion (§§ 97 and 29). See also below.
21. For the relation of the monads to the One see also *El. Theol.* 108−110; cf. 181, and for the souls being analogous to the gods see 185 and 201.

the same respective position as members in the other series. Such analogous members, i.e., with in part the same set of attributes[22], can be used in our anagogic analogies[23].

In a slightly less technical context, the different elements of this ontological justification of analogies as discerned by Proclus come out beautifully: in the 12th essay of his *Commentary on the* Republic, when he interprets the Divided Line. Regarding the second cut of the line (i.e., cutting each of the two unequal parts in two), Proclus first of all points out that the geometrical proportion, i.e., that each of the line sections resulting from the first cut, is itself cut "analogous to the original line"[24], serves to bring out the nature of emanation as a combination of declension with identity − the secondary is inferior to the primary, but through sameness[25]. Subsequently, he states that "analogy is the identity of relation and the most beautiful bond" − which seems a general statement about analogies, but by its reference to *Timaeus* 31c brings in the horizontal and entirely immanent ontological analogy between the elements[26]. And finally, he calls analogy the "judgement of Zeus". This is a reference to *Laws* VI 757b, where that judgment is described as consisting in genuine equality ("much to the great and less to the less great"), which leads to true friendship. This addition emphasizes the way horizontal analogy can teach us about vertical analogy. Or in Proclus own explanation (*In R.* I 289.3−6):

> Just as this cosmos has been crafted according to analogy, with all things obtaining an indissoluble friendship to one another, so too all things came forth, tied together by it and agreeing with one another[27].

So with this feat of Platonic exegesis, Proclus makes the perceptible *horizontal* analogies of the physical world and political life, an indication of the vertical analogy between the sensible and the intelligible.

Although the phenomenon of analogy built on ontological relations of prior and posterior may be more often associated with the *analogia entis* of Christian

[22]. See Van Riel 2016 for a fascinating elaboration of declension among the henads.
[23]. See further *El. Theol.* 21, 24, 63, 97, and 125.
[24]. *In R.* I 288.26−7: τούτων δὲ τῶν ἀνίσων δύο τμημάτων ἑκάτερον ἀνάλογον τέμνει τῇ ἐξ ἀρχῆς γραμμῇ.
[25]. *In R.* I 288.27−9: τῆς ἀναλογίας πάλιν ταύτης ἐναργῶς δηλούσης τὴν διὰ ταυτότητος τῶν δευτέρων ἀπὸ τῶν πρότερον ὕφεσιν.
[26]. *In R.* I 288.29−289.2: λόγου γάρ ἐστιν ἡ ἀναλογία ταυτότης καὶ δεσμῶν ὁ κάλλιστος, ὡς ἐν Τιμαίῳ [31c] μεμαθήκαμεν.
[27]. ὡς οὖν ὁ κόσμος οὗτος δεδημιούργηται <κατὰ> τὴν ἀναλογίαν, πάντων φιλίαν ἄλυτον πρὸς ἄλληλα λαβόντων, οὕτω καὶ τὰ πάντα προῆλθεν συνδεθέντα διὰ ταύτης καὶ ὁμολογήσαντα ἀλλήλοις.

metaphysics[28], it is central to late ancient pagan Platonism as well (and even occurs in middle Platonism, see below)[29]. An important ancient objection to the ontological foundation is the rejection of an important assumption on which it is built: that there is a one-on-one relation between the intelligible and the sensible. In other words, according to critics the presumed universal behind analogical reasoning about the intelligible on the basis of the sensible is not just unknown, it is non-existent. This criticism, which echoes Plato's own "large largeness" and Aristotle's third man, is unjustified. It was in fact quite clear to Neoplatonic thinkers (pagan and Christian) that the use of analogical reasoning in theology, and assumptions concerning vertical analogies, are to some extent problematic. Among them, as Chiaradonna has shown, Plotinus held an extreme position, demonstrating that analogy fails to capture the differences between the sensible and the intelligible, and arguing that the two realms are not related by analogy[30]. Proclus, instead has a more traditional view, in that he does embrace the metaphysical analogy, but he too discusses the limitations of using analogical reasoning concerning the transcendent.

In Proclus' writings, the best place to look for the role of analogical reasoning as a method based on this metaphysical relation of analogy, is found precisely in a theological work, the *Platonic Theology*. We will turn to that next, which will also allow us to address the epistemological problem mentioned above. Before turning to the exegesis of passages however, there are two important caveats to be made: first of all, it is possible that in different writings we find different notions of analogy. Although Proclus is a very systematic philosopher, his writings do not need to contain a fully coherent system. And second, one reason why we might find different notions, is that Proclus is often limited by the texts he is interpreting, such as Platonic dialogues or Orphic Hymns. I do think, however, that despite these caveats, it is still possible to find a coherent notion of analogy and analogical reasoning in Proclus' writings.

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[28]. On this topic see Ashworth 2009. We will leave aside the medieval tradition in this chapter, but it does make sense to point out here that the three medieval senses of analogy distinguished by Ashworth, analogy of proportionality (comparison of relations), analogy of attribution (cf. Aristotle's focal meaning), and analogy of imitation or participation (between God and creation), the first and last come together in Proclus. As we will see, the participation of lower levels in the higher is a proportional relation. In a sense the analogy of attribution is part of this picture as well, as there is always a highest level to which a term belongs in its truest sense, but this is semantically a bit different from the transcategorial attribution in Aristotle.

[29]. One of the sources of inspiration for Proclus' view discussed in this chapter was Iamblichus' Pythagorean approach (for which see e.g., Simpl. *In Cat.* 116.25−33).

[30]. Plot. *Enn.* VI 3 [44], 2. See the discussions in Chiaradonna 2002, pp. 227–305 and Chiaradonna 2016, p. 201. As Chiaradonna shows, Plotinus criticized the assumption of those who attempted to harmonize Plato and Aristotle, that there was an *aph'henos-pros hen* relation between the intelligible and the sensible; he moreover criticized the use of analogy as obfuscating the irreducible differences between these two realms.

The Role of Analogy in Theology

In the *Platonic Theology*, Proclus lays out Plato's theology, which is not the topic of one specific dialogue, but spread throughout Plato's writings. In II 5 of the *Platonic Theology*, Proclus presents what he takes to be Plato's two paths human beings might take to gain some form of cognitive access to the One, namely the road by analogy (or, to use its medieval name, the *via analogiae*), which is represented in the *Republic*, and the road by negation (*via negativa*), for which the *Parmenides* is the main source[31]. We will mostly disregard the latter, and focus on the former, which is first introduced by Proclus as follows: "Well now, I say that [Plato] sometimes exhibits that [i.e., the One/Good] through analogy and the sameness of the secondary things"[32].

Analogia here refers to the *method* Proclus thinks Plato chose for exhibiting (although not really describing) the One, and that the "sameness" refers to the underlying metaphysical relations justifying the use of that method[33]. Just like Alcinous before him, Proclus mentions as the prime example of this method the analogy of the sun from Plato's *Republic*: "For in the *Republic* he reveals the unspeakable character and existence of the Good through the analogy to the sun"[34]. Where Alcinous merely mentions analogy, and elaborates the example, Proclus almost does the inverse: he summarize the example, to then elaborate extensively on the method. The summary is more informative than it might seem at first sight, however, so we will return to it below. First let us take a closer look at Proclus'

[31]. Note, however, that analogical reasoning also plays a role in the *Parmenides*. See below. On the broader history of these two roads, and the additional third road, the *via eminentiae*, see O'Rourke 2016. As O'Rourke points out, the three roads are already discussed by Alcinous. See also below. Plotinus also discusses different ways in which we can know the first cause, including among others analogy and negation, *Enn*. VI 7. 36.6−9. See also below. On Plotinus on analogy, see above and Smith 2012.

[32]. *Theol. Plat.* II 5.37.12−3: λέγω τοίνυν ὅτι ποτὲ μὲν δι' ἀναλογίας αὐτὸ καὶ τῆς τῶν δευτέρων ὁμοιότητος ἐμφανίζει. On this passage see also Beierwaltes 1979, p. 330. Saffrey and Westerink translate: "fait voir le premier principe par le moyen de l'analogie et par la ressemblance avec lui de ce qui lui est inférieur".

[33]. The same pattern is used in the description of the second method of negation. There too we get the method used first, and the underlying reality second: "and sometimes displays its transcendence through negations and the excess with respect to all beings together" (*Theol. Plat.* II 5.37.13−15: ποτὲ δὲ διὰ τῶν ἀποφάσεων τὸ ἐξῃρημένον αὐτοῦ καὶ ἀφ' ὅλων ὁμοῦ τῶν ὄντων ἐκβεβηκὸς ἐπιδείκνυσιν).

[34]. *Theol. Plat.* II 5.37.15−17: ἐν Πολιτείᾳ μὲν γὰρ διὰ τῆς πρὸς τὸν ἥλιον ἀναλογίας τὴν ἄφραστον ἰδιότητα καὶ ὕπαρξιν τοῦ ἀγαθοῦ μεμήνυκεν. Cfr. Alcin. *Did.* 10.5: "The second way of conceiving it is that of analogy, as follows: the sun is to vision and to visible objects (it is not itself sight, but provides vision to sight and visibility to its objects) as the primal intellect is to the power of intellection in the soul and to its objects; for it is not the power of intellection itself, but provides intellection to it and intelligibility to its objects, illuminating the truth contained in them." (tr. Dillon 1993, modified)

further description of how analogy works, namely through a "representation of reversion" (*Theol. Plat.* II 5.38.2−8):

> By the other method [i.e., through analogy] the return to it [i.e., the One] of the things that have proceeded from it is represented; through sameness to that [i.e., the One], there is a monad for each order of beings, that is analogous to the Good, and which is to the whole [series] conjoined with it, what the Good is to all the orders of the Gods; of that sameness itself, the return of the wholes to the One is altogether the cause[35].

At first sight, this passage may be a bit unclear, because Proclus here describes a couple of steps, and moves between the method and its metaphysical underpinnings. First (in the order of reality, not of the text), there is the emanation of a monad (a universal, a whole) from the Good (which we may understand to be the One/Good in its emanative capacity), heading the orders of being and orders of the Gods respectively. These are real, ontological relations. Second, there is a return of the monads/wholes to the One (or the One/Good in its final capacity), i.e., an ontological *epistrophe* of the "wholes" to the One, which causes an (ontologically founded) sameness − and it is this sameness, which founds the analogical relationship. This is not just a reference to the structure of reality − i.e., the repetition in functioning as a monad, or being a lower kind of One with their own chain of emanation − but also to the power patterns at lower metaphysical levels have to help us revert. This analogy is why Plato can use an analogous, lower entity, as an *image* of the higher in a philosophical or literary representation (*apeikonizetai*), for the sake of our ascent (*anagoge*) to the One[36]. Note that this description is not specific to the analogy of the Sun − although Proclus probably did pick that example as a very clear, or the standard, illustration of this method − but is apparently intended to function as applicable to all uses of the method of analogy[37].

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[35]. κατὰ δὲ αὖ τὸν ἕτερον τὴν πρὸς αὐτὸ τῶν προελθόντων ἐπιστροφὴν ἀπεικονίζεται· διὰ γὰρ τὴν ὁμοιότητα τὴν πρὸς ἐκεῖνο καθ' ἑκάστην τῶν ὄντων τάξιν ἀνάλογον ὑπέστη τῷ ἀγαθῷ μονάς, τοῦτο οὖσα πρὸς ὅλον τὸν σύζυγον αὐτῆς ὃ πρὸς ἁπάσας ἐστὶ τὰς τῶν θεῶν διακοσμήσεις τἀγαθόν· τῆς δὲ ὁμοιότητος ταύτης ἡ πρὸς ἐκεῖνο τῶν ὅλων ἐπιστροφὴ πάντως αἰτία.

[36]. On the methods of negation and analogy as linked to emanation and reversion respectively, see also Baltzly *et al*. 2022, pp. 12–3.

[37]. This shows, I think, from the distance between the illustration and the description (there are 8 lines between them, in which the other method is described) and the general terms used in the descriptions, e.g. the use of expressions like "all other things" (*Theol. Plat.* II 5.37.19−20). To give two further theological examples: like the relation between the demiurge and all encosmic things, the relation between the One and all beings is an analogous relation (*In Prm.* I 642.15−16); and in the *Elements of Theology* Proclus argues for the existence of the henads on the basis of analogical reasoning: in a nutshell, every plurality is analogous to its cause, and as the intellects have Intellect as their cause, the divine series has the One as its

To return to the analogy of the sun, however: the phrasing used for it, already suggests that we may run into problems when using this method in theology: Plato "reveals the unspeakable character and existence of the Good through the analogy to the sun" − "reveals" and "unspeakable" are clear indications that what we are dealing with here is not exactly the same as what we find in mathematical analogies. In fact, as we will see, the method of analogy in this context merges with the *via eminentiae* (mentioned as a third method by Alcinous, but not by Proclus)[38]. This becomes clear when we look into Proclus' immediate problematization of a misconception of the method of analogy. After rejecting unjustified criticism of the *via negativa* (based in wrongly understanding the negations as privations), Proclus goes on to address and reject unjustified criticism of analogy – or perhaps a misguided use of analogies (*Theol. Plat.* II 5.38.15−18):

> And let noone try to misrepresent that upward journey to the first principle by defining analogy as consisting in identity (*tautotes*) of proportions (*logoi*) and proportions as consisting in relations (*en schesesin*)[39].

At first sight, Proclus' picture of the "misrepresentation" seems to contradict what I said above about the method of analogy being rooted in geometry, so let's pick this passage apart. The issue addressed seems to have several layers: the first problem would be that if analogies consist in *logoi* and *logoi* consist in relations, then presenting a functional analogy would imply describing a relation of the lower to the One Good, which is impossible by definition due to the latter's complete transcendence. In addition, as a second problem Proclus brings in that analogies are said to consist in *identity* of *logoi* − not just similarity or partial sameness, but identity, which would put the One and whatever emanates from it on the same ontological level[40]. As we read near the end of the chapter, Proclus refutes this

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cause, therefore the divine series has to consist of unities (113, cf. 151). Admittedly, however, this is not a case of reversion, as the *Elements of Theology* is set up in descending order.

[38]. For the three roads in Alcinous see note 000. Cf. O'Rourke 2016, p. 525 for a similar merging of analogy and eminence in Celsus, and Chiaradonna 2016, p. 76 for Iamblichus and Simplicius.

[39]. καί μοι μηδεὶς [...] μήτε τὴν ἀναλογίαν ἐν λόγων ταὐτότητι τοὺς δὲ λόγους ἐν σχέσεσιν ἀφοριζόμενος διαβάλλειν ἐπιχειρείτω τὴν ἀναγωγὸν ταύτην πορείαν ἐπὶ τὴν πρωτίστην ἀρχήν. Note that Proclus here seems to disagree with Plotinus, for whom analogy is not a path to the first principle, but a teaching about it (*didaskousin*, *Enn*. VI 7 [38], 36.6−8). The path to the first principle instead is provided through certain activities and states, such as purification and virtue (VI 7 [38], 36.8−10). Moreover, Plotinus distinguishes between analogies and "things derived from it" (γνώσεις τῶν ἐξ αὐτοῦ), which suggests that he is thinking of literary analogies, where for Proclus instead the analogies that we need are precisely about "things derived from it".

[40]. For the difference between similarity, partial sameness and identity, see above and Martijn and Gerson 2016, p. 53, note 43.

criticism, in both its elements, by showing that Plato's method of analogy neither expresses identity of, nor a relation between the One and the things after it (*Theol. Plat.* II 5.39.6−9):

> Analogies are only used through *indication* of the *sameness* (*homoiôseôs*) of secondary things to the first, and neither a proportion, nor a relation, nor a communion, of the first principle to the things after it, is displayed by them[41].

Again, this might seem to undercut an important part of my thesis, namely that analogies work because of a metaphysical relation of proportionality, but if we keep on reading, we will see it does not. The main point Proclus is here making is that the analogies that help us "reach" the One rely on two kinds of relations of transcendence of cause to effect (*Theol. Plat.* II 5.39.9−15). If you recall, chapter 5 of book II of the *Platonic Theology*, as we saw above, not just contains an explanation of analogy, but also an example, on the One Good and the monads. Let us return to that example to see how it solves our problem. Importantly, in the context of the *Platonic Theology*, the example primarily serves, not so much as illustration of analogy, but as part of the argument for the transcendence of the One. The example is expressed as an analogy consisting of proportional relations, namely "the monad [...] is to the whole [series] conjoined with it, what the Good is to all the orders of the Gods". Or to render this a bit more schematically:

monad : series : : Good : orders of Gods.

This analogy suggests that the relation between the monad or unparticipated universal and its chain is the same as that between the Good and the orders of the Gods. And it is, in some respects, but importantly not in all − the relation is itself one of analogy. As Proclus goes on to say: "But all of intellect and every god has a lower superiority to the inferior of which it is the cause, then the first has to each of the beings"[42]. In order to understand this addition, we have to point out that monads exist

[41]. αἱ δὲ ἀναλογίαι δι' ἔνδειξιν μόνον τῆς πρὸς ἐκεῖνο τῶν δευτέρων ὁμοιώσεως παραλαμβάνονται, καὶ οὔτε λόγος οὐδεὶς οὔτε σχέσις οὔτε κοινωνία τῆς πρωτίστης ἀρχῆς πρὸς τὰ μετ' αὐτὴν ἐκ τούτων ἀναφαίνεται. As we will see, such "indication" can also be found in applying names that belong to sensible objects to intelligible entities "by mere analogy". See e.g., *In Prm.* VII 1200.32−1201.2.

[42]. *Theol. Plat.* II 5.39.15−17: ἀλλὰ πᾶς ὁ νοῦς καὶ πᾶς θεὸς ὑφειμένην ἔλαχεν ὑπεροχὴν πρὸς τὰ καταδεέστερα καὶ ὧν ἐστιν αἴτιος ἢ τὸ πρῶτον πρὸς ἕκαστον τῶν ὄντων. It is unfortunate for the systematic rendering that Proclus in this passage moves from the contents of the analogy to a more general

at different levels, and that Proclus here introduces what he calls the second and third levels (gods and intellects), but in the inverse order (first intellect, then gods) − perhaps because it would be easier to understand for his readers how intellect is inferior to the One than how a god would be inferior to the One. Moreover, the "orders of Gods" of the second half of the analogy are in fact headed by the very monads of the first half of the analogy. To illustrate with a mathematical example, the kind of analogy we are working with here has three terms, not four: it is not similar to 4 : 8 :: 1 : 2, but **2** : 4 :: 1 : **2**. The analogy, then, both illustrates the partial sameness between any monad and the Good, and provides the space to point out the relevant difference between them, in how they relate to their inferiors[43]. It thereby reinforces Proclus' explanation for why analogy does not express *relations* in the case of the One.

In order to ascend to the One through analogy, we use an analogy in which some source domain figures and offers something knowable (in this case the ontological analogy or proportional relation between monad and its series), which has a sameness − in some but importantly not all respects! − to something we wish to know (but is essentially unknowable) about our target domain: the One.

Schematically, we might render the analogy in the following table:

source domain			*unknowable target domain*		*sameness*	*difference*
monad	2	*analogous to*	One	1	unity	metaphysical level
| *schesis*		***analogous to***	/*no schesis*			relation vs. transcendence
series	4	*analogous to*	Gods (and all beings)	2	emanated plurality of ones	metaphysical level

Table 1

Two things are worth noting here. First of all, it seems problematic that the relation between "1" and "2" is different from the relation between "2" and "4". After all, the

point regarding the transcendence of the One. I have tried to make up for this transition in the below table by including both the term from the analogy and the more general point about "all beings".

[43]. Procl. *Theol. Plat.* II 5.39.9−20: καὶ γὰρ τὸ ἐξῃρημένον αὐτῆς οὐ τοιοῦτόν ἐστιν οἷον ἐπὶ τῶν δευτέρων καὶ τρίτων θεωρεῖται διακόσμων, ἀλλὰ πολλῷ μειζόνως τἀγαθὸν ὑπερέχει τῶν ὅλων ἢ ὁ νοῦς τῶν μετ' αὐτόν, εἴτε ὁ δημιουργικὸς οὗτος εἴτε ὁ τοῦ παντὸς κόσμου νοῦς εἴτε ἄλλος τις εἴη τῶν θείων προσαγορευομένων· ἀλλὰ πᾶς ὁ νοῦς καὶ πᾶς θεὸς ὑφειμένην ἔλαχεν ὑπεροχὴν πρὸς τὰ καταδεέστερα καὶ ὧν ἐστιν αἴτιος ἢ τὸ πρῶτον πρὸς ἕκαστον τῶν ὄντων, ὃ πάντων ὁμοίως ἐκβέβηκε καὶ οὐ τῶν μὲν μᾶλλον τῶν δὲ ἧττον· οὕτω γὰρ ἂν αὐτῷ σχέσιν πρὸς τὰ δεύτερα μείζω καὶ ἐλάττω προσάγοιμεν.

whole point of using an analogy is that it allows us to conclude something on the basis of patterns shared by the known and the unknown. Proclus may take this to be resolved by what we will term, modifying a distinction made by Keynes, a combination of positive and negative analogy[44]: what holds in the source domain, holds in the target domain, *but not in the same manner* — the pattern is the same, but with an adverbial modification[45]. Let me illustrate that with the use of the second point: the merging of the method of analogy and the method of eminence mentioned above. It comes out quite clearly in Proclus' statement, reminiscent of the "true equality" from the *Laws*, that the (One) "Good exceeds the wholes by much more" (πολλῷ μειζόνως τἀγαθὸν ὑπερέχει τῶν ὅλων, *Theol.Plat.* II 5.39.9−20, "the wholes" referring to the first level below the Good). So there is a proportional relation, but perhaps it is better to render it as 0:1 :: 1:2. Significantly, even though the mathematical relation is broken up by the adverbial distinction, it is also maintained by the use of the verb "to exceed" (*hyperechein*), which is also used in the context of mathematical analogies[46].

Proclus on the Didactics of the Analogy of the Sun

The tension we found in the use of analogies between incomparable realms could be considered a subtype of two of the main problems of analogical reasoning: the ontological and the epistemological problem of justification. As Bartha points out, it is very hard to find a general justification for the reliability of reasoning by analogy, without it justifying arguments that are actually bad arguments[47]. We may also recall Plato's own eloquent and quite informative warnings against the pitfalls of analogies, which relate to the epistemological problem (Pl. *Phdr*. 262a−b and *Sph*. 231a):

> Therefore, if you are to deceive someone else and to avoid deception yourself, you must know precisely the respects in which things are similar and dissimilar to one another. - Yes, you must. - And is it really possible for someone who doesn't know what each thing truly is to

[44]. Bartha 2013, § 2.2 refers to Keynes 1921 for this distinction, that highlights that analogies may refer to propositions holding for both source domain and target domain, but also to propositions holding for one domain but not for the other. See also Lloyd 1966, p. 175.
[45]. This concept is inspired by Siorvanes 1996, p. 126. For a similar concept in Syrianus (*In Metaph.* 115.5−9) and Opsomer 2004, who phrases it as a combination of homonymy and synonymy. For this combination see also below.
[46]. See e.g., *In Eucl.* 122.3−5, Archyt. DK47B2 *apud* Porph. *In Harm*. *Ptol*. 92.
[47]. Bartha 2013, § 4.

> detect a similarity - whether large or small - between something he doesn't know and anything else[48]?

and

> And a wolf [is similar to] a dog, the wildest to the gentlest. If you're going to be safe, you have to be especially careful about similarities, since the type we're talking about is very slippery[49].

The first warning, from the *Phaedrus*, is in fact a variation on Meno's paradox, as analogical reasoning is used to draw conclusions about things we do not know on the basis of things we do know, but as long as we do not know the former ("know what each thing truly is"), we may not be able to discern a similarity between it and something else. The second warning, from the *Sophist*, is an example of this problem. If we do not really know what a wolf truly is (or a sophist for that matter), we may conclude that it is similar to a dog (or a philosopher) in more respects than it actually is − thereby not seeing salient differences, such as that the wolf is a wild animal (and the sophist a fraud)[50]. The importance of knowing differences is also brought out by Proclus himself, in his discussion of the analogy with the sun in the *Commentary on the* Republic, and more specifically in the 11th Essay, the topic of which is the *Republic*'s discussion of the Good. He introduces the analogy of the sun as Socrates "clarifying [the Good] through an image (*eikon*)"[51]. "Through an image" here refers not to the method of analogy[52], but to the sun, which is an image of the Good in an ontologically heavy sense. This interpretation is based on similarities with a chapter early in the *Platonic Theology*, namely I 4, with its distinction of four methods of speaking about the divine. Of the two methods that are not "unveiled" but "by indication" (*di'endeixeos*), one speaks about the divine "through images" (*di'eikonon*) (*Theol. Plat.* I 4.20.1–5). This method is further elaborated as "teaching through mathematics [...] and a treatise about the gods from ethical or physical *logoi*" (*Theol. Plat.* I 4.19.6– 13). Although the *Timaeus* would perhaps come to mind first, and is in

[48]. Tr. Nehamas and Woodruff 1995. δεῖ ἄρα τὸν μέλλοντα ἀπατήσειν μὲν ἄλλον, αὐτὸν δὲ μὴ ἀπατήσεσθαι, τὴν ὁμοιότητα τῶν ὄντων καὶ ἀνομοιότητα ἀκριβῶς διειδέναι. − Ἀνάγκη μὲν οὖν. − Ἦ οὖν οἷός τε ἔσται, ἀλήθειαν ἀγνοῶν ἑκάστου, τὴν τοῦ ἀγνοουμένου ὁμοιότητα σμικράν τε καὶ μεγάλην ἐν τοῖς ἄλλοις διαγιγνώσκειν;

[49]. Tr. White 1993. καὶ γὰρ κυνὶ λύκος, ἀγριώτατον ἡμερωτάτῳ. τὸν δὲ ἀσφαλῆ δεῖ πάντων μάλιστα περὶ τὰς ὁμοιότητας ἀεὶ ποιεῖσθαι τὴν φυλακήν· ὀλισθηρότατον γὰρ τὸ γένος.

[50]. Cfr. also *Euthd.* 298c5: "you are joining flax and not-flax together", also discussed in Lloyd 1966, p. 395.

[51]. *In R.* I 273.21−22: διὰ τῆς εἰκόνος ὁρμήσας ποιῆσαι σαφές.

[52]. Pace Baltzly *et al*. 2022, pp. 142–4, who seem to take "image" to refer to "analogy".

fact explicitly mentioned by Proclus[53], the *Republic* fits this method rather well. And it is significant, I think, that in the 11th Essay, Proclus paraphrases Glaucon's request to Socrates to recount what the Good is, as a request to "give an indication" what the Good is[54].

Before Proclus provides us with his reading of the analogy, he first discusses the risk involved in using analogies as a means of teaching (*tropos didaskalias*, *In R.* I 274.21−22). Plato, he argues, is warning the reader of that risk, but fitting the methods "by indication", his warning is presented in a way that will only be understood by readers who are "not cursorily listening": he uses word play[55]. The more advanced and careful readers will understand that the main risk in using analogies for teaching purposes lies in the fact that the audience has to select the features of the source domain that are shared by the target domain. Because of course, not all features are shared, but the whole point of analogy or any *logos* is that they "are not understood according to the difference of all things, but quite the opposite, according to their sameness"[56]. Analogies, that is, work because of a sameness of the things compared to one another. That sameness is never complete, however, as there is *always* also some difference between its members. In the case of the sun and the Good, Proclus emphasizes, the sun's being a cause is the relevant feature, but some readers might think the sun's being *caused* is also included, and would wrongly conclude that the Good is caused as well (*In R.* I 274.24−275.2). Moreover, because of the nature of emanation, Proclus emphasizes, it would in any case not be possible to compare the Good to anything in all its respects (I 275.2−5): there is nothing that is identical to it.

Proclus takes the risk of overinterpretation of analogies very seriously, and a couple of pages later, when presenting his own interpretation of the analogy of the sun, underscores it once more (*In R.* I 277.7−9):

[53]. On these methods in relation to the *Timaeus*, see Martijn 2006.

[54]. *In R.* I 273.16−21: μετὰ δὲ τοῦτο τὸ ἀγαθόν, διορισάμενος τὸν περὶ αὐτοῦ λόγον ὡς μείζονα ὄντα τῶν προκειμένων, τοῦ Γλαύκωνος κινήσαντος ὅπως ἂν ἐνδείξηται τί τὸ τῷ ὄντι ἀγαθόν, ὥρμησεν μέν, ὡς καὶ αὐτός φησιν, οὐχ ὅτι ἐστὶν εἰπεῖν τὸ ἁπλῶς ἀγαθόν, ἀφ' οὗ καὶ τὸ ἐν ἡμῖν, ἀλλὰ τί τὸ ἐοικὸς αὐτῷ. Cf. Glaucon's requests in the *Republic*: καὶ περὶ τοῦ ἀγαθοῦ διέλθῃς (VI 506d5) and Ἀλλ', ἔφη, λέγε· εἰς αὖθις γὰρ τοῦ πατρὸς ἀποτείσεις τὴν διήγησι (506e6−7).

[55]. For the word play see *Resp.* VI 507a, Proclus' explanation at I 274.17−22, and Baltzly *et al*. 2022, p. 144; p. 154 note 29.

[56]. *In R.* I 275.10−12: αἱ δὲ ἀναλογίαι καὶ οἱ λόγοι οὐχὶ κατὰ τὸ ἀνόμοιον λαμβάνονται πάντων, ἀλλ' αὐτὸ τοὐναντίον κατὰ τὸ ὅμοιον. It is not entirely clear how semantically specific *logos* is in this sentence, which is why I have left it untranslated, so it can refer to e.g., proportions or arguments

> As we said before, in analogies we should look at each of the things taken in the same relations (*logoi*), insofar as they are the same, not insofar as they are different and outside of the same relations of the things assumed[57].

The formulation is a bit awkward, especially due to the repeated phrase "the same *logoi*". The point is clear, however: when comparing things that display (or that we take to have) sameness in their relations − i.e., the relations within the two pairs, in our example specifically the relation between the sun and the sensible and the relation between the Good and the intelligible − we should focus on the sense in which these two relations are the same, and by no means on whatever it is that they are different in[58]. What Proclus does not mean here, of course, is that the differences are irrelevant. Quite the opposite. We should see the difference as such − and hence not equate the two things in all respects. In the case of the analogy of the sun, he says, there is only one aspect in which the sun is "analogous" to the Good, namely in its being a cause of light (καθ' ἓν δὲ μόνον τὸ αἴτιον εἶναι φωτός, I 276.26−27). Proclus seems to propose that we consider all other aspects of the analogy (light being analogous to truth, their causal roles, the respective transcendence of sun and Good, the eye being analogous to the mind) as following from this one point. In addition, there are points in which the sun is not analogous to the Good, such as the fact that the sun has a body, and a physical place, and can be moved (I 276.25−26) − these are not part of the analogy. (Note that both sameness and difference are located in the lower entity, as we learn about the Good by thinking about the sun − and the analogy starts from we thinking about the right properties of the sun). In addition, of course, the analogy reveals its own limitations: it shows us that the Good is the cause of truth, and therefore transcends truth, and therefore cannot be understood by the intellect. In other words, the analogy shows us that it itself cannot be the final tool for "knowing" the Good − for that we need dialectic and the negative method, of *aphairesis* (I 280.8−281.8). And now we finally understand the point of Proclus' emphasis on the risk of overinterpreting analogies. Emphasizing once more that analogies should not "transgress to other particularities"[59], Proclus starts his introduction of that negative

[57]. δεῖ γὰρ ὡς εἴπομεν ἐν ταῖς ἀναλογίαις ᾗ ὅμοια βλέπειν ἕκαστα τῶν λαμβανομένων ἐν τοῖς αὐτοῖς λόγοις, ἀλλ' οὐχ ᾗ ἀνόμοια καὶ τῶν αὐτῶν ἐστι λόγων τῶν εἰλημμένων ἐκτός.

[58]. Baltzly *et al*. 2022 translate "τοῖς αὐτοῖς λόγοις" and "τῶν αὐτῶν λόγων" as "the discussion itself", but I think this stronger "proportional" sense does more justice to the overall idea.

[59]. Apparently with a nod to Aristotle's principle of *metabasis*: μήποτε οὖν ἐνταῦθα τῶν περὶ τῆς ἀναλογίας ἡμῖν προδιῃρημένων μνημονεύειν καιρός, ἐν οἷς ἐλέγομεν δεῖν μὴ μεταβαίνειν ἐπ'ἄλλας ἰδιότητας τῶν τοὺς λόγους ἐχόντων, ἀλλ'ἐμμένειν χρῆναι ταῖς ἐξ ἀρχῆς, καθ'ἃς ἡ ἀναλογία θεωρεῖται (*In*

method as applied to the Good with a *difference* between the sun and the Good: the sun is both ungenerated and generated, albeit in different senses, which might lead us to believe, overinterpreting the analogy, that the Good is both not existing and existing. This, however, is not true.

If we return to the discussion from the *Platonic Theology*, we have to emphasize that for analogy to work in reversion, such *negative* analogy is just as important as the positive one: we have to understand, not only how the One is like a monad, or how the Good is like the sun, but also in what sense they are not. Otherwise, we can never transcend an intellectual approach[60].

Taking a step back, let us return to the bigger issue highlighted by Plato: we are still stuck with the problem that to use analogy in reasoning, and, by extension, in reversion, we must, in fact, already know precisely in which sense the things compared are similar and dissimilar, in order for it not to deceive us. The easy way out for a Platonist is, of course, the theory of recollection, and the *a priori* knowledge which lies dormant within us, and allows us to recognize correct analogies. In other words, the underlying universal is not really unknown, we just need to remember it. This is why Proclus can say, in his discussion of the proportional "bond" imposed by the Demiurge between the physical elements (*Ti.* 31c,) "From whence do we get the conception of such a 'bond' and of what is it a symbol?"[61]. His answer is that we get the conception from the very things of which it is a symbol: the One, the One-Being, and the Living Being. In other words, the source of our very concept of analogy is at the same time that which any analogy will "teach" us about.

Perhaps more interestingly than the reliance on recollection, however, is the fact that the use of analogies seems to be justified especially in expositions by experts. It is the teacher who selects the relevant analogies on the basis of their prior knowledge; and good students (no doubt with the help of a commentator) are capable of collecting and interpreting all relevant traits from those analogies[62]. In other words:

R. I 281.12−16). The principle of *metabasis* says that demonstrations should remain within the domain of one science, and not cross over to another scientific domain (*APo.* 75a38−39).

[60]. Cf. Aubenque's 1981 discussion of the Neoplatonic notion of analogy, with some references to Proclus − although Aubenque seems less confident that the Neoplatonists themselves realized analogy's limitations for knowing the first principle.

[61]. *In Ti.* III 22.19–20 Van Riel (II 16.13−14 Diehl): πόθεν οὖν ὁ τοιοῦτος ἐπινοεῖται δεσμός, καὶ τίνος ἐστὶ σύμβολον;

[62]. See for example *In R.* II 201.5−8: ἃ δὴ πάντα συλλαβὼν ἔχεις τὰς ἰδιότητας τοῦ κοσμικοῦ ... <60> ... τὸν κόσμον ... <16> ... καὶ οὐ θεωρήσῃς τὴν [διαφοράν, ἀλ]λὰ καὶ τὴν κοινωνίαν καὶ τὴν ὑπεροχήν. There are some textual problems, but the overall point is clear: Proclus says that from the analogy in the Myth of Er, of the ray of light passing through heaven and earth like the ropes keeping together the hull of a trireme, we can understand how that cosmic light relates to (is identical to, different from, and superior to) the world

analogical reasoning is primarily a didactic tool. This is confirmed by the use of another kind of analogies, which I will call "remotely anchored analogies".

Remotely Anchored Analogies

After the ontological and the epistemological problems, which were in part solved by the "anchor" of the metaphysical system of emanation (more specifically through declension and recollection respectively), in part by the merging of the method of analogy with the method of eminence and the addition of the method of negation, and in part by making analogy a didactic tool, an additional issue I want to put before you is that sometimes Proclus' use or interpretation of analogy in *epistrophe* seems not so well-anchored. And not being well-anchored, i.e., not having a clear or reliable relation to that system, might render the analogies useless. There are some analogies − for now, let us call them superficial and or purely literary analogies − to which the universal principle of proportionality does not seem to apply. Perhaps not by chance, some of these less manifestly anchored forms of analogy show up especially in theological and theurgic contexts; this is a specific type that, as we will see, nonetheless does have a remote ontological foundation. Moreover, this type provides an alternative solution to the problem of the limitations of direct analogy as applied to the divine: instead of talking about the divine indirectly through the *via negativa*, we can also name them (and summon them) by remote analogy.

In Proclus' *Commentary on the* Cratylus, we find an example of what at first sight seems an unanchored kind of analogy. In this commentary Proclus is rather interested in analogies. This is not surprising, because he is, among others, countering the Porphyrian/Dexippean view of names adhering primarily to the sensible realm and only secondarily ("by analogy and homonymously") to the intelligible. The Porphyrian/Dexippean notion of analogy here is a weak one, we could say, according to which words that originally indicate the sensible, are applied to the intelligible as well, but only metaphorically[63]. Like Plotinus, Proclus maintains that names adhere to the intelligible primarily. Unlike him, however, as we saw, Proclus does not reject the relation of image and original between sensible and intelligible, and does want to leave room for names of the sensible that apply to the intelligible, but not directly and

[63]. Van den Berg 2008, p. 72; p. 166, quoting Dexipp. *In Cat*. 41.25–30.

not metaphorically either. The main disagreement with Porphyry and Dexippus seems to lie in the meaning of "by analogy".

The most emphatic discussion of the notion of analogy in the *Commentary on the* Cratylus is found in chapter 56 (on *Cra*. 389d−390a), where Proclus rejects the two options of the debate, saying that analogy is neither a relation of image to Form, nor a matter of convention[64]. At *Cra*. 389b1−7 Plato has Socrates speak of a human made shuttle and its Form, "the Shuttle itself by itself", which would suggest the existence of intelligible versions of human artefacts. This is a conclusion Proclus emphatically wants to avoid. Instead, he says, we should understand this as an analogy. Proclus explains what this means in four steps of increasing precision. First (LVI 24.17−20), he points out that Plato is presenting an *eikon*, an image of the "discriminative" powers of the gods, in the sense that it carries a *synthema*, a "signal", of the divine "discriminative" order[65]. In the second step, Proclus further clarifies this by explaining how "image" and "signal" should *not* be understood, connecting Plato's statement in the *Cratylus* to the practices of theologians: when theologians speak of shuttles, the image they use is neither an image of a Form − there is no Form of "Shuttle" − nor a mere matter of convention or a symbol − "because it would be strange for science (*episteme*) to use names arbitrarily, especially concerning the gods"[66]. Instead, they are speaking *kat'analogian*. The reference to *episteme* is not arbitrary, as we understand when looking at Proclus' subsequent elaboration with the use of the mathematically inspired formula: "what the shuttle is to the art of weaving, discrimination is to the creation of Forms"[67]. The theologians, that is, are not relying on something like inspiration, but on knowledge and reasoning.

In the third step, Proclus seems to emphasize especially the intentional and real nature of Plato's analogy (*In Cra.* LVI 24.28−25.1):

> Analogy is neither a relation of image to Form, nor merely by convention, for example when Plato calls such powers of the souls "horses": this is neither by chance, nor because he is calling them Forms of perceptible horses, but using analogy[68].

[64]. *In Cra.* LVI 24.28−25.1, see also below. The following builds on van den Berg's discussion of analogy in the *Commentary on the* Cratylus, esp. Van den Berg 2008, pp. 148–50.
[65]. On such signals see also Sheppard 1980 and Van den Berg 2001.
[66]. *In Cra.* LVI 24.24−25: πῶς γὰρ οὐκ ἄτοπον τὴν ἐπιστήμην, ὡς ἔτυχεν, χρῆσθαι τοῖς ὀνόμασι, καὶ ταῦτα ἐπὶ θεῶν;
[67]. *In Cra.* LVI 24.26−28: ὃ γάρ ἐστιν κερκὶς ἐπὶ τῆς ὑφαντικῆς, τοῦτο ἡ διάκρισις ἐπὶ τῆς δημιουργίας τῶν εἰδῶν.
[68]. ἡ δ'ἀναλογία οὔτ' ἰδέας ἐστὶν πρὸς εἴδωλον σχέσις, οὔτε θέσει μόνον, οἷον ὡς ὁ Πλάτων <ἵππους> καλεῖ τὰς τοιάσδε τῶν ψυχῶν δυνάμεις, οὔτε ὡς ἔτυχεν, οὔτε ἰδέας λέγων τῶν αἰσθητῶν ἵππων ἐκείνας, ἀλλὰ τῇ ἀναλογίᾳ χρώμενος. The reference is to *Phdr*. 246a ff.

The reformulation of "convention" as "chance" and the example of the *Phaedrus*' famous chariot do not really clarify what "analogy" means here. The horses, it seems, merely help make the two negative qualifications more obvious: of course (human) soul powers are not Forms of perceptible horses. And of course Plato does not use the example of horses "by chance" or even "mere" convention (θέσει μόνον). The latter emphasizes, I would say, that the "horses" were Plato's choice, and that he chose them for a reason. And the former instead seems to suggest that there is a real relation between the example from the source domain and that from the target domain, but it is more complex than mere image and Form (and hence linguistically more complex than the same term applied to two levels): although "shuttle" is not an image of a Form of Shuttle, or "horse" of a Form of Horse, a shuttle does have something to do with divine powers, and a horse with soul powers. In addition, the fact that the example from the *Phaedrus* is wedged in between a discussion about the practices of theologians and those if the initiated (see below) suggests that it is on a par with them: Platonic dialogues are all to some extent theological texts, as we also know from the *Platonic Theology*.

In the fourth and probably most important step, we learn what a shuttle has to do with divine powers, and how, despite appearances, the vertical analogy Plato has in mind does in fact have an ontological anchor, and a special one at that (*In Cra.* LVI 25.1−7):

> For this reason the initiated (*telestai*) make (*poiountes*) things down here sympathetic (*sumpathe*) to the gods through such a connection (*oikeiotetos*), and use these instruments as "passwords" (*hos sunthemasi*) of the divine powers, for example, the shuttle of the discriminative powers, the mixing bowl of the lifegiving power, the scepter of the leading powers, and the key of the guardian powers, and in this way call upon them and others with the use of analogy[69].

The most important elements in this addition for our purposes are first of all the connection and sympathy between the divine and things down here, i.e., instruments, and second that this sympathy is established by the initiated "through a connection".

[69]. ὅθεν δὴ καὶ οἱ τελεσταί, διὰ τῆς τοιαύτης οἰκειότητος συμπαθῆ τὰ τῇδε ποιοῦντες τοῖς θεοῖς, χρῶνται τοῖς ὀργάνοις τούτοις ὡς συνθήμασι τῶν θείων δυνάμεων, οἷον κερκίδι μὲν τῶν διακριτικῶν, κρατῆρι δὲ τῶν ζωογονικῶν, σκήπτρῳ δὲ τῶν ἡγεμονικῶν, κλειδὶ δὲ τῶν φρουρητικῶν, καὶ οὕτως ἐπὶ τῶν ἄλλων ἀναλογίᾳ χρώμενοι καλοῦσιν.

The connection between a shuttle and divine discriminative power or a mixing bowl and lifegiving power may not be obvious right away, but there are three reasons to accept them. First, as van den Berg points out, assuming that theurgy works, the connections are evident from the fact that apparently the things down here are indeed effective in ritual contexts[70]. Second, in what a shuttle *does*, we do see a similarity with the much more elevated divine powers. And third, even such analogies are not just the result of a human-made comparison, but rely on the tracing of causal lineage[71]. The above passage does not deny the existence of such a natural connection. The word *poiountes* suggests that it is established by the initiated, but that they do it through an *oikeiotês* adds the qualification that they do that on the basis of an observed and real similarity. Ultimately, the very practical and material power of dividing threads comes from a divine and intellectual power of dividing intelligibles[72], and the theurgists art lies in using that connection.

What goes for human made tools, goes *a fortiori* for "names", i.e., for our linguistic tools, as long as they have been selected by the wise. True names are images of the intelligibles[73]. At the same time, however, our language is a limited tool, as it belongs to the level of discursive thought[74]. This is why it cannot completely capture the essence of the intelligible. In addition, most of our language does not actually describe the intelligible itself, but the sensible. Since there is a causal analogy between the transcendent and the appearances, however, one may still indirectly indicate the transcendent using the names of the appearances, even if as such those names do not literally apply to the transcendent (*In Cra.* CXIII 65.20−23 and LXVI 16−19):

[70]. Van den Berg 2008, p. 150.
[71]. *In Cra.* LIII 22.15−19: εἴ τις οὖν, ὅπερ ἔλεγον, κατὰ ταύτας τὰς ἀναλογίας ἐκπεριτρέχων τὰς μὲν τῶν θεῶν δυνάμεις αἰτίας ἐπονομάζοι τῶν τεχνῶν τούτων, τὰ δ' ἀποτελέσματα αὐτῶν ἐκλάμψεις τῶν δυνάμεων τούτων δι' ὅλου τοῦ κόσμου φοιτώσας, ὀρθῶς ἂν λέγοι. Van den Berg 2008, p. 149 seems in two minds about this: he explains the analogies as the result of "some sort of comparison", and points to their a posteriori justification through their effectiveness in ritual contexts, but although he quotes this passage in note 45, he does not seem to see (or accept) its importance as revealing the real causal connection behind the analogy.
[72]. Cf. *In Prm.* IV 847.19−29, where we find the same combination of production and underlying paradigm.
[73]. *In Cra.* XLVII−XLVIII and the analogical relation between language and things at *In Crat.* LVII 1−3: ὅτι ἀναλόγως ὡς ἔχουσι τὰ πράγματα πρὸς ἄλληλα, οὕτως καὶ τὰ ἐπ' αὐτοῖς ὀνόματα πρὸς ἄλληλα ἔχει κατά τε τιμὴν καὶ κατὰ δύναμιν. For Proclus' theory of language see further Van den Berg 2008, esp. pp. 131−3.
[74]. Cf. *In Cra.* LIII 23.23−25.

> For [intelligible beings] are not by nature knowable through names, but the theologians indicate (*semainousi*) them from afar (*porrothen*), on the basis of the analogy of the appearances to them[75].

"Analogy" in this case has a double meaning: at the level of reality there is a causal top to bottom relation, which results in the appearances being "analogous" to the intelligible, which translates at the linguistic level into a metonymic bottom to top relation. Therefore, on the basis of an analogical argument (e.g., as shuttle is to warp and weft, Athena is to universal and particular things) we can speak of the divine shuttle, or say that Athena is weaving[76], with the connection or relation lying in "discriminative power".

Recall that Plotinus criticized philosophers who maintained that our names for sensible things apply analogically to the Forms. Although it seems that Proclus here ignores Plotinus' criticism, in fact he found a way to acknowledge it, that allows him to keep analogies while doing justice to divine transcendence: our application of words, which actually belong to the sensible, to the intelligible is a "remote indication", in which the two are tied together by a semantic relation. Think of "shuttle" used for the discriminative power at divine and practical levels[77].

Conclusion: Analogies in the Parmenides

Finally, let me briefly turn to the *Commentary on the* Parmenides, the most theological dialogue of all, to sketch the application of different kinds of analogy, and how Proclus has them contribute to philosophical progress, starting from what is

[75]. τὸ πέρας τῶν νοητῶν θεῶν μόνον ὀνόματι δηλῶσαι δεδύνηνται, τὰ δ'ἐπέκεινα δι' ἀναλογίας μόνης, ἄρρητα ὄντα καὶ ἄληπτα, σημαίνουσιν. (...) οὐδὲ γὰρ δι'ὀνομάτων γνωρίζεσθαι πεφύκασιν, ἀλλὰ οἱ θεολόγοι πόρρωθεν αὐτὰ σημαίνουσι ἐκ τῆς τῶν φαινομένων πρὸς ἐκεῖνα ἀναλογίας. Duvick's 2007 translation ("by analogy with the visible entities related to them") seems to be misconstructing the direction of the analogy: it is the visible entities that are analogous to the intelligible ones, not the other way around. Cf. also the way we saw Proclus describe the sun's being analogous to the Good above.

[76]. As in *Orph*. fr. 178, quoted at *In Cra.* LIII 21.25−26.

[77]. A passage that does not fit this interpretation is found in the fourth book of the *Commentary on the* Parmenides. There Proclus says we should not name the divine "by analogy", as those do who consider names mere matters of convention (*In Prm.* IV 849.16−853.12). Instead, he maintains, we should assume that names have a primarily natural meaning, and that Plato's use of them for intelligible and sensible alike combines homonymy and synonymy. Proclus here seems to use the terminology differently, and take "by analogy" in the sense of people like Hermogenes in the *Cratylus*, but the overall point remains the same: real names apply first to the intelligible. Assuming that our names for the sensible apply to the intelligible is wrong, but there are instances where real names for the intelligible derivatively and in a different sense – with adverbial modification – apply to the sensible.

arguably the most problematic type of analogies in Proclus' writings: apparently far-fetched and ad hoc literary analogies.

The *via negativa* is the most important method in the *Parmenides* according to Proclus, but if we look at the way *analogia* and related terms are used in the *Commentary on the* Parmenides, we find a hierarchy of types of analogies, ultimately leading, as in the *Platonic Theology*, to that method of negation, and the negation of analogy[78]. The first, lowest kind of analogies are the numerous cases in his exegetical writings where he uses a concept of "analogy" to explain every single detail of a Platonic text (fitting the principle of *heis skopos* according to which all details of each dialogue fit the one overall aim of that dialogue). Such far-fetched and rather *ad hoc* interpretations seem anything but anchored by an ontological relation. Think, for example, of the numerous pages of exegesis spent on the analysis of the prefatory materials of Platonic dialogues as revealing analogies between characters and ontological layers: Socrates might stand for the Intellect, Parmenides for Being, Zeno for Intellect − the whole first book of the *Commentary on the* Parmenides is littered with such analogies, and Proclus even proposes different possible interpretations[79]. I will follow Sheppard and assume that even such analogy, which seems completely unanchored, "never loses its metaphysical overtones"[80]. There is always an ontological foundation, based in the patterns pervading reality[81]. In addition, as Proclus tells us, such analogies have a propaedeutic function in theology: they are useful in a preparatory phase, before "entering on the mysteries"[82] They "should not be taken as unimportant", because they are a form of exercise, to help the soul move "from phenomena to being" and "imagine immaterial nature"[83]. Maybe coming up with different possible interpretations of the characters is, from this perspective, even better than finding just one interpretation.

These literary analogies are found mainly in the prefatory materials in the first book, and in the transition to the second part of the *Parmenides*, in the fifth book[84]. In

[78]. Again, let me emphasize that in part, Proclus' understanding of analogy here is given by the Platonic text.
[79]. For this specific example see *In Prm.* I 627.27−629.25. See also Luna and Segonds 2007–2021.
[80]. Sheppard 2002.
[81]. A true application of the principle that "everything is in everything, but appropriately" − not in Anaxagoras' sense, but in this sense of repeated patterns. On this principle see Martijn and Gerson 2016, pp. 58–61. To use Asworth's 2007, p. 327 interpretation of the difference between metaphor (not based on causal relation) and analogy in ancient and medieval logic: for Proclus there are no metaphors, only analogies.
[82]. *In Prm.* I 663.35−664.3
[83]. *In Prm.* I 675.30−676.2
[84]. For the prefatory materials see above. For book V, see *In Prm.* 1021.7−1032.6.

book 2, Proclus introduces Zeno's philosophical method, which allows us to move from any plurality (whether sensible or intelligible) to its related unity, and "seeing the analogous everywhere"[85], and fittingly, analogies identified by Proclus in books 2 and 3 are more abstract than those in book 1. For example, as likeness is analogous to limit, unlikeness is to the unlimited; and "there is a certain analogy" between the irrational souls and their causes in this world and between the immortal souls and their transcendent causes[86]. In book 4[87], in commenting upon the first mention of *to ison*, we find the emphatic identification of "Equality" as the cause of all harmony and analogy, both in the sensible world and in the intelligible[88]. And finally, this concept is used to demonstrate that analogical relations exist between different divine realms and opposite properties: as similarity (a property of lower hypercosmic gods) is to sameness (of higher demiurgic gods), dissimilarity is to difference (VII 1193.6−1194.1). More importantly, the ultimate cause of all analogy is also used to demonstrate that the One exceeds all equality and inequality − and therefore also analogy (VII 1202.25−1204.31). Thanks to the preparatory analogical exercises in the first and fifth book, and to Zeno's method, students of Platonism are primed to reach a thorough understanding of the fundamental analogical structures in the intelligible, and learn to use analogical reasoning in theology, but also understand its limitations. In the end, this strengthens the role of the *via negativa* in the hypotheses of the *Parmenides*[89].

To conclude, let me collect the different kinds of analogy and analogical reasoning, and their roles in theology, as we encountered them in Proclus.

The first and fundamental kind of analogy is ontological or causal analogy in the vertical plane, going down from higher realities to lower ones, with proportional declension. These vertical causal analogies are the core of the whole Proclean system. Second, there are ontological analogies in the horizontal plane, e.g., between the elements or mathematical entities (also vertical?). This type in the horizontal plane, we saw, exists because there is an underlying vertical analogy: for example, a

[85]. *In Prm.* II 724.9−725.11.
[86]. *In Prm.* II 734.15−16 plus II 749.11−15 and III 820.22−24 respectively.
[87]. Book 4 begins with a discussion of analogies that does not really fit the upward path I am sketching, although it does interestingly address the limitations of analogies. Proclus there analyzes the different Platonic analogies for the relation of participation: reflections, imprints, copies (IV 839.16−842.11). These analogies have but limited use, he says: they are for beginners, and do not have any scientific value.
[88]. *In Prm.* IV 868.23−869.11; 871.9−10. Cfr. *In Ti.* III 26.10–14 Van Riel (= II 19.1−3 Diehl).
[89]. After all, the main method used in the *Parmenides* is the method of negation, see *Theol. Plat.* II 5.37.12−19.

“horizontal” analogy between the elements depends on the vertical causal relation of emanation.

Third, there are what we may call directly reverting, scientific, analogies, going up from knowledge of the lower to knowledge of the higher. The first, vertical or causal analogies provide a means of *epistrophe* precisely because of the fact that human knowledge fits snugly into the web of causal relations (*anamnesis*). These analogies adopt the geometrical formula, but are risky because prior knowledge of the relations and properties involved is required to correctly assess them. They should therefore be used by knowledgable teachers. In theology, analogical reasoning can be used to teach about the divine, if it is merged with the *via eminentiae* and, because of its limitations, ultimately gives way to the *via negativa*.

Fourth, there are different kinds of remotely anchored analogies. One kind is what we may call hieratic or indirect analogy (or signals), established by those who are initiated on the basis of their knowledge of a more remote causal relation. This type of analogy is very similar to the literary analogies that function as preparatory training for the mind entering into a scientific or dialectical study. In both cases, the initiated − a priest or an author − establishes an effective analogy from a range of options.

Overall, to solve the problem in theology that the divine is “incommensurable”, so to say, analogical reasoning merges with the *via eminentiae* and prepares for the *via negativa*; or alternatively, to avoid the pitfall of naming the divine directly and therefore inappropriately, it approaches the divine through remotely anchored analogies.

Bibliography

E. J. Ashworth, *Metaphor and the Logicians from Aristotle to Cajetan*, in «Vivarium», 45 (2007), pp. 311-27.

E. J. Ashworth, *Medieval Theories of Analogy*, in «The Stanford Encyclopedia of Philosophy» (2009), https://plato.stanford.edu/archives/fall2009/entries/analogy-medieval (seen August 1 2025).

P. Aubenque, *Neoplatonisme et Analogie de l'Être*, in *Néoplatonisme. Mélanges Offerts à Jean Trouillard*, Fontenay-aux-Roses, ENS Editions, 1981, pp. 63-76.

P. Aubenque, *Sur la naissance de la doctrine pseudo-aristotélicienne de l'analogie de l'être*, in «Les Études Philosophiques», 3–4 (1989), pp. 291–304.

D. Baltzly – F. Finamore – G. Miles (eds.), *Proclus. Commentary on Plato's* Republic, Vol. 2, Cambridge, Cambridge University Press, 2022.

P. Bartha, *Analogy and Analogical Reasoning*, in «The Stanford Encyclopedia of Philosophy» (2013), https://plato.stanford.edu/archives/fall2013/entries/reasoning-analogy/ (Seen August 1 2025).

W. Beierwaltes, *Proklos: Grundzüge seiner Metaphysik*, Frankfurt am Main, Klostermann, 1979.

R. Chiaradonna, *Sostanza, movimento, analogia. Plotino critico di Aristotele*, Napoli, Bibliopolis, 2002.

R. Chiaradonna, *Plotino critico dell'analogia*, in «Archivio di Filosofia», 84.3 (2016), pp. 75–86.

J. M. Dillon (ed.), *Alcinous, The Handbook of Platonism*, Oxford, Clarendon Press, 1993.

B. Duvick (ed.), *Proclus On Plato* Cratylus, London, Cornell University Press, 2007.

F. Fronterotta, *ΑΝΑΛΟΓΙΑ in Platone*, in «Archivio di Filosofia», 84.3 (2016), pp. 49–64.

S. Gersh, Κίνησις ἀκίνητος*: A Study of Spiritual Motion in the Philosophy of Proclus*, Leiden, Brill, 1973.

K. Holyoak – D. Gentner – B. Kokinov, *Introduction: The Place of Analogy in Cognition*, in D. Gentner – K.Holyoak – B. Kokinov (eds.), *The Analogical Mind: Perspectives from Cognitive Science*, Cambridge, MA: MIT Press, 2001, pp. 1-19.

J. M. Keynes, *A Treatise on Probability*, London, Hawk Press, 1921.

G. E. R. Lloyd, *Polarity and Analogy*, Cambridge, Cambridge University Press, 1966.

M. Martijn, *Theology, Naturally: Proclus on Science of Nature as Theology and the Aristotelian Principle of* Μετάβασις, in M. Perkams – R. M. Piccione (eds.), *Proklos: Methode, Seelenlehre, Metaphysik*, Leiden, Brill, 2006, pp. 49-70.

M. Martijn, *Proclus' Geometrical Method*, in P. Remes – S. Slaveva-Griffin (eds.), *The Routledge Handbook of Neoplatonism*, London / New York, Routledge, 2014, pp. 145–59.

M. Martijn – L. Gerson, *Proclus' System*, in P. d'Hoine – M. Martijn, *All From One: A Guide to Proclus*, Oxford, Oxford University Press, 2016, pp. 45−72.

G. R. Morrow (ed.), *Proclus. A Commentary on the First Book of Euclid's* Elements, Princeton, Princetone University Press, 1970.

J. Opsomer, *Syrianus on Homonymy and Forms*, in C. Steel – G. Van Riel – C. Macé – L. Van Campe (eds.), *Platonic Ideas and Concept Formation in Ancient and Medieval Thought*, Leuven, Leuven University Press, 2004, pp. 31-50.

F. O'Rourke, *The Triplex via of Naming God*, in «The Review of Metaphysics», 69 (2016), pp. 519– 54.

W. Pannenberg, *Analogie und Offenbarung: eine kritische Untersuchung zur Geschichte des Analogiebegriffs in der Lehre von der Gotteserkenntnis*, Göttingen, Vandenhoeck & Ruprecht, 2007.

A. Sheppard, *Studies on the 5th and 6th Essays of Proclus' Commentary on the* Republic, Göttingen, Vandenhoeck und Ruprecht, 1980.

A. Sheppard, *Image and analogy in Neoplatonism*, in T. Kobusch – M. Erler (eds.), *Metaphysik und Religion, Zur Signatur des spätantiken Denkens*, Wurzburg, De Gruyter, 2002, pp. 639–47.

L. Siorvanes, *Proclus: Neo-Platonic Philosophy and Science*, Edinburgh, Edinburgh University Press, 1996.

A. Smith, *Image and Analogy in Plotinus*, in «Proceedings of the Boston Area Colloquium in Ancient Philosophy», 7.1 (2012), pp. 1–27.

R. M. Van den Berg, *Proclus' Hymns: Essays, Translations, Commentary*, Leiden and Boston and Köln, Brill, 2001.

R. M. Van den Berg, *Proclus' Commentary on the* Cratylus *in Context: Ancient Theories of Language and Naming*, Leiden, Brill, 2008.

G. Van Riel, *The One, the Henads, and the Principles*, in P. d'Hoine – M. Martijn (eds.), *All From One: A Guide to Proclus*, Oxford, Oxford University Press, 2016, pp. 73−98.

N. P. White (ed.), *Plato:* Sophist, Indianapolis, Hackett Publishing Company, 1993.

A. Nehamas – P. Woodruff (eds.), *Plato:* Phaedrus, Indianapolis, Hackett Publishing Company, 1995.

ABSTRACT

Marije Martjin: *Relations that Are Given and Established. Proclus' Use of Analogical Reasoning in Theology*

This chapter addresses the pivotal role of analogical reasoning in Proclus's understanding of theology. Analogy, defined as drawing conclusions about a less known "target domain" based on a similarity with a better known "source domain," is central to scientific reasoning in all sciences, and a fortiori in theology. The chapter will discuss Proclus's definition of analogy, its relation to mathematical analogy or proportionality, and the different kinds of analogy he distinguishes within theological contexts. A main challenge for analogical reasoning is the justification of the assumed similarity between the source and target domains. Proclus's primary justification is ontological, relying on the processes of emanation and reversion as themselves establishing necessary analogical relations. However, the author notes that Proclus's framework also contains problematic forms of analogy, specifically symbolic and literary analogies, for which the ontological justification does not hold. In these cases, Proclus seems to introduce a separate category: a real relation of similarity that does not pre-exist but is established by an initiate (a philosopher or priest), underscoring a different, creative dimension of theological understanding.

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L'ΑΡΙΘΜΟΣ DIVINO NEL SISTEMA METAFISICO DI PROCLO

Nel libro I della *Teologia Platonica*, Proclo fa corrispondere al più elevato livello di conoscenza, che concerne gli dei, quattro diversi modi di insegnamento teologico (τοὺς τρόπους διαστησώμεθα καθ' οὓς ὁ Πλάτων τὰ μυστικὰ περὶ τῶν θείων ἡμᾶς ἀναδιδάσκει νοήματα)[1]: il più elevato riguarda il modo manifesto (ἀπαρακαλύπτως)[2] e si distingue in modo ispirato (ἐνθεαστικῶς), secondo il modello iniziatico caldaico (κατ'ἐπίπνοιαν)[3], e in modo scientifico (κατ'ἐπιστήμην), secondo il modello dialettico platonico (διαλεκτικῶς); il livello inferiore parla degli dei in modo allusivo (δι' ἐνδείξεως) o attraverso simboli (συμβολικῶς), come fa la teologia orfica[4], o per mezzo di immagini (ἀπὸ τῶν εἰκόνων) come nei Pitagorici[5]. La *Teologia* di Proclo è scientifica perché ordinata, ovvero costruita in livelli di realtà in cui ogni proprietà dell'essere è disposta in una scala di processione dalla sua forma "unitaria" alla sua forma "molteplice". Questo sistema poggia su una distinzione tra totalità unitaria, molteplicità determinata e molteplicità indeterminata, che Proclo articola anche da un punto di vista lessicale, distinguendo tra μονάς, πλῆθος e ἀριθμός. In questo sistema è d'altra parte particolare l'uso che Proclo fa del termine ἀριθμός, "numero", e che corrisponde da un lato ad un termine tecnico della gerarchia ontologica, designante la "serie" molteplice di una monade; dall'altro ad un particolare livello d'essere, quello che corrisponde all'introduzione dell'alterità a partire dal mondo intelligibile-intellettivo. Tale confluenza di significati è inoltre giustificata da un tentativo di conciliare tradizioni teologiche diverse, tra queste quella appunto pitagorica. Nel *Commento al* Timeo I 11.10–15 Van Riel (= I 7.25–30 Diehl), Proclo rapporta il modo di insegnamento teologico dei Pitagorici alla categoria del sublime e anche al modo di espressione assertivo (ἀποφαντικός) in un tono ispirato:

> In linea con lo stile abituale (συνήθεια) dei Pitagorici nell'opera vi è l'elevatezza di pensiero (ὑψηλόνουν), il carattere intellettivo (τὸ νοερόν), il tono ispirato (τὸ ἔνθεον), l'abitudine a mettere tutto (πάντα) in dipendenza (τὸ ἐξάπτον) degli intelligibili, a definire (τὸ ἀφοριζόμενον) il Tutto (τὰ ὅλα) per il tramite dei numeri (ἐν ἀριθμοῖς), ad esprimere (τὸ ἐνδεικνύμενον) le realtà (τὰ πράγματα) per mezzo di simboli

[1]. Procl. *Theol. Plat.* I 4.17.15.
[2]. Procl. *Theol. Plat*. I 4.20.3.
[3]. Procl. *Theol. Plat*. I 4.20.5.
[4]. Procl. *Theol. Plat*. I 4.20.7.
[5]. Procl. *Theol. Plat*. I 4.20.8.

(συμβολικῶς) e in forma misterica (μυστικῶς), la tendenza all'elevazione (τὸ ἀναγωγόν), il superamento (τὸ ὑπεραῖρον) delle comprensioni parziali, il tono affermativo (τὸ ἀποφαντικόν).

Nel presente studio mi propongo di mostrare come Proclo riutilizzi la tradizione numerologica pitagorica per strutturare la sua teologia scientifica ordinata e, d'altra parte, risolvere il problema del divario uno-molti. A tale scopo, procederò in quattro fasi: la prima sarà consacrata allo studio del lessico dell'unità per distinguere il primo impiego del termine ἀριθμός in quanto "unitario" (ἑνιαῖος) e "divino" (θεῖος); la seconda identifica nel "numero divino" un particolare livello di unità, che Proclo definisce "enadi"; la terza propone uno studio del termine πλῆθος con riferimento particolare alla dimensione intelligibile, mostrando come il πλῆθος νοητόν sia il primo "unificato di enadi"; la quarta è infine un'indagine sull'influenza pitagorica nella definizione intelligibile-intellettiva del numero in quanto alterità. Queste quattro sezioni del presente studio si basano d'altra parte su una considerazione della dottrina dell'ordine, τάξις, nella prospettiva di una lunga tradizione: il *Parmenide* di Platone, che Proclo considera il primo grande antecedente di sistematicità scientifica; Plotino, che ha saputo ben organizzare il reale, rispettando l'ordine di priorità dell'unità sul numero; Giamblico con cui si confronta e da cui attinge molto; e in ultimo il suo maestro Siriano, cui Proclo attribuisce la maggior parte delle innovazioni del suo sistema. L'elemento costante che caratterizza tutte le concettualizzazioni sull'ordine ontologico degli autori neoplatonici è la gerarchia. Esso è anticipato dalla configurazione plotiniana del reale organizzata nelle tre ipostasi, Uno, intelletto e anima,[6] che il filosofo non chiama propriamente ordini, ma "nature"[7]. Plotino fa infatti riferimento al sistema di ipostasi discendenti dall'Uno come ad un "ordine conforme alla natura":[8] inoltre, quando parla di una sequenza delle "cose divine" (τὰ θεῖα)[9], anticipa quella che da Giamblico e Proclo è definita la gerarchia del divino. Τάξις è anche usato da Plotino nel senso più generale di ordine naturale dell'universo[10]: si tratta, in questo caso, di un ordine imposto da un principio ordinante sull'ordinato[11], che recupera il modello platonico della fabbricazione dell'universo da parte del demiurgo come "organizzazione della materia" e passaggio

6. Sulla definizione del termine "ipostasi" in Plotino e nei suoi successori si veda Achard–Narbonne 2012, pp. XCVIII–CXIII, in cui è discussa la tesi secondo cui l'uso del termine "ipostasi" non sia riconducibile a Plotino, che, per riferirsi all'Uno, all'intelletto e all'anima utilizza piuttosto "tre nature", ma a Porfirio, che sarebbe, al contrario, l'autore del titolo di *Enneade* V 1 [10]. Plotino, per Achard e Narbonne, non avrebbe disdegnato l'impiego di questo termine: ciò è per loro evidente a partire dall'impiego che Plotino già fa in ben quattro passaggi della sua opera degli aggettivi cardinali τρεῖς e τριττά per riferirsi all'Uno, all'intelletto e all'anima.
7. In riferimento all'impiego del termine "natura" in Plotino cf. *Enn*. V 1 [10], 8.27 impiegato in riferimento alle tre "ipostasi". Cf. Achard 2006, pp. 381–8, in part. p. 387.
8. Plot. *Enn*. II 9 [33], 1.15: "Questo è l'ordine conforme a natura (τάξις κατὰ φύσιν); né c'è da porre null'altro, né di più né di meno, nella realtà intelligibile. Se si pone di meno, bisognerà proclamare identici o l'Anima e l'Intelligenza, o l'Intelligenza e il Primo" (trad. Faggin).
9. Si veda ancora il passo sopra citato *Enn*. II 9 [33], 1.15; e V 1 [10], 7.49.
10. Plot. *Enn*. II 9 [33], 7.27–35. Cf. anche II 9 [33], 13; III 2 [47], 4; III 2 [47], 13; IV 3 [27], 16.
11. Plot. *Enn*. II 4 [12], 15.4.

"dal disordine all'ordine"[12], e che attribuisce all'ordine sensibile la funzione di imitazione della perfezione intelligibile[13]. Sia riguardo all'ordine delle cose divine sia dell'universo, Plotino sembra anticipare i suoi successori in merito all'interpretazione di τάξις come gerarchia[14] di elementi disposti tra di loro in un rapporto di anteriorità e di posteriorità, in una sequenza ordinata di generante e generato, e in una sorta di scala in cui l'inferiore è in funzione del superiore[15]. Questa configurazione, come si mostra nelle pagine successive, è esplicitamente introdotta da Giamblico nella distribuzione triadica del genere divino, e ampiamente sviluppata poi da Proclo in un sistema del reale organizzato in una duplice gerarchia.

Il numero come "serie" discendente dalla monade corrispondente

La complessa organizzazione della scala del reale, che discende dall'Uno e si muove verso il molteplice, è articolata in livelli di unità decrescenti, che rispondono all'esigenza di mediare il distacco incommensurabile tra l'Uno e il molteplice. Ciò fa del sistema metafisico di Proclo un sistema della continuità per mediazione, in cui le enadi, che hanno un ruolo cardine nella determinazione e estensione dei livelli dell'essere, rappresentano la mediazione per eccellenza in quanto "ponte" tra l'Uno e il molteplice. In questa sezione mi occupo delle monadi che corrispondono, da un punto di vista degli "ordini" di realtà, ai διάκοσμοι, i mondi divini e sensibili, che focalizzano certi tipi di esistenza. Proclo si riferisce ad essi in quanto "monadi" per porre l'accento sul loro carattere di "cause" di entità particolari. D'altra parte, il termine significa due gradi di realtà differenti: da un lato quello di "causa" universale e trascendente, dall'altro quello di "insieme" di entità partecipate e riunite sotto la medesima proprietà.

Dal punto di vista dell'ordinamento verticale degli enti, il concetto di monade è legato a quello di serie/ἀριθμός: la monade è la causa di un insieme di entità accomunate dalle medesime caratteristiche. Proclo ne enumera tre, precisando che si tratta di monadi-cause al di là dei corpi, alle quali corrisponde una serie coordinata: la monade dell'anima (ψυχή) cui corrisponde la serie psichica (ἀριθμὸς ψυχικός); la monade intelletto (νοῦς) cui corrisponde la serie intellettiva (ἀριθμὸς νοερός); la monade dell'unificazione massima al di sopra dell'intelletto (ὑπὲρ νοῦν ἕνωσις), l'Uno appunto, cui corrisponde la serie uniforme (ἀριθμὸς ἑνοειδής)[16] delle enadi. La produzione della

[12]. Plot. *Enn*. II 4 [12], 15 e III 2 [47], 4, in cui Plotino recupera *Ti*. 30a5.
[13]. Plot. *Enn*. I 2 [19], 1.43. Sull'ordine come imposizione dell'intelligibile sul sensibile si veda anche *Enn*. III 6 [26], 11.
[14]. Si veda a tal proposito Collette-Dučić 2007, pp. 157–80.
[15]. Plot. *Enn*. IV 4 [28], 17.32.
[16]. Procl. *Theol. Plat*. I 3.14.5–15.

molteplicità a partire dalla monade è spiegata da Proclo nei termini di una diminuzione di unità: ciò che resta nascosto nell'unità originaria della monade, progressivamente, si divide e si moltiplica. Alla definizione di monade come causa risultano allora affiancati gli avverbi ἑνοειδῶς, συνεπτυγμένως e κρυφίως, che designano rispettivamente il carattere uniforme, contratto e nascosto della monade, in rapporto alla serie, che è invece διηρημένως, "divisa"[17].

Ciò che mi interessa a questo punto precisare è che nella prospettiva presa in considerazione, quella dell'ordinamento verticale monade-serie, c'è per Proclo una differenza tra πλῆθος e ἀριθμός: l'ἀριθμός è la molteplicità definita dalla sua appartenenza alla monade, ovvero la serie delle entità riunite grazie alla loro comune somiglianza, organizzata secondo una gradazione gerarchica e secondo un λόγος, ovvero secondo il principio dettato dalla monade; il πλῆθος è invece la molteplicità indeterminata, il punto più lontano dalla monade. Nella proposizione 21 degli *Elementi di teologia* Proclo propone, a questo proposito, una chiara definizione di questi differenti livelli di unità:

– la monade ha la funzione di principio (ἀρχῆς ἔχουσα λόγον) e riunisce in un unico ordinamento e in un'unica serie (μία σειρὰ καὶ μία τάξις) il molteplice. Essa è causa unica anteriore e comune a tutte le entità riunite nel medesimo ordine (ὁμοταγής), ed è grazie ad essa che l'ordine degli esseri riceve una coerenza interna e un carattere identico (τὸ ταὐτόν). La monade fornisce a tutto ciò che è ordinato in un ordine (τεταγμένοις) la legge unica (τὸν ἕνα λόγον) della relazione delle entità tra loro e in rapporto alla totalità (πρός τε ἄλληλα καὶ πρὸς τὸ ὅλον);

– la molteplicità propriamente detta si trova al limite estremo della monade poiché si è allontanata completamente da essa: Proclo parla esplicitamente di una discesa, ὑπόβασις;

– al movimento di discesa dalla monade verso la molteplicità si contrappone quello di risalita e unificazione della molteplicità verso la monade (ἀνάγεται πάλιν εἰς μίαν αἰτίαν);

– ciascun ordine è costituito da una catena di esseri ordinati secondo una certa legge (εἰρμός), comunanza, coesione e identità (καθ'ἑκάστην τάξιν ἐστί τις καὶ κοινωνία καὶ συνέχεια καὶ ταὐτότης) tra gli esseri dello stesso rango (ὁμοταγῆ) o di un altro (ἑτεροταγῆ)[18].

Da queste considerazioni emerge una gerarchia verticale scandita da un doppio movimento di salita della molteplicità verso la monade e di discesa dell'unità verso la molteplicità: è il processo

[17]. Procl. *Theol. Plat*. III 2.8.10.

[18]. I termini impiegati da Proclo a proposito della produzione della serie ad opera della monade sono i sostantivi ποίησις e ἐνέργεια: per un approfondimento sul concetto di ποίησις nel sistema filosofico di Proclo faccio riferimento all'articolo di Trouillard 1977, pp. 69–84. Inoltre, il filosofo parla di rapporti di predominanza (ἐπικράτεια) in virtù dei quali le entità più simili alla monade ne riflettono il carattere universale e produttivo, trasponendolo sulle entità successive.

di generazione e conversione delle entità componenti una serie verso la propria causa[19]. La monade-causa è monade trascendente (ἐξῃρημένη) e immagine dell'Uno, cioè è per la sua serie quello che l'Uno è per l'insieme degli esseri: la causa totale e impartecipabile (ἀμέθεκτος) del tutto, che trascende ugualmente tutti gli esseri (ὁμοίως ἐξῃρημένον τῶν ὄντων) ed è produttrice di tutti in modo unitario (ἑνιαίως παρακτικόν). D'altra parte, per Proclo la serie è anche una monade, di secondo livello, non nel senso di causa, ma nel senso di "insieme" di entità accomunate dalle medesime caratteristiche, partecipato (μετεχομένη) dal livello successivo[20]. I due tipi di monade ricoprono due ruoli differenti: la monade trascendente è la causa delle entità ad essa simili; la monade partecipata sancisce il legame della sua causa con la monade successiva e dissimile.

La superiorità di un certo tipo di monade, evidenziata nella *Teologia Platonica* nel carattere dell'impartecipabilità, è espressa nel *Commento al* Parmenide dal concetto di universalità trascendente. Alla gerarchia tra monade impartecipabile e monade partecipata corrisponde, infatti, quella tra la monade-causa, che ricopre il ruolo di categoria universale trascendente e la serie, in quanto entità particolari definiti da caratteri interni comuni[21]. Ciò è messo in evidenza dall'impiego dei termini inassegnato/assegnato, per designare, da un lato, con ἀκατάτακτος, l'universale trascendente, dall'altro, con κατατεταγμένος, l'universale immanente in ciascun soggetto, che ne è causa della sua determinazione e, al tempo stesso, della sua definizione. A questo proposito, in II 724.5–9 del *Commento al* Parmenide, Proclo distingue dei "molti indeterminati" (τὰ πολλά/τὰ ἀόριστα), non ancora unificati, e una molteplicità determinata in virtù della sua definizione all'interno di una classe di realtà. Il passaggio dall'indistinzione del molteplice alla distinzione delle sue proprietà in una monade non è tuttavia automatico. Inoltre, in V 980.30–981.21, Proclo escogita un doppio meccanismo di definizione, che consiste dapprima nella determinazione interna di un'entità particolare – che costituisce il passaggio dall'indeterminatezza dei molti alla definizione di cosa sia ciascuna entità in sé –, poi nella sua determinazione esterna, che avviene nel confronto di ciascuna entità con entità simili. Applicando il rapporto monade assegnata/monade inassegnata alla prospettiva verticale possiamo pertanto evidenziare come la monade assegnata sia l'idea interna ad una singola entità, che ha il ruolo di misura determinante, che Proclo identifica nel τι κοινόν e nel τί ἐστιν, ovvero nel principio di dimostrazione (ἀπόδειξις) e di definizione (ὁρισμός) di un'entità concreta, a partire dalle sue proprietà. La monade trascendente, invece, costituisce il principio di

[19]. Il punto di congiunzione tra la monade e la serie è essenziale. Cf. Achard-Narbonne 2012, pp. CLXVIII–CLXI in cui ne è ricostruita la storia, attribuendo l'insegnamento riguardante il legame tra la monade aritmetica e la serie a Moderato, citato da Stob. *Anth.* I 49.32.54 (Achard cita a questo proposito le osservazioni di Dodds 1963, pp. 208–10 in cui l'autore commenta Procl. *Elem. Theol*. 21); e riconoscendo in Teone di Smirne la definizione di numero come un sistema di monadi di-scendenti progressivamente verso il molteplice, che risale a sua volta verso la monade. Per Proclo, d'altra parte, come cercherò di dimostrare in seguito, la monade è al principio di tutte le serie che ordinano il reale.
[20]. Cf. Procl. *Theol. Plat*. III 2.10.15–11.5.
[21]. In *Theol. Plat.* III 2.11.5–10 Proclo parla espressamente di genere e di forma.

definizione totale (λόγος ὅλος, come è del resto evidenziato dalla proposizione 21 degli *Elementi di teologia*), che riunisce in modo uniforme (ἑνοειδῶς) tutte le entità aventi definizioni simili (*In Prm*. V 981.8: οἱ δὲ τῶν ἐν τοῖς καθ'ἕκαστα κοινῶν ὁρισμοί), e presentandone in atto (V 981.20: κατ'ἐνέργειαν) tutte le distinzioni: seguendo l'esempio proposto da Proclo, la monade assegnata riguarda l'insieme dei caratteri interni a Socrate che lo definiscono in quanto "uomo"; la monade inassegnata "uomo" è la categoria di appartenenza universale da cui dipende la serie degli uomini particolari, in virtù delle loro definizioni simili.

Alla luce di ciò, la serie non è una realtà semplice ma un sistema complesso e molteplice di insiemi di entità aventi proprietà e definizioni simili, riunite sotto un'unica causa-categoria universale. La serie, costituita da monadi assegnate (ciascuna una definizione che dice "cosa è" un'entità), è definita, nella sua interezza, da una monade trascendente che la definisce nel complesso, ovvero comprendendone tutte le identità che accomunano le sue entità e tutte le differenze che le distinguono[22].

Il θεῖος ἀριθμός in quanto "unitario" (ἑνιαῖος) e il πλῆθος νοητόν in quanto primo "unificato di enadi"

L'elemento che costituisce il punto di maggiore innovazione del sistema di Proclo è il principio della somiglianza, secondo cui la sommità simile del livello successivo è legata al limite simile del livello precedente. Seguendo questo principio, Proclo utilizza l'espressione πλῆθος ἀριθμός per indicare un'unità assolutamente simile all'Uno, che, come è evidente in *Teologia Platonica* III 3.12.20, è sovraessenziale: "la primissima serie (πρώτιστος ἀριθμός) è al contempo, in quanto connaturata all'Uno (τῷ ἑνὶ συμφυόμενος), uniforme (ἑνοειδής), ineffabile (ἄρρητος), sovraessenziale (ὑπερούσιος) e assolutamente somigliante al suo principio causale (πάντῃ τῷ αἰτίῳ προσόμοιος)". Il πλῆθος ἑνιαῖον riproduce, in effetti, le medesime caratteristiche dell'Uno, in forma di numero molteplice. Perciò è chiamato θεῖος ἀριθμός in *Teologia Platonica* III 3.13.3, e θεῖος ἀριθμὸς ἑνιαῖος nella proposizione 113 degli *Elementi di teologia*, pur non essendone esplicitamente chiarito il contenuto. Possiamo, d'altra parte, desumerne alcune caratteristiche a partire da proposizioni significative degli *Elementi di teologia*: nella dimostrazione condotta nella proposizione 25 il piano universale dell'Uno, che è causa unitaria di tutti gli enti – è cioè τἀγαθόν (25.24), μία τῶν ὅλων ἀρχή (25.22), πάντων ἐστὶν ἑνιαίως ὑποστατικὴ τῶν ὄντων (25.23–24) – è riprodotto in forma particolare dalle entità che Proclo definisce "frazioni di unità", che sono

[22]. Procl. *In Prm.* II 729.14–26.

τελειότης τἀγαθοῦ μοῖρά τίς (25.27–28). La proposizione 133 traccia la medesima linea di demarcazione universale-particolare tra l'Uno e l'ἑνιαῖος: da una parte l'Uno è ἁπλῶς τἀγαθὸν καὶ ἁπλῶς ἕν (133.10), dall'altra l'ἑνιαῖος ἀριθμός è un insieme costituito da bontà e unità particolari (133.11: τὶς ἀγαθότης ἐστὶ καὶ τὶς ἑνάς). Lo scarto tra l'Uno e il θεῖος ἀριθμὸς ἑνιαῖος si verifica, d'altra parte, anche dal punto di vista della causalità, poiché è determinata dal carattere del livello successivo: l'Uno fa sussistere ἑνιαίως un gruppo di entità ἑνιαῖοι; le entità ἑνιαῖοι fanno invece sussistere ὑπερουσίως il livello dell'οὐσία. Il carattere sovraessenziale del πλῆθος ἑνιαῖον è stabilito anche nel passo III 3.13.4–7 della *Teologia Platonica*, che lascia, tuttavia, intravedere una connessione tra il piano dell'essenza e quello dell'unitarietà:

> Prima degli enti (Πρὸ τῶν ὄντων), dunque, l'Uno fa sussistere (ὑφίστησι) le enadi degli enti (τὰς ἑνάδας τῶν ὄντων). Ed in effetti ancora in un altro modo è necessario che a loro volta quelli che sono in senso primario enti (τὰ πρώτως ὄντα) attraverso le loro enadi contigue (διὰ τῶν προσεχῶν αὐτῶν ἑνάδων) partecipino della primissima Causa (πρωτίστης αἰτίας).

L'aggettivo προσεχής evidenzia il rapporto di congiunzione che sussiste tra i τὰ πρώτως ὄντα, ovvero il primo grado del piano dell'οὐσία, e un certo tipo di enadi, in cui si articola il πλῆθος ἑνιαῖον. La distinzione tra l'ἑνιαῖος e l'οὐσία emerge, d'altra parte, dalla proposizione 135 degli *Elementi di teologia*, in cui Proclo definisce l'οὐσία simile ma non identica all'ἑνιαῖος, poiché non è un'unità, al di là dell'essere, pura e semplice (ovvero enade, secondo la dimostrazione della proposizione 133), ma è un "unificato". Malgrado questa distinzione, possiamo rintracciare dei casi in cui Proclo si riferisce a due livelli di πλῆθος ἑνιαῖον, il primo, è un sovraessenziale che corrisponde al primo livello della prima triade intelligibile, poiché riceve i caratteri dal livello puramente sovraessenziale; il secondo, in senso proprio essenziale, dal momento che consiste nell'azione unificatrice, ἑνιαίως, della terza triade intelligibile, in quanto ricettore dei caratteri delle sue due sommità. Questa differenziazione è d'altronde desumibile dall'analisi di alcuni passi della *Teologia Platonica*. In IV 31.93.1–7, mettendo in rapporto la triade intelligibile con quella intelligibile-intellettiva, Proclo distingue da una parte tre forme di unità intelligibile, dall'altra tre forme di moltiplicazione determinata, ovvero, di numero intelligibile-intellettivo. Nel mondo intelligibile, il primo livello è l' "uno" degli unitari (τὸ ἓν τῶν ἑνιαίων), il secondo è l' "uno" delle potenze generative (τὸ ἓν τῶν γεννητικῶν), il terzo l' "uno" delle forme essenziali (τὸ ἓν τῶν οὐσιωδῶν); nel mondo intelligibile-intellettivo, il primo livello è dei "primissimi numeri" (πρώτιστοι ἀριθμοί), il secondo dell'alterità (ἑτερότης), il terzo dell'ente (τὸ ὄν).

Prima di passare all'analisi della moltiplicazione determinata intelligibile-intellettiva, è importante comprendere cosa Proclo intenda per "molteplicità intelligibile". Essa è nella

proposizione 133 degli *Elementi*, un insieme di unità e bontà particolari (ἕκαστος τὶς ἀγαθότης ἐστὶ καὶ τὶς ἑνάς) che hanno ciascuna una proprietà divina particolare (ἰδιότης ἡ θεία). Nella proposizione 6 Proclo precisa che il piano ontologico è organizzato in classi di unificati (ἐξ ἡνωμένων), disposti in una gerarchia di quattro livelli di unità: la molteplicità pura (πλῆθος μόνον), l'unificato (ἡνωμένον), l'enade (ἑνάς) e l'Uno in sé (τὸ αὐτοέν). L'unificato è ciò che riceve l'unità per partecipazione (εἰ μὲν μετέχον τοῦ ἑνός), l'enade è invece il costituente semplice del primo unificato (εἰ δὲ ἐξ ὧν τὸ πρώτως ἡνωμένον), che è il primo partecipante dell'Uno in sé (τὸ πρώτως αὐτοῦ μετέχον).

D'altra parte, in *Teologia Platonica* III 24.84.4–9 Proclo sembra frapporre tra l'Uno impartecipabile e il primo unificato intelligibile un'enade partecipata che corrisponde al primo livello intelligibile, l'essenza (ἑνάς ἐστι μετεχομένη μὲν ὑπὸ τοῦ ὄντος). In virtù di questa partecipazione, che per Proclo corrisponde già ad una prima forma di aggiunzione e moltiplicazione (πλεονάσασα τῇ προσθήκῃ τῆς μεθέξεως ταύτης τοῦ πρώτως ἑνός) l'enade-essenza intelligibile si distingue dall'Enade-Uno. In *Teologia Platonica* III 26.89.5–90.6, risulta d'altra parte evidente la composizione di questa triade intelligibile in due enadi partecipate, ovvero due livelli di unità ancora assolutamente semplici – l'unità nascosta (τὴν κρύφιον ἕνωσιν τῆς πρώτης)[23] e la distinzione diadica (τὴν τῆς δευτέρας δυαδικὴν διάκρισιν)[24]– e un primo unificato, che è una molteplicità costituita di parti, un insieme appunto di enadi (ἡ τῆς τρίτης ἀπογεννᾶται πρόοδος, ἐκ μερῶν μὲν ἔχουσα τὴν ὑπόστασιν)[25].

Il non-numero, il numero e il numerato

Finora ho mostrato come l'ἀριθμός riceva nella prospettiva verticale, rispetto alla monade, la definizione di molteplicità determinata, ovvero di serie di entità collocate in livelli di unità decrescenti. Nel mondo intelligibile-intellettivo, tuttavia, Proclo assegna al termine ἀριθμός un significato specifico, che è quello di "alterità" (ἑτερότης), che, come mostro a seguire, ha una collocazione e una funzione precisa nella gerarchia ontologica.

Dal punto di vista della metafisica dell'Uno, Proclo identifica l'ἀριθμός con il διακρινόμενον, ovvero con il primo grado di divisione introdotto dalla congiunzione καὶ nella formula "essere e uno" con cui è identificato il mondo intellettivo. L'ἀριθμός è dunque l'ἑτερότης che si oppone all'identità unica, ἑνικός, del mondo intelligibile, dal momento che determina la

[23]. Procl. *Theol. Plat.* III 26.89.7–8.
[24]. *Ibid*.
[25]. Procl. *Theol. Plat.* III 26.89.10.

separazione dell'essere dall'unità[26] e il suo passaggio verso il "già diviso" (τὸ διακεκριμένον)[27] e il "già moltiplicato" (τὸ πεπληθυσμένον). Inoltre, nel passaggio IV 27.79 della *Teologia Platonica*, Proclo assegna una collocazione precisa al numero nel primo livello del mondo intelligibile-intellettivo, seguito dal "tutto composto di parti" (τὸ ὅλον τῶν μερῶν), e dal "completo" (τὸ τέλειον). La triade intelligibile-intellettiva è posta in relazione analogica con il mondo intelligibile, dal momento che riproduce in maniera degradata la stessa articolazione: nel mondo intelligibile l'uno-che-è, l'intero e la molteplicità illimitata sono messi in corrispondenza nel mondo intelligibile-intellettivo con il numero, l'intero e il perfetto[28]. Proclo distingue a questo punto la molteplicità intelligibile da quella intelligibile-intellettiva, affermando che la prima è "in modo unitario" (ἡνωμένως) e la seconda "in modo diviso" (διακεκριμένως), la prima è caratterizzata dalla "potenza" che è causa del numero in quanto monade-diade, la seconda è invece introduzione della prima alterità tra la monade *e* la diade.[29] È a questo punto che Proclo pone in un rapporto di superiorità il πλῆθος intelligibile in quanto unitario e non numerabile rispetto all'ἀριθμός intelligibile-intellettivo in quanto molteplicità determinata, ovvero quantificata.

La triade intelligibile presenta nella sua sommità la prima espressione essenziale dell'uno (in quanto insieme di limite e illimitato), che è, d'altra parte, per il tramite della potenza, seconda triade intelligibile, e per il tramite del πλῆθος, terzo livello intelligibile[30], il principio causale della sommità intelligibile-intellettiva, ovvero del numero, che Proclo fa corrispondere al pari e al dispari. In questo caso, poiché il πλῆθος intelligibile è superiore rispetto all'ἀριθμός intelligibile-intellettivo, i due termini sono posti in una relazione inversa rispetto a quanto mostrato nella prospettiva verticale della monade, a proposito della superiorità di ἀριθμός in quanto serie determinata, rispetto al πλῆθος indeterminato. Il principio di continuità della gerarchia degli enti prevede in effetti quanto già estrapolato da *Teologia* IV 39.113 e dalla proposizione 112 degli *Elementi*, ovvero che la sommità di un livello inferiore riceve i caratteri del livello superiore per il tramite del limite di quest'ultimo: applicando questo principio ai rapporti di continuità tra il mondo intelligibile e il mondo intelligibile-intellettivo, possiamo constatare che il πλῆθος, essendo il limite del mondo intelligibile, è la causa della processione dei caratteri del mondo intelligibile nella sommità del mondo intelligibile-intellettivo, ovvero nell'ἀριθμός. In questa prospettiva, il πλῆθος e l'ἀριθμός non sono considerati come gradi di unità (come lo sono, del resto, nel caso della

[26]. Ciò è affermato esplicitamente da Proclo stesso in *Theol. Plat*. IV 27.80.10: "è naturale, dunque, che Platone abbia dato inizio alle negazioni dell'uno partendo da questa: infatti è in questo livello che si trovano i molti per via dell'alterità che ha determinato la separazione dell'uno e dell'essere (διὰ τὴν ἑτερότητα τὴν διαστήσασαν τὸ ὂν καὶ τὸ ἕν)"; tr. Abbate 2019.
[27]. Si veda per l'abbassamento di grado dell'unità in forma tripartita *Theol. Plat*. III 14.52.5–10.
[28]. Procl. *Theol. Plat.* IV 27.78.19–79.16.
[29]. Per tutto questo passaggio cf. *Theol. Plat.* IV 28.
[30]. Procl. *Theol. Plat*. IV 28.81.3–20.

prospettiva verticale: monade-serie-molteplice), ma come livelli ontologici, il primo incluso nella monade intelligibile, il secondo nella monade intelligibile-intellettiva. D'altra parte, il πλῆθος intelligibile denota un tipo di molteplicità superiore a tutte le altre, non è infatti né una molteplicità determinata (ἀριθμός), né una molteplicità illimitata che non partecipa dell'uno e dunque dell'essere (ἀπειράκις ἄπειρον secondo *Theol. Plat.* II 1.5.9 e *El. Theol*. 1), ma è il primo misto, immediatamente prodotto dal primo πλῆθος ἑνιαῖον (l'essenza intelligibile), e dalla δύναμις ἑνιαία (la vita intelligibile). Al mondo intelligibile nel suo complesso, Proclo attribuisce un tipo di molteplicità nascosta (κρύφιος), indistinta, che non è ancora divisa e non è numerabile (ἀνάριθμος). D'altronde, Proclo fa riferimento alla proprietà intelligibile dell'innumerabilità in passi sporadici delle sue opere, in particolare in *Teologia Platonica* IV 29.85.20 la oppone alla proprietà intellettiva dell' "essere numerato" (τὸ ἠριθμημένον); in *Teologia Platonica* IV32.97.14 la definisce "primissima forma di molteplicità" (τὰ πρώτιστα πλήθη), unitaria, indifferenziata e non numerabile (ἑνιαῖα δέ ἐστι καὶ ἀδιάκριτα καὶ ἀνάριθμα). Nell'esegesi del *Parmenide*[31], sebbene questa proprietà non sia menzionata da Platone tra gli attributi negati dell'Uno nella prima ipotesi[32], Proclo applica all'Uno, l'essere "senza molteplicità" e "senza numero"[33]. È importante anche sottolineare che Proclo non si riferisce mai al mondo intelligibile con il termine ἀριθμός. Le uniche volte in cui l'intelligibile e le enadi sono chiamati ἀριθμός, lo sono nel senso di "serie" discendente dall'Uno e non in riferimento ad un particolare grado d'essere. Ciò effettivamente è evidente dal momento che nelle opere di Proclo i termini ἀριθμός e πλῆθος, se riferiti alla monade intelligibile o alla totalità delle enadi sono intercambiabili. Nella proposizione 113 degli *Elementi di Teologia*, l'ἀριθμός è la pluralità ordinata discendente dalla rispettiva causa: la pluralità delle enadi dall'Uno, degli intelletti particolari dalla monade dell'intelletto, delle anime dalla monade dell'anima[34]. Nel passo III 3.12 della *Teologia Platonica* le enadi sono invece definite la "molteplicità unitaria" (πλῆθος ἑνιαῖον), "serie assolutamente simile" (ἀριθμὸν τῇ αἰτίᾳ συγγενέστατον) all'Uno, e "primissima serie" (πρώτιστος ἀριθμός), che ci fa dunque supporre ne esistano altre. In effetti in *Elementi* 64.5–12 Proclo fa riferimento all'ἀριθμός delle enadi, degli intelletti, delle anime e anche dei corpi fisici, dimostrando in cosa consista la sua duplice articolazione interna. La stessa distinzione dei quattro ἀριθμοί si trova in *Teologia Platonica* III 3.12.8–14, in riferimento al rapporto di somiglianza che la

[31]. Procl. *In Prm*. VI 1075.23–1076.28.

[32]. Questa proprietà non è menzionata nel *Parmenide* tra gli attributi negati dell'Uno nella prima ipotesi, ma, nella seconda ipotesi (*Prm*. 143a4–144a9) si dimostra che se l'Uno esiste, il numero esiste. Il numero per Proclo è tra le negazioni dell'Uno anche in *In Prm.* VI 1083.1–20; 1076.23–27, e in *In R.* I 285.21–22.

[33]. *In Prm*. VI 1075.26–28: "l'Uno ha fatto sussistere tutta la molteplicità perché esso è senza molteplicità (ἀπλήθυντον), e il numero perché è senza numero (ἀνάριθμον), e la figura perché è senza figura (ἀσχημάτιστον)". Qualche riga dopo *In Prm*. VI 1076.24–28: "diciamo che la monade è senza numero (ἀνάριθμον τὴν μονάδα) non in quanto è inferiore ai numeri e indefinita, ma perché essa genera i numeri e li definisce (γεννῶσαν τοὺς ἀριθμοὺς καὶ ὡς ὁρίζουσαν), intendo la monade assolutamente prima (τὴν πρωτίστην μονάδα) di cui noi diciamo che contenga tutte le specie dei numeri (πάντα ἔχειν φαμὲν τὰ εἴδη τῶν ἀριθμῶν)".

[34]. Procl. *El. Theol*. 113.5–15.

monade-causa intrattiene con la propria serie: la natura in modo naturale (ἡ φύσις φυσικῶς), l'anima in modo psichico (ἡ ψυχὴ ψυχικῶς) e l'intelletto in modo intellettivo (ὁ νοῦς νοερῶς) generano le realtà da essi derivate (ἀπογεννᾷ). Dunque, l'Uno in forma unificata (καθ' ἕνωσιν) è causa della totalità del reale (αἴτιόν ἐστι τῶν ὅλων) ed al contempo la processione a partire dall'Uno è uni-forme (ἑνοειδής). Nella *Teologia*, il numero è un attributo che compare, come nel caso dell'Uno del *Parmenide*, tra le negazioni del mondo intelligibile. Questa constatazione è rilevante soprattutto in funzione della determinazione del carattere, per certi versi, innovativo della dottrina delle enadi di Proclo rispetto ai suoi predecessori. Nell'espressione "numero divino", indubbiamente di derivazione giamblichea, non possiamo in realtà intravedere un'anticipazione della dottrina delle enadi nel modo in cui è concepita da Proclo[35], dal momento che questa espressione, riferita alle enadi indica solamente la serie discendente dall'Uno-Dio che ne rispecchia le proprietà (essendo a sua volta divina), ma non ne esprime il suo effettivo statuto metafisico. Dall'altro canto, il numero, in quanto entità astratta, ha una collocazione precisa nel sistema metafisico di Proclo, poiché è posto nel livello del mondo intelligibile-intellettivo, garante della continuità tra il mondo intelligibile e il mondo intellettivo. Il θεῖος ἀριθμός intelligibile-intellettivo consiste, in effetti, nel principio di distinzione che mantiene il legame (σύνδεσις)[36] tra l'ἀνάριθμος intelligibile e l'ἠριθμημένος, il numerato intellettivo, che è "già diviso" (διακεκριμένος)[37]. In virtù di questa funzione di mediazione, il numero astratto intelligibile-intellettivo è costituito da una natura duplice: dal mondo intelligibile riceve una potenza elevatrice, dal momento che ha la capacità di riunire in unità i molti (κατὰ τὸ συναγωγὸν τῶν πολλῶν εἰς ἕνωσιν)[38]; per vicinanza con il mondo intellettivo, ha invece una propensione degradante verso la distinzione. Proclo dice infatti che il numero intelligibile-intellettivo genera i molti: κατὰ τὸ γεννητικὸν τῶν πολλῶν[39].

La τάξις ontologica risente complessivamente della presenza del numero. Benché non sia un'entità numerica a tutti gli effetti, il mondo intelligibile pre-contiene, nella contrazione della sua natura (κρύφιος), la prima espressione del numero, che nella *Teologia* è collocata solamente nel terzo livello intelligibile, in cui le idee sono riunite nella totalità uniforme e completa dell'intelletto. La molteplicità intelligibile (πλῆθος νοητόν) è pertanto la prima espressione di numero e principio, in forma unitaria e nascosta, del numero divino intelligibile-intellettivo. Infatti, anche nella proposizione 64 degli *Elementi*, la serie delle enadi, che costituisce la monade partecipata dell'Essere immediatamente contigua all'Uno, è a sua volta distinta in una serie duplice, quella delle realtà effettive, ovvero le enadi sovraessenziali, che sono, a nostro giudizio, effettivamente

35. Cf. O'Meara 1997, pp. 135–47.
36. Procl. *Theol. Plat.* IV 29.86.2.
37. Procl. *Theol. Plat.* IV 29.85.21.
38. Procl. *Theol. Plat.* IV 29.86.5.
39. Procl. *Theol. Plat.* IV 29.86.6.

non numerate, e quella delle entità intelligibili che ereditano i caratteri delle enadi per il tramite delle loro irradiazioni, e che sono la prima espressione del numero in forma unitaria (ἑνιαίως) e intelligibile, in quanto causa, e totalità assolutamente contratta (ὁ ἀριθμὸς νοητῶς ἐστι καὶ κατ' αἰτίαν μοναδικῶς)[40]. La scelta di riferire al terzo livello intelligibile, livello delle forme pure, un numero sovraessenziale che precede il numero essenziale intelligibile-intellettivo e le successive entità numerate, che dall'intellettivo si diramano fino al mondo sensibile, complica ulteriormente il quadro teorico attorno alla questione delle enadi. L'apparente contraddizione tra la numerabilità o non numerabilità intelligibile è, tuttavia, giustificabile alla luce dello sfondo dottrinale di Proclo, influenzato principalmente dalla lettura di *Parmenide* 144a–c, in cui è dimostrato che il numero è strettamente e necessariamente legato alla dimensione dell'essere[41], e può averne funzione di misurazione; e di *Timeo* 53b in cui il numero assume un ruolo ordinatore sulla dispersione fenomenica della materia originaria, e da cui Proclo estrapola, non a caso, la citazione "per mezzo di numeri e forme" (ἀριθμοῖς καὶ εἴδεσι)[42] per riferirla all'ultimo livello di estensione dell'ἀριθμός nella gerarchia degli enti, che precede la dispersione del molteplice nell'indeterminatezza non partecipe di unità e, dunque, non-essere[43]. L'essere singolo (ἑνικός) intelligibile viene così ad essere smembrato in due parti, l'una vicina all'Uno, l'altra all'ἀριθμός, ovvero al grado d'essere costituito dalla prima espressione dell'alterità (ἑτερότης). Possiamo, dunque, distribuire il numero nella seguente gerarchia degli enti, escludendo il sovraessenziale e includendo l'ultimo livello dell'intelligibile nel novero delle realtà in cui è rintracciabile il numero, in quanto determinazione essenziale. Proclo stesso, d'altra parte riferisce all'intelletto intelligibile la proprietà dell'essere primo unificato (τὸ ἡνωμένον) precedente il numero stesso (πρὸ ἀριθμοῦ)[44], in quanto composto da enadi che sono "elementi primi" (στοιχεῖα) irriducibili[45].

[40]. Procl. *Theol. Plat*. IV 29.88.6.
[41]. Nel momento dell'elaborazione di *Theol. Plat*. IV 29.84–89 Proclo ha probabilmente sott'occhio questi passi del *Parmenide* in cui, come nella *Teologia Platonica*, è questione della cosiddetta generazione dei numeri, in quanto "prova della loro esistenza" o di una giustificazione di alcuni numeri. È inoltre possibile che nel discorso di Proclo, come in quello di Platone, a proposito della distinzione dei "primissimi numeri" intelligibili-intellettivi, ovvero il pari e il dispari, da cui discendono il parimenti pari e il parimenti dispari vi sia un'allusione alla sistematica, e, per certi versi, simile, classificazione dei numeri presentata da Nicom. *Ar*. 8.10.13–13.8 ed. R. Hoche. Cf. Iambl. *In Nic.* 20.1–22.5 ed. H. Pistelli.
[42]. Procl. *Theol. Plat.* IV 29.86.16.
[43]. Procl. *El. Theol*. 1 e 2. Cf. *Theol. Plat*. IV 29.86.15.
[44]. Procl. *Theol. Plat.* IV 29.88.12.
[45]. Cf. *El. Theol*. 6.

L'aritmologia come immagine del piano ontologico

La costruzione della gerarchia ontologica procliana ordinata secondo il numero ha il modello della teologia aritmetica di Giamblico. Il frammento 37 del suo *Commento al* Timeo[46] distingue i due livelli (intelligibile, sensibile) secondo le proprietà dell'ordine, della compiutezza e dell'intelligenza da una parte (τεταγμένος, τέλειος, νοερός); dall'altra del disordine, dell'incompiutezza, dell'assenza di intelligenza (ἄτακτος, ἀτελής, ἀνόητος) ovvero delle proprietà che costituiscono la materia originaria di *Timeo* 30a5 che il frammento riproduce in questo modo:

> considera l'intera struttura corporea (τὴν ὅλην σωματοειδῆ σύστασιν) separatamente da sé, in tutta la sua casuale disorganizzazione (ὅπως ἐστὶ πλημμελὴς καὶ ἄτακτος), proprio perché, una volta visto l'ordine proveniente dall'anima (ἀπὸ ψυχῆς τάξιν) e la disposizione introdotta dal demiurgo (τὴν δημιουργικὴν διακόσμησιν), si possa distinguere quale sia la natura assegnata al corporeo in sé e per sé[47].

La διακόσμησις è dunque la διακόσμησις δημιουργική, l'azione ordinatrice del demiurgo che, come in *Timeo* 53b5 (εἴδεσί τε καὶ ἀριθμοῖς)[48], imprime al mondo un ordine matematico. Il fram- mento pone, d'altra parte, l'accento sulla dimensione temporale della διακόσμησις, che è continua nel tempo, poiché tiene perpetuamente insieme le due dimensioni nel κόσμος ("se entrambi convergono, ne consegue necessariamente che il cosmo è eternamente (διαιωνίως) beneficiario della sua attività creativa")[49]. A questo stesso proposito, i frammenti 63 e 67–68[50], che sono tratti da una lunga citazione giamblichea che Simplicio introduce nel *Commento alla* Fisica 793.30–795.1, sottolineano la posizione "all'origine del tempo" della διακόσμησις[51], proponendo una gerarchia ontologica e temporale su tre livelli: il paradigma intelligibile è posto nell'eternità[52]; la διακόσμησις consiste nell'operosità (διάκοσμον ἅμα οὐρανὸν ποιεῖν)[53] e nell'essenza del tempo

[46]. Si veda per la citazione Procl. *In Ti*. II 258.1–8 Van Riel (= I 382.12–20 Diehl). Per il frammento 37 si veda Dillon 1973, p. 139 (traduzione) e p. 310 (commento).
[47]. Iambl. *In Ti.* fr. 37.14–18 Dillon (= Procl. *In Ti.* II 258.13–15 Van Riel [= I 382.25–28 Diehl]).
[48]. *Ti*. 53b–c il passaggio dal disordine all'ordine avviene nel momento in cui la divinità inaugura la struttura matematico-geometrica del tutto, configurando l'universo "secondo forme e numeri" (εἴδεσί τε καὶ ἀριθμοῖς). Sul ruolo della matematica nell'organizzazione dell'universo nel *Timeo* si veda Brisson 2000a, pp. 295–315. Per un approfondimento sui commentari successivi, si veda Ferrari 2000, pp. 171–224. Sull'espressione εἴδεσί τε καὶ ἀριθμοῖς si veda Isnardi Parente 1997, pp. 187–93.
[49]. Iambl. *In Ti.* fr. 37.9–10 Dillon (= Procl. *In Ti.* II 258.7–8 Van Riel [= I 382.19–20 Diehl]).
[50]. Cf. Dillon 1973, pp. 172–3, 180–3 (traduzione); pp. 345–7, 351–4 (commento).
[51]. *Ti*. 37d–38b.
[52]. Simpl. *In Ph.* 794.23–24: "per queste ragioni il tempo è anche "il più simile possibile a sé stesso secondo il paradigma della natura eterna (κατὰ τὸ παράδειγμα τῆς διαιωνίας φύσεως)".
[53]. Simpl. *In Ph.* 793.33: "la frase 'simultaneamente alla disposizione che fa del cielo (διάκοσμον ἅμα οὐρανὸν ποιεῖν)' significa questo, che la venuta all'esistenza del Tempo (ἡ τοῦ χρόνου ὑπόστασις) è congiunta alla messa in ordine che parte dal demiurgo (τῇ διακοσμήσει τῇ ἀπὸ τοῦ δημιουργοῦ προελθούσῃ)".

(οὐσίαν αὐτοῦ τὴν κατ᾽ ἐνέργειαν[54] τῇ προϊούσῃ διακοσμήσει)[55], ed è la causa dell'unità insieme di tutte le cose[56]; il mondo sensibile che corrisponde al "venire a sussistere del tempo" (ἡ τοῦ χρόνου ὑπόστασις)[57] è un ordine "diviso" secondo "potenze o movimenti! (τὴν μεριστῶς κατὰ λόγους ἢ κινήσεις ἢ ἄλλας διωρισμένας δυνάμεις ἀφωρισμένην)[58], La citazione giamblichea anticipa un contenuto fondamentale del sistema procliano, che consiste nella tripartizione del rapporto anteriore-posteriore, articolato nei seguenti livelli: il modo superiore causale e "trascendente" (ἐξῃρημένος)[59] ovvero l'essenza-οὐσία; il modo secondario "coordinato" (συνταττόμενος) al primo, ovvero l'ἐνέργεια-δύναμις produttrice e ordinatrice (τάττουσα); il terzo venuto a essere e "ordinato" (ταττόμενος) dall'intermediario. Questa tripartizione corrisponde all'organizzazione della gerarchia ontologica di III 8.34–35 della *Teologia Platonica*, in cui Proclo presenta la triade essenza / limite, potenza / illimitato, misto[60]: in essa si articola ciascun livello, e ad essa corrisponde la distribuzione graduale della causalità enunciata nella proposizione 65 degli *Elementi di teologia*[61] in cui il primo livello è causale e "originario" (ἀρχοειδῶς), il secondo rispecchia le proprietà del primo per partecipazione, il terzo nella propria sussistenza[62], "in forma di copia" (εἰκονικῶς).

Il frammento 46[63], come il frammento 37, distingue tra una dimensione sensibile, divisa, molteplice e in movimento, e una dimensione indivisa, unitaria ed eterna, come si riscontra, del resto, anche nel cosiddetto *De Mysteriis* (*Risposta a Porfirio*) III 28.127.2–12. Nella stessa opera, d'altra parte, il livello della διακόσμησις demiurgica sembra essere attribuito al livello delle forme

[54]. La stessa distinzione κατ'ἐνέργειαν è riferita al livello delle forme intellettive nel passo di *Myst*. I 19.59.2–5: "C'è dunque anche nelle operazioni intellettive (κατὰ τὰς νοερὰς ἐνεργείας) il comune legame (ὁ κοινὸς σύνδεσμος) indivisibile che si trova tra queste forme, ma esiste anche nelle loro comuni partecipazioni (κοινὰς μετουσίας) alle forme, poiché nulla introduce alcuna distinzione in questa partecipazione e non c'è in essa alcun intermediario".

[55]. Simpl. *In Ph*. 793.30: "la sua essenza nell'attività la poniamo allo stesso livello (συντάττομεν) di questa disposizione che procede (τῇ προϊούσῃ διακοσμήσει) ed è unita alle sue creazioni (συνταττομένῃ πρὸς τὰ δημιουργούμενα) ed inseparabile (ἀχωρίστῳ) dalle cose portate a compimento da essa".

[56]. Simpl. *In Ph*. 794.20: κατὰ τὸ αἴτιον καὶ ἓν ὁμοῦ δὴ πάντων τούτων. La stessa espressione, sebbene riferita al livello intelligibile, ricorre in Iambl. *Myst*. I 19.59.1.

[57]. Simpl. *In Ph*. 793.33. In effetti in questo caso, ὑπόστασις designa il passaggio dall'essenza all'esistenza. Con lo stesso significato, questo termine è usato anche da Proclo. Negli *Elementi di teologia*, in cui ὑπόστασις si trova impiegato con una certa frequenza (si veda in particolare *El. Theol*. 64; 114; 153; 154; 162), non è pertanto mai connotato con il significato plotiniano di livello specifico e gerarchizzato d'essere. Al contrario, in Proclo, come nel passo di Simplicio, il termine significa il fatto puro e semplice di esistere. Ciò spiega anche il largo impiego che Proclo fa anche dell'aggettivo ὑποστατικός con il significato di "fare sussistere" (l'aggettivo è impiegato 12 volte su 16 con questo significato, in particolare in *El. Theol.* 20; 21; 25; 56; 57; 113; 137). Per un approfondimento sul significato post-plotiniano di ὑπόστασις si veda Achard – Narbonne 2012, pp. CXIII–CXXXVII. Per un approfondimento sui differenti significati del termine nella tardo-antichità si veda Taormina 1994, pp. 101–31; Combès 1994, pp. 131–49.

[58]. Simpl. *In Ph.* 794.2–7.

[59]. Simpl. *In Ph*. 794.10.

[60]. Questa triade nel libro III della *Teologia* è la triade per eccellenza essenza-vita-intelletto e per cui si rimanda a Hadot 1960, pp. 105–41 (= Hadot [1999] pp. 127–81].

[61]. Procl. *El. Theol*. 65.13–14: "Tutto ciò che sussiste in qualsiasi modo (τὸ ὁπωσοῦν ὑφεστός) o è in modo originario (ἀρχοειδῶς) nel suo essere causa (κατ'αἰτίαν), oppure in forma di immagine (εἰκονικῶς) nel suo modo d'essere (καθ' ὕπαρξιν) o nella sua partecipazione (κατὰ μέθεξιν)".

[62]. In questo caso, il termine ὕπαρξις, che in Proclo esprime un particolare modo d'essere potrebbe coincidere con l'ὑπόστασις, nel senso di venire a sussistere in forma di icona. Si veda su ciò anche Hadot 1973, 101–15.

[63]. Procl. *In Ti*. II 342.5–10 Van Riel (= I 440.15–20 Diehl); cf. Dillon 1973, pp. 148–51.

intellettive cui corrisponde l'ἐνέργεια[64]. Giamblico fa, in effetti, riferimento a tre livelli di dei: gli intelligibili che sono paradigmi per gli ultimi (ἀπὸ τῶν νοητῶν θείων παραδειγμάτων)[65], e che hanno una propria διακόσμησις, che consiste in un'"unica unità" (κατὰ μία ἕνωσιν)[66] e "in una perpetua superiorità" (κατὰ τὴν διαιωνίαν αὐτῶν ὑπερβολήν)[67]; gli intellettivi (θεῖα νοερὰ εἴδη) che preesistono "in modo separato" (χωριστῶς) agli dei encosmici visibili, mettendoli in comunione con i primi[68]. Perciò, Giamblico anticipa, anche in questo caso, un elemento fondante del sistema procliano, ponendo tra le due realtà inconciliabili un "comune legame" (κοινὸς σύνδεσμος)[69], che rende possibile la processione, per Proclo, la σειρά. Allo stesso modo, il frammento 44 tratto dal Commento di Giamblico al *Timeo* presenta la διακόσμησις in quanto legame intermedio, che unifica il livello intellettivo al paradigma intelligibile con il livello demiurgico, per il tramite dell'unificazione[70]. Lo scarto tra la dimensione intelligibile e quella sensibile, colmato da una διακόσμησις demiurgica che, sul modello del *Timeo*, imprime un ordine matematico all'universo, è anche il contesto di affermazione privilegiato per una teologia aritmetica, iniziata da Nicomaco di Gerasa, che lascia delle tracce importanti anche nel sistema filosofico di Proclo. L'antecedente diretto anche in questo caso è Giamblico, seppure Proclo intervenga su una dottrina rimaneggiata fortemente da Siriano[71].

Nel *Commento alla* Metafisica di Aristotele, Siriano infatti antepone alla scansione del reale tre presupposti ontologici fondamentali, che Proclo introdurrà nel suo sistema filosofico: il primo consiste nel fatto che il numero, secondo il precetto pitagorico, è il paradigma dell'universo e delimita gli ordini divini[72], permettendo così la lottizzazione del divino e la definizione di ciascun ordine secondo le sue proprietà; il secondo in base al quale ogni ordine è presente in ogni livello secondo la sua modalità d'essere[73], il terzo è il concetto di "molteplicità ordinata" (πλῆθος τὸ

64. Iambl. *Myst*. I 19.44.3.
65. *Myst*. I 19.43.18.
66. *Myst*. I 19.43.24.
67. *Myst*. I 19.44.1–2.
68. *Myst*. I 19.43.25–26.
69. *Myst*. I 19.44.3.
70. Iambl. *In Ti.* fr. 44 Dillon 1973, pp. 148–9 = Procl. *In Ti*. II 328.1–5 Van Riel (= I 431.23–25 Diehl): "il divino Giamblico prende una posizione intermedia tra questi due, collegando e unendo (συνάπτων καὶ ἑνίζων) il paradigma al demiurgo attraverso l'unità dell'intelletto (διὰ τὴν ἕνωσιν τὴν τοῦ νοῦ) con l'intelligibile".
71. Su ciò si veda Coulter 1976. Questa legge è la stessa che si riscontra in *El. Theol*. 103.
72. Syr. *In Metaph*. 145.25: ῥητέον οὖν ὅτι δυνάμει μὲν ἄπειρα τὰ θεῖα, ἀριθμῷ δὲ χρῆται μὲν πεπερασμένῳ.
73. *In Metaph*. 145.32: πάντα ἐν πᾶσιν. Questa legge neoplatonica di omologia, "tutto in tutto", esprime la convinzione che l'insieme della realtà si riflette in ciascuno degli esseri reali. Appare chiaramente già in Porph. *Sent*. 31, in cui gli incorporei, ovvero intelletti e anime (piuttosto che i sensibili, che possono comunque corrispondere al Tutto in virtù dell'analogia tra microcosmo e macrocosmo) riflettono il Tutto a proprio modo secondo una scala graduale in cui Dio è "ovunque e in nessun luogo rispetto a tutto ciò che da lui procede" (πανταχοῦ καὶ οὐδαμοῦ τῶν μετ' αὐτὸν πάντων); l'intelletto "è in Dio" (ἐν μὲν θεῷ), ed è "ovunque e in nessun luogo rispetto a ciò che da lui procede" (πανταχοῦ δὲ καὶ οὐδαμοῦ τῶν μετ' αὐτόν); l'anima è in Dio e nell'intelletto (ἐν νῷ τε καὶ θεῷ) e "ovunque e in nessun luogo rispetto al corpo" (ἐν νῷ τε καὶ θεῷ, πανταχοῦ <δὲ> καὶ οὐδαμοῦ ἐν σώματι); il corpo è "nell'anima, nell'intelletto e in Dio" (ἐν ψυχῇ καὶ ἐν νῷ καὶ ἐν θεῷ). Su ciò si veda Coulter 1976. Questa legge è la stessa che si riscontra in *El. Theol*. 103. Si tratta di una legge invariabile del Neoplatonismo, che di fatto si appoggia e sviluppa a partire dalla convinzione

τεταγμένον)[74] "continua" (συνεχής)[75] dei livelli dell'essere, che sembra corrispondere alla "molteplicità determinata" di Nicomaco di Gerasa (*Ar*. 7.9.7: πλῆθος ὡρισμένον). Questa molteplicità comprende il livello intelligibile, intellettivo, dianoetico (διανοητικός) e sensibile (φυσικός)[76]. Ciascun livello, inoltre, esprime le proprietà che gli corrispondono "in modo appropriato" (οἰκείως)[77]. Siriano presenta una struttura del reale fortemente gerarchica, che accorpa da una parte negli "ordinamenti divini"[78] il livello intelligibile e intellettivo, d'altra fa svolgere al livello dianoetico la funzione di ponte per l'anima[79] verso il divino. Dunque, la gerarchia del divino presenta la seguente scansione: (i) il piano intelligibile, che è presso gli dei, paradigmatico e perfetto[80], "riempie"[81] gli ordinamenti divini (θεῖαι διακοσμήσεις); (ii) il piano intellettivo è demiurgico[82] e "contempla" (θεωρεῖται) il precedente; (iii) il piano dianoetico "imita"[83] l'ordinamento divino, e "si innalza" (μετεωροπολεῖ) verso il divino[84]. Inoltre, la scansione del reale è messa in corrispondenza delle strutture matematiche in due passi del *Commento alla Metafisica*. In 125.19–126.25, recuperando la dottrina pitagorica secondo cui "ogni cosa che esiste è numero" (125.27: τὰ ὄντα ἄρα ἀριθμοί), Siriano distingue tre livelli che precedono il mondo sensibile: (i) l'uno e la diade, entrambi numeri originari, che si manifestano a tutti i livelli dell'essere, e costituiscono l'ἑνιαῖος ἀριθμός[85]; (ii) l'ἀριθμὸς μοναδικός[86], che consiste nel mondo intelligibile, che è la triade originaria di tutti gli altri numeri, che precontiene la molteplicità in modo nascosto (è

platonica fondamentale che le realtà sensibili posseggono la loro controparte nelle Idee in un modo eterno e realmente esistente (cf. *Ti*. 39e5–6). Dam. *In Phlb.* 130.2–4 (= Iambl. *In Phlb*. fr. 5 Dillon) attribuisce la dottrina della presenza del Tutto nelle parti a Porfirio e Giamblico: "Allora tutti gli elementi che sono nell'universo sono anche in noi, e tutti quelli che sono in noi sono anche nell'universo? Come si distingue allora il tutto dalle parti? Porfirio e Giamblico affrontano questo problema dicendo che 'tutte le cose sono ovunque (πάντα εἶναι πανταχοῦ), ma in modi diversi in luoghi diversi (ἄλλως μέντοι καὶ ἄλλως)'". Si veda Lecerf 2017, pp. 187–223, che a partire da *De anima* sez. 6–7 Dillon-Finamore (= Stob. *Anth*. I 49.32), sottolinea che questa legge del "tutto in tutto" è attribuita da Giamblico già a Numenio, e abbassa l'anima al livello delle parti (in quanto "anima parziale", μερικὴ ψυχή), in quanto "omologa" all'intelletto, ma in forma inferiore (cf. Porph. *Sent.* 10 e 11).

[74]. *In Metaph*. 81.36.

[75]. Questo carattere è già attribuito da Giamblico alla διακόσμησις intelligibile, in *Myst.* I 19.43.23.

[76]. Syr. *In Metaph*. 82.4–5 dice che i tre livelli dell'essere (intelligibile-dianoetico-sensibile) sono uniti tra di loro: εἰ γάρ ποτε τὰ τρία ταῦτα συντρέχει καὶ ἑνοῦται πρὸς ἄλληλα.

[77]. *In Metaph.* 82.2.

[78]. *In Metaph*. 82.13.

[79]. *In Metaph.* 82.15.

[80]. *In Metaph*. 82.3.

[81]. *In Metaph*. 82.10.

[82]. *In Metaph*. 82.14.

[83]. *In Metaph*. 82.15.

[84]. Cf. Iambl. *Myst*. V 15.163.16. Il verbo μετεωροπολέω esprime l'uscita fuori dal corpo per elevarsi verso l'intelletto. D'altra parte, l'immagine è chiaramente tratta da *Phdr*. 246c, che costituisce poi la rivoluzione celeste recuperata da Proclo nel libro IV della *Teologia Platonica*.

[85]. Si veda *In Metaph*. 126.17. Dal numero unitario di Siriano, composto dall'uno e dalla diade, Proclo attinge la sua dottrina delle enadi, che è effettivamente applicata in *El. Theol*. 113.

[86]. Cf. *In Metaph*. 126.2.

il κρύφιος ἀριθμός)[87]; (iii) le forme demiurgiche (ἰδέαι δημιουργικαί)[88]; (iv) le entità encosmiche (ἐγκόσμια)[89]; Siriano presenta così uno schema dei mondi divini modellato perfettamente sul numero pitagorico, cui fa corrispondere una proprietà e un'azione: la triade è il primo perfetto, ed è semplicemente contemplata: τὸ πρώτως τέλειον ἐθεωρήθη[90]; la tetrade è l'insieme originario che comprende le realtà encosmiche (la καθολικὴ διακόσμησις di *Theol. Ar*. 28.13): ἀρχοειδῶς πάντα τὰ ἐγκόσμια περιείληπται (*In Metaph*. 146.1); l'ebdomade è la provvidenza che ordina le realtà generate, non essendo sottoposta a generazione: ἀγενήτως τὰ γενητὰ προνοίας ἀξιοῦται (*In Metaph*. 146.2); la decade è l'insieme del reale già diviso e intellettivo: διακεκριμένως, μᾶλλον καὶ ἤδη νοερῶς πάντα προείληπται (*In Metaph*. 146.2–3). Egli mostra, dunque, una certa tendenza a pitagorizzare Platone, conciliandolo con le letture di Aristotele.

Nel libro IV della *Teologia Platonica*, Proclo riutilizza la tradizione aritmologica pitagorica per giustificare lo spostamento semantico del "numero" da quello generico di serie alla definizione di un ordine divino preciso, quello occupato dagli dèi intelligibili-intellettivi. In questo livello, nella definizione del numero in quanto introduzione della prima alterità tra la monade e la diade, Proclo recupera la tradizione pitagorica sul dispari, che riunisce e connette, e il pari che fa procedere e distingue, per mostrare in cosa consista la forza ontologica del numero in quanto legame della gerarchia degli enti: esso tiene insieme la prima serie di opposti (τὰς δύο συστοιχίας ὑφίστησιν)[91]. Il numero intelligibile-intellettivo è prima di tutto composto da due potenze originarie in base alle quali esso svolge il ruolo di forza riunente in unità tutte le entità: il dispari, simbolo del limite, dell'unitarietà e della razionalità, connette la molteplicità intellettiva verso gli intelligibili; il pari, simbolo di illimitatezza, pluralità e irrazionalità, fa procedere la molteplicità nei livelli successivi. Dall'opposizione del pari e del dispari, sul modello pitagorico[92], Proclo fa procedere una serie di coppie di opposti: generatrici di vita (ζωογονικάς) / immutabili (τὰς ἀτρέπτους), feconde (γονίμους) / produttive (ποιητικάς), indivisibili (τὰ ἀμέριστα) / divisi (τὰ μεριστῆς προϊστάμενα ζωῆς ἢ ποιήσεως), intellettive e unitarie (τὰ νοερώτερα καὶ ἑνικώτερα) / irrazionali e moltiplicate (ἀλογώτερα καὶ πληθυόμενα)[93]. Queste coppie presiedono alla frammentazione del numero nell'ordine intellettivo: ad ogni genere diviso degli dèi intellettivi corrisponde un numero particolare. In questo modo, Proclo costruisce una gerarchia del numero, ponendo alla due estremità da una parte il numero sovraessenziale, quello rappresentato dai primi due livelli intelligibili – le

87. *In Metaph*. 126.23. Per Siriano, come per Proclo, il livello della molteplicità contratta e "nascosta" non corrisponde alle enadi-principi ma al livello intelligibile, che è il primo effettivo numero, ovvero la prima composizione risultante dalla triade inizio-mezzo-fine, come è detto in *Theol. Ar*. 17.15.
88. *In Metaph*. 126.18; 8.23.
89. *In Metaph*. 126.20.
90. *In Metaph*. 145.32.
91. Procl. *Theol. Plat*. IV 29.84.26.
92. La testimonianza è di Arist. *Metaph*. A 5.986a22–26.
93. *Theol. Plat*. IV 29.85.1–7.

enadi sovraessenziali – e dall'altro il numero periodico – quello che regola l'ordinamento di tutto il cosmo e che scandisce la discesa e la risalita delle anime[94]. Il Licino pone le forme e il numero in un rapporto di priorità ontologica: la generazione delle forme nel livello intelligibile precede quella del numero nel mondo intellettivo, in questo modo, nella logica del sistema ontologico procliano secondo il quale il livello inferiore deve contenere necessariamente i livelli che lo precedono, "ogni numero è forma, mentre non ogni forma è numero"[95]. D'altra parte, da *Timeo* 39e10, "i numeri sono quattro", Proclo deriva che nella tetrade intelligibile, vi è il "numero delle forme", una monade che è causa della moltiplicazione e della differenziazione dei numeri intellettivi; dalla seconda ipotesi del *Parmenide*[96] in cui l'Uno-che-è risulta formato da molteplici componenti numerabili, è inoltre derivato il fatto che "il numero viene dopo la molteplicità"[97]. Tre sono dunque le concezioni del numero che emergono dal libro 4 della *Teologia Platonica*, consacrato interamente all'introduzione dell'alterità nella gerarchia dell'essere: il numero come molteplicità contratta, il numero come principio di distinzione all'origine delle coppie di opposti e il numero come molteplicità dispiegata nella serie degli dei divisi (paterni, demiurgici, ipercosmici, non vincolati, encosmici). Il termine ἀριθμός scandisce dunque tutta la gerarchia dell'essere: la monade sovraessenziale delle enadi; la tetrade delle forme intelligibili; la diade del pari e del dispari nel mondo intelligibile-intellettivo; l'ebdomade degli dei intellettivi; la serie delle coppie coordinate degli enti.

Nel *Commento al* Parmenide, Proclo sottolinea che i διάκοσμοι più vicini alla monade sono caratterizzati da una parte da una molteplicità più contratta (*In Prm*. IV 890.32–33: πλῆθος ἔχει συνεσταλμένον) e da un'esistenza limitata dal numero *(In Prm*. IV 890.34: ποσῷ πεπέρασται), sono cioè "numeri monadici" (μοναδικοί); dall'altra possiedono una potenza maggiore (*In Prm*. IV 890.33: δύναμιν δὲ ὑπερηπλωμένην τῶν ποῤῥωτέρω) che dipende dalla dominazione del principio enadico e dalla sua capacità di estensione ai livelli successivi. Le enadi sono, dunque, un principio che penetra nei διάκοσμοι, i quali, a loro volta, quanto più ne sono impregnati, tanto più dominano sui numeri successivi[98]. L'originarietà delle enadi rispetto ai numeri è, inoltre, messa in luce in un altro passo del *Commento al* Parmenide in cui Proclo parla della moltiplicazione delle ipotesi del dialogo platonico e della loro corrispondenza con i livelli del reale. Poiché la prima ipotesi è l'Uno, ἀνυπόθετος,[99] la seconda ipotesi, che Proclo fa corrispondere alla diade, coincide con il numero divino enadico,[100] da cui dipendono la triade, la tetrade e la dodecade[101], ovvero il livello

94. Proclo fa qui riferimento al passaggio del *Ti.* 37d6 e 53b5.
95. Procl. *Theol. Plat*. IV 29.87.25–26.
96. In particolare, *Prm*. 143a1ss. e 144a–c.
97. Procl. *Theol. Plat*. IV 29.88.15.
98. Procl. *In Prm*. IV 890.36–37: κρατούμενα ὑπὸ τῆς ἑαυτῶν ἑνάδος κρατοῦντα δὲ ἁπάντων τῶν ἐφεξῆς ἀριθμῶν.
99. Procl. *In Prm*. I 622.32. Si veda *Resp*. VI 510b7; 511b6 e il commento di Procl. *In R*. I 283.6–284.25; *In Alc.* 128.4 e Westerink, 1954, p. 105 nota 8.
100. Cf. l'accostamento delle enadi alla diade successiva all'Uno in *Theol. Plat.* III 7.30.10.

intelligibile, il livello intellettivo-intelligibile e il livello intellettivo (Zeus, ultimo della triade paterna intellettiva presiede il corteo dei dodici dei giovani, gli stessi di *Fedro* 247a).

Conclusioni

Come ho cercato di mostrare, il presupposto che fonda la scientificità del sistema teologico di Proclo è il rigoroso (ἀκρίβεια)[102] modo in cui sono collocati i livelli di essere rispetto all'Uno. Tale modo è prima di tutto numerico. Proclo definisce la teologia come ἕξις, una "condizione"[103] che ha un carattere duplice: da una parte "rivela" (ἐκφαίνω) le τάξεις[104], ed è perciò scienza, dall'altro "mantiene la segretezza" sull'Uno (διαφυλάσσω)[105]. Questa duplicità si manifesta nella distinzione di due tipi di evidenza: quella immediata dell'Uno, che è inconoscibile, e quella discorsiva delle τάξεις, che appartengono al piano del conoscibile. L'unico procedimento che, secondo Proclo, si può applicare alla τάξις dell'essere, che, al tempo stesso, determina la scientificità e l'incontrovertibilità della teologia, è quello diairetico[106] κατ'ἄρθρα, che attinge "all'ordine geometrico o delle altre matematiche" (ἐν γεωμετρίᾳ τάξεως ἢ τοῖς ἄλλοις μαθήμασι)[107]. Difatti, l'Uno è un concetto immediato e indimostrabile, la verità concernente la τάξις è invece un risultato, il "divenire evidenti" (κατάδηλα γίνεται)[108]. Si tratta di una distanza non solo conoscitiva ma anche metafisica che si riflette sulla struttura di due opere centrali nel progetto di validazione scientifica della teologia operata da Proclo: la *Teologia platonica* e gli *Elementi di teologia*. In queste due opere la verità è presentata secondo uno statuto triplice: in base al suo contenuto, è assolutamente trascendente se riguarda l'Uno; trascende le proposizioni di cui è condizione di esistenza se riguarda l'intelletto; è una verità proposizionale se è rivolta ai livelli successivi. L'evidenza che riguarda l'Uno è perciò una verità originaria e paradigmatica che è incoglibile, indicibile e indimostrabile per la ragione umana, è un assioma e una nozione comune innata e eterna

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[101]. Procl. *In Prm*. I 623.15.
[102]. Il termine ἀκρίβεια indica un criterio di esattezza scientifica connesso generalmente all'esercizio delle scienze matematiche, in quanto scienze esatte. Si veda Iamblichus, *Comm. Math*. 29.9 ed. Klein. La forma aggettivale, ἀκριβής, è attribuita frequentemente al modo di vita pitagorico: Porfirio lo impiega in *Plot*. 15.23 e in *V. Pyth.* 36.5. Anche Giamblico lo usa in riferimento al modo di vita pitagorico: Iambl. *VP* 29.163. Cf. su queste testimonianze O'Meara 1989, pp. 30–52. Il concetto di esattezza scientifica arriva fino a Simplicio che nel commento sulle *Categorie* di Aristotele attribuisce alla conoscenza intelligibile (τὴν νοερὰν θεωρίαν), un criterio ἀκριβέστερον: si veda su ciò Simpl. *In Cat.* 2.12. Cf. anche Simpl. *In Ph*. 4.10–11, riguardo alla *Fisica* di Aristotele come ἀκρόασις, e 5.17–18 (ἀκριβὴς κατανόησις). Su queste occorrenze cf. Hoffmann 2000, p. 479.
[103]. Procl. *Theol. Plat*. IV 4.17.10.
[104]. Procl. *Theol. Plat*. I 10.45.15; I 12.55.13; II 5.38.11; II 11.65.10; II 12.72.13.
[105]. Procl. *Theol. Plat*. III 1.5.6–16.
[106]. Procl. *Theol. Plat*. I 10.45.15.
[107]. Procl. *Theol. Plat.* I 10.45.26.
[108]. Procl. *Theol. Plat*. I 10.45.22.

nell'anima, poiché è una conoscenza già data, cui si può pervenire solo con la συναφή, con il contatto, anche se non diretto, con l'Uno[109]. D'altra parte, l'evidenza che riguarda l'intelletto è il risultato di un percorso argomentativo, ovvero è la certezza derivante dalla prova dimostrativa affrontata dall'anima. Infine, l'evidenza che possiede l'anima consiste nella discorsività che si estende sul piano del visibile, essendo il λόγος chiaro e coerente che attualizza le κοιναὶ ἔννοιαι in un tempo e in una struttura argomentativa determinati[110], che non è, tuttavia, autonomo e universalmente valido, ma interno al meccanismo di funzionamento della dialettica. L'attività dell'anima è, dunque, quella dell'approfondire la sua familiarità con l'Uno[111] attraverso la conoscenza dei livelli ad essa superiori. Nella proposizione 162 degli *Elementi* questa presa di conoscenza dell'Uno, che è τὸ θεῖον καὶ ἄγνωστον, è connotata come un risultato cui si perviene (γνωρίζεσθαι συμβαίνει) a partire dalle proprietà di quelli che ad esso partecipano (ἀπὸ δὲ τῆς τῶν μετεχόντων ἐξαλλαγῆς καὶ τὰς ἐκείνων ἰδιότητας)[112].

Bibliografia

M. Abbate, *Il "linguaggio dell'ineffabile" nella concezione procliana dell'uno-in-sé*, in «Elenchos», 22 (2001), pp. 305–27.

M. Abbate (ed.), *Proclo. Teologia Platonica*, Milano, Rusconi, 2019.

M. Achard – J.-M. Narbonne – L. Ferroni (ed.), *Plotin. Œuvres complètes. Traité 1 (I, 6), Sur le beau*, t. I., Vol. I, Paris, Les Belles Lettres, 2012.

M. Achard, *Le Traité 54 de Plotin. À propos d'une traduction récente*, in «Laval théologique et philosophique», 62 (2006), pp. 381–8.

L. Brisson, *Le rôle des mathématiques dans le* Timée *selon les interprétations contemporaines*, in A. Neschke-Hentschke (ed.), *Le* Timée *de Platon. Contributions à l'histoire de sa réception*, Louvain-Paris, Peeters, 2000, pp. 295–315.

B. Collette-Dučić, *Plotin et l'ordonnancement de l'être*, Paris, Vrin, 2007.

J. Combès, Ὕπαρξις *et* ὑπόστασις *chez Damascius*, in F. Romano and D. P. Taormina (eds.), Hyparxis *e* hypostasis *nel neoplatonismo*, Firenze, Olschki, 1994, pp. 131–49.

[109]. L'Uno è infatti, secondo Proclo, più ineffabile dell'ineffabile stesso (*Theol. Plat*. II 11.65.15). Tale considerazione è quella che verrà poi sviluppata da Damascio che anteporrà all'Uno un massimamente Indicibile. Su ciò Abbate 2001, pp. 305–27.

[110]. Cf. Taormina 2000, pp. 29–46 per un approfondimento sul concetto di "evidenza" nella *Teologia*, e sull'importanza della coerenza, come prova di validità.

[111]. Si veda per l'impiego di questo verbo nel senso di "presa di familiarità con l'Uno" *Theol. Plat*. II 10.64.3; I 3.15–17.

[112]. *El. Theol*. 162.1–4.

J. A. Coulter, *The Literary Microcosm: Theories of Interpretation of the Later Neoplatonists*, Leiden, Brill, 1976.

J. M. Dillon, (ed.), *Iamblichi Chalcidensis in Platonis dialogos commentariorum fragmenta*, Leiden, Brill, 1973.

E. R. Dodds (ed.), *Proclus.* The Elements of Theology, Oxford, Oxford University Press, 1963.

F. Ferrari, *Ordine del mondo e ordine dell'anima. Il* Timeo *e la desocratizzazione dell'etica socratica*, in M. Migliori – L. M. Napolitano Valditara (ed.), *Plato Ethicus. La filosofia è vita*, Brescia, Morcelliana, pp. 129–42.

P. Hadot, *Être, Vie, Pensée chez Plotin et avant Plotin*, in *Les Sources de Plotin, Entretien sur l'Antiquité classique* 5, Vandoeuvres-Genève, Fondation Hardt, 1960, pp. 105–41.

P. Hadot, *L'être et l'étant dans le néoplatonisme*, in «Revue de théologie et de philosophie», 23 (1973), pp. 101–15.

P. Hadot, *Plotin. Porphyre. Études néoplatoniciennes*, Paris, Les Belles Lettres, 1999.

Ph. Hoffmann, *La triade chaldaïque* erôs, alètheia, pistis *: de Proclus à Simplicius*, in A.-Ph. Segonds – C. Steel (eds.), *Proclus et la* Théologie Platonicienne, Leuven / Paris, University Press / Les Belles Lettres, 2000, pp. 459–89.

M. Isnardi Parente, *Idee e numeri nel* Timeo, in Thomas Calvo and Luc Brisson (eds.), *Interpreting the* Timaeus-Critias, Sankt Augustin, Academia Verlag, 1997, pp. 187–93.

A. Lecerf, *L'évolution du concept de principe dans le premier néoplatonisme. Un bref parcours*, in M.-A. Gavray – A. Michalewski (eds.), *Les Principes cosmologiques du platonisme. Origines, influences et systématisation*, Turnhout, Brepols, 2017, pp. 187–223.

D. J. O'Meara, *Pythagoras Revived. Mathematics and Philosophy in Late Antiquity*, Oxford, Clarendon Press, 1989.

D. J. O'Meara, *Structures hierarchiques dans la pensée de Plotin. Étude historique et interprétative*, Leiden, Brill, 1997.

D. P. Taormina, *Anima e realtà del conoscere. Ὑπόστασις e ὕπαρξις nei commentari tardoantichi al* De anima, in F. Romano – D. P. Taormina (eds.), Hyparxis *e* hypostasis *nel neoplatonismo*, Firenze, Olschki, 1994, pp. 101–31.

D. P. Taormina, *Procédures de l'évidence dans la* Théologie platonicienne, in A.-Ph. Segonds – C. Steel (eds.), *Proclus et la* Théologie Platonicienne, Leuven, University Press - Paris, Les Belles Lettres, 2000, pp. 29–46.

J. Trouillard, *Les degrés du ποιεῖν chez Proclus*, in «Dionysius», 1 (1977), pp. 69–84.

ABASTRACT

Miriam Cutino, *The Divine* Arithmos *in Proclus' Metaphysical System*

This investigation studies the articulation of the universal ontological hierarchy (*taxis*) in Proclus's Platonic Theology, a metaphysical system defined by the fundamental existential value of unity. The *taxis* is comprised of three different forms of order—*seira*, *diakosmesis*, and *diakosmos*—and the chapter aims to show how the structure of Proclus's divine *diakosmoi* is articulated on a mathematical analogy. This analogy establishes a correspondence between the Pythagorean divine number (*arithmos*) and each level of ontological reality. The analysis focuses on the ontological significance of the term *diakosmos* as both number and paradigmatic cause of unification. The essay also concentrates on the term *arithmos* itself, which is presented as the first "otherness" (*heterotes*) introduced by the intelligible-intellectual world. This interpretation connects Proclus's use of *arithmos* to its meaning in the works of Iamblichus and Syrianus's *Commentary on Aristotle's* Metaphysics, highlighting how number serves to mediate the initial unity of being by introducing multiplicity or difference.

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LES STRUCTURES NUMERIQUES DES ORDRES DIVINS DANS LA *THEOLOGIE PLATONICIENNE* DE PROCLUS. DE LA TRIADE A L'HEBDOMADE ET A LA DODECADE.

Pour contribuer à l'étude de la liaison entre science et théologie dans l'œuvre de Proclus, qui est l'objet de notre colloque, je me propose de parcourir son œuvre majeure, la *Théologie Platonicienne*, afin de mettre en évidence, dans la perspective de la liaison de la théologie scientifique et de l'arithmétique (ou de l'arithmologie), les *structures numériques* à l'œuvre dans la *Théologie Platonicienne* (désormais *Theol. Plat.*)[1]. Il s'agira d'un examen de ce que l'on pourrait appeler une macrostructure arithmétique, qui surplombe les structurations numériques plus fines dont la *Théologie Platonicienne* est tissée – car les nombres sont partout dans cette somme théologique et servent aussi à l'harmonisation entre la théologie de Platon et les théologies orphique et chaldaïque, que nous devrons laisser ici de côté[2].

Pour la clarté de mon propos, je commencerai par en donner une vision synoptique.

La διάκρισις du Réel, au fur et à mesure que se déploie la πρόοδος à partir de l'Un-Bien, se structure selon une multiplicité structurelle croissante, et à chaque fois définie, qui correspond à une progression numérique 1 – 3 – 7 – 12, les diverses διακοσμήσεις divines, celles des dieux intelligibles (νοητοί), intelligibles-intellectifs (νοητοὶ καὶ νοεροί) et intellectifs (νοεροί), puis des dieux hypercosmiques, « séparés » (ἀπόλυτοι) et encosmiques, s'organisant selon ces nombres *qui sont à chaque niveau un principe de détermination maîtrisant la multiplicité*. Après le Premier principe (l'Un-Bien), l'organisation des plans intelligible et intelligible-intellectif est constituée de triades (dont chaque terme est lui-même triadique), avant que ce noyau triadique n'engendre l'hebdomade intellective construite par une duplication et l'addition d'une unité. Au plan intellectif, on obtient ainsi *deux triades* augmentées d'*une monade*, c'est-à-dire la triade des « pères » (Cronos/Rhéa/Zeus), flanquée de la triade anonyme des dieux « immaculés » (ἄχραντοι), *plus* une

[1]. Je suivrai pour l'essentiel les excellentes traductions en français de Saffrey – Westerink 1968–1997, et renvoie le lecteur aux introductions magistrales et aux riches annotations de ces volumes, qui sont une base pour toute recherche. Une autre traduction très utile (avec texte grec) est celle de Abbate 2019. Cette communication s'insère dans plusieurs travaux sur la fondation de la scientificité de la théologie proclienne : Hoffmann 2021, pp. 961–1005 ; Hoffmann 2024, pp. 41–80 ; Hoffmann – Gavray 2024. Le présent travail interrogera la dimension arithmétique de la science théologique chez Proclus.

[2]. Lewy 2011, pp. 481–5 (*Excursus VII. Proclus' exposition of the Chaldæan system of the noetic entities*); et par exemple Brisson 2000, pp. 109–62 (tableau en Annexe, pp. 161-2).

monade « séparative », donc : [3 x 2 = 6] + 1 = 7. Le noyau triadique de cette hebdomade (la triade des « pères ») dépend étroitement de triades antérieures ce qui permet d'assurer la continuité « mimétique » du système processif. Après l'hebdomade intellective apparaît ensuite, au niveau des dieux hypercosmiques et des dieux hypercosmiques-encosmiques (les dieux « séparés ») une structure dodécadique (4 x 3) inspirée par le cortège des dieux dans le *Phèdre*. Nous prêterons une attention toute particulière aux analyses du livre VI de la *Theol. Plat.*, afin de comprendre comment cette structure dodécadique est construite par Proclus dans le souci de maintenir une stricte continuité mimétique avec les diverses triades qui organisent les plans transcendants, mais aussi avec l'organisation hebdomadique du diacosme intellectif, dont dépend *immédiatement*, et en parfaite continuité, la dodécade des dieux hypercosmiques. La simplicité des ordonnances transcendantes fait place alors à une relative complexité des plans hypercosmiques et hypercosmiques-encosmiques, qui ne fait que refléter les progrès de la pluralité tout en reproduisant et combinant des structures d'ordre transcendantes. C'est ainsi que se bâtit une somptueuse architecture du divin, dont tous les plans sont reliés par une parfaite continuité les uns avec les autres au sein même de la dégression (ὕφεσις), le schème triadique apparaissant du haut en bas de l'édifice comme un principe de structuration majeur.

Le plan intelligible, structuré par des triades, est étudié dans le livre III de la *TP*, chapitres 7–28. Un exemple très clair suffira pour décrire ce niveau de réalité : les chapitres 15–19, consacrés à l'enseignement des Intelligibles dans le *Timée*, énumèrent les trois ordres (τάξεις) des Intelligibles que sont, dans l'ordre ascendant, le Vivant-en-Soi qui est le modèle de l'Univers (τὸ αὐτοζῷον), l'Éternité (ὁ αἰών), et l'un (τὸ ἕν) ou la monade intelligible, qu'il ne faut pas confondre avec l'Un absolu imparticipable qui transcende toutes choses (*Theol. Plat*. III 18.59.8–10).

Cette construction est présentée en ces termes par Proclus (*Theol. Plat.* III 18.58.13–23) :

> Trois degrés d'intelligibles (τρεῖς […] τῶν νοητῶν τάξεις) viennent donc de se révéler (…) à nous d'après l'enseignement du *Timée* : le Vivant-en-soi (τὸ αὐτοζῷον), l'éternité (ὁ αἰών), l'un (τὸ ἕν). Ainsi, à cause de cet un-là et de son installation stable en lui (σταθερὰν ἵδρυσιν), l'éternité a fixé le royaume intelligible (τὴν νοητὴν […] βασιλείαν), et, à cause de l'éternité, le Vivant-en-soi délimite éternellement de la même façon la limite inférieure des dieux intelligibles (τὸ πέρας τῶν νοητῶν θεῶν). Et le Vivant-en-soi, qui a procédé sous quatre formes (τετραδικῶς προελληλυθός), dépend de la dyade qui est dans l'éternité (τῆς ἐν τῷ αἰῶνι δυάδος) (car l'éternité est le 'toujours' accompagné de l'être [τὸ γὰρ ἀεὶ μετὰ τοῦ ὄντος ὁ αἰών])[3], tandis que la dyade qui est dans l'éternité participe de la monade intelligible, que Timée a appelé un *un*, précisément pour cette raison qu'elle est monade et principe de tout le plan intelligible.

[3]. Arist. *Cael.* A 9.279a27–28.

Chaque ordre est lui-même triadique (3 x 3). La monade intelligible, qui est aussi la « première triade », est l'*unité* de tous les intelligibles (ἕνωσις πάντων τῶν νοητῶν, *Theol. Plat.* III 18.59.8) et elle assure la fonction de « limitant » (πέρας) : « Timée a eu raison d'appeler *un* la première triade qui est essentiellement caractérisée par le limitant (κατὰ τὸ πέρας), en la dénommant d'un nom qu'il tire de l'idée de limitant (ἀπὸ τοῦ πέρατος) » (*Theol. Plat.* III 18.58.23–59.3). Vient ensuite l'Éternité, qui est la deuxième triade (ἡ δευτέρα τριάς, *Theol. Plat.* III 18.59.16), qui correspond, selon la triadologie proclienne, aux valeurs de vie, de procession, de puissance, d'illimitation, et qui est donc pour cela dyadique (le nombre *deux* étant du côté de l'illimité) puisque αἰών est la concrétion de ἀεί et ὄν (comme l'enseigne Aristote), et que l'αἰών est « *puissance* intelligible » (δύναμις νοητή), à la fois « limitant » et « illimité », πέρας et ἄπειρον : « Timée a eu raison d'appeler la triade intermédiaire *éternité*, nom qui exprime la dualité (δυαδικῶς) par une combinaison de mots (τὰ ὀνόματα συμπλέκων), parce que cette triade est déterminée par la puissance intelligible (κατὰ τὴν δύναμιν [...] τὴν νοητήν ἀφώρισται) [*Theol. Plat.* III 18.59.3–5]. Proclus précise : « Quant à la deuxième triade, elle est la mesure directe (μέτρον προσεχές) de tous les êtres, et elle est coordonnée (συντεταγμένον) avec ce qu'elle mesure. En elle, il existe à la fois du limitant (πέρας) et de l'illimité (ἄπειρον) ; en tant qu'elle mesure les intelligibles, du limitant, mais en tant qu'elle est cause de la sempiternité (ἀιδιότης), c'est-à-dire du 'toujours' (ἀεί), de l'illimité » (*Theol. Plat.* III 18.59.16–20). La troisième triade, quant à elle (ἡ τρίτη τριάς : *Theol. Plat.* III 18.59.5 ; 61.1 et 9), est le tout premier Vivant, le Modèle intelligible qui est *tétradique* (*Theol. Plat.* III 18.58.19 τετραδικῶς προεληλυθός et *Theol. Plat.* III 19.64.14–67.19)[4] car il contient τὰ πρωτουργὰ καὶ νοητὰ παραδείγματα, qui sont au nombre de quatre : il constitue donc une tétrade complète (παντελής *Theol. Plat.* III 19.64.19), et cette *tétrade* procède de la *dyade* de l'Éternité. Participant de l'αἰών, qui est Vie, le Modèle est le Vivant-en-soi : « (…) la troisième triade est remplie de vie intelligible (πεπλήρωται μὲν ζωῆς νοητῆς), et c'est pour cette raison qu'elle est un vivant intelligible (ζῷον νοητόν) et le tout premier vivant (πρώτιστον ζῷον) » (*Theol. Plat.* III 18.61.1–3). Ce troisième terme étant le paradigme au sens propre, c'est-à-dire *stricto sensu* l'Intelligible, donne son nom à l'ensemble de la triade intelligible : « (…) quant à la troisième [triade], il a eu raison de l'appeler *Vivant-en-soi* (αὐτοζῷον), transférant ainsi à la triade tout entière le nom du terme inférieur » (*Theol. Plat.* III 18.58.23–59.7). La triade intelligible est constituée de trois triades, en même temps que l'on observe une séquence 1-2-4 (un, éternité [dyade], Vivant-en-soi [tétrade]). La liaison de la dyade et de la tétrade, principes de « fécondité »,

[4]. Cf. Pl. *Ti.* 39e10–40a2 : les quatre espèces de vivants. – Proclus précise que cette tétrade des Formes se décompose en une monade *plus* une triade (*Theol. Plat.* III 19.65.23). Sur la tétrade, voir l'exposé détaillé dans *In Ti.* III 104.23–116.21 Diehl (= IV 135.10–150.11 Van Riel) ; tr. Festugière 1968, pp. 137–52.

se retrouvera, beaucoup plus bas, dans l'analyse de la dodécade qui structure les plans hypercosmique et hypercosmique-encosmique (voir infra, p. 000).

Une même structure fondamentale triadique s'observe, à un niveau plus important de pluralité, dans l'ordre intelligible et intellectif étudié dans le livre IV de la *Théologie Platonicienne* à partir de l'enseignement du *Phèdre* (IV 4–26) et du *Parménide* (IV 27–39). L'organisation de ce diacosme est fondée (*Theol. Plat.* IV 4.18.1–21) sur la topographie du mythe du *Phèdre* (246e4–248c2)[5] qui distingue entre le Ciel *suprasensible* et sa révolution (οὐρανός), le lieu supracéleste (ὑπερουράνιος τόπος 247c3–e6) et la voûte subcéleste (ὑπουράνιος ἀψίς 247a8–b1), le Ciel (qui devient suprasensible dans l'interprétation de Proclus : *Theol. Plat.* IV 5, spéc. 21.9–22.8) occupant la position médiane. (Il occupe par ailleurs, structurellement, la position médiane dans l'ensemble des trois ordres divins intelligible, intelligible-intellectif et intellectif). Chacun des *trois* plans divins correspond à la triade de l'être, de la vie, de l'intellect (avec prédominance à chaque niveau du principe qui lui confère sa détermination)[6] : il est ainsi lui-même *triadique* (*Theol. Plat.* IV 1.9.10–10.6 ; IV 4.17.16 : αἱ τρεῖς νοηταὶ καὶ νοεραὶ τριάδες), et par exemple le lieu supracéleste est triadique (*Theol. Plat.* IV 14 : Science/Sagesse/Justice ; IV 15 : Plaine de la Vérité/Prairie/Nourriture des dieux ; IV 16), tandis que la deuxième triade (le Ciel) est elle aussi triadique (IV 21 : dos du ciel/profondeur du ciel/voûte céleste). Une correspondance est établie entre ces trois niveaux divins et trois entités révélées par les *Oracles Chaldaïques*, car Platon s'accorde avec les *Oracles* (*Theol. Plat.* IV 9) : le lieu supracéleste est le lieu des dieux « rassembleurs » (συναγωγοὶ θεοί) qui correspondent aux Iynges (ἴυγγες) [cf. *Or. Chald.* 76–77 dP : ce sont des formes intelligibles sous le mode propre à l'ordre intelligible-intellectif] ; le Ciel (qui occupe le rang médian) est le lieu des dieux « mainteneurs » (συνεκτικοὶ θεοί) qui correspondent aux συνοχεῖς chaldaïques ; enfin la voûte subcéleste est le niveau des « dieux-chefs de la perfection » (οἱ τῆς τελειότητος ἡγεμόνες ou τελεσιουργοὶ θεοί, *Theol. Plat.* IV 9.27.19–28.1) qui sont les « Télétarques » chaldaïques (τελετάρχαι)[7]. Dans la direction de la remontée anagogique, Proclus pose parallèlement une tripartition des étapes ou phases mystériques – le rituel éleusinien étant réinterprété – : τελετή (introduction au mystère), μύησις (initiation) et ἐποπτεία (révélation ultime, comme dans le discours de Diotime) [*Theol. Plat.* IV 26.77.9–19][8].

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[5]. La longue histoire de l'exégèse du mythe du *Phèdre* dans la tradition platonicienne est étudiée de manière approfondie par Saffrey – Westerink 1981, pp. IX-XLV.

[6]. Voir Saffrey – Westerink 1968, pp. LXV-LXVI (structure des dieux intelligibles–intellectifs).

[7]. Sur le sens de ces appellations dans la théologie chaldaïque, voir par exemple : Kroll 1894, pp. 39–44 (traduction en français par Saffrey 2016, pp. 57–64) ; Lewy 2011, pp. 129–37, 155–6, 345–53, 633 [*τελετάρχης] ; Seng 2016, pp. 76–8, 116, 124. Sur les correspondances entre ces entités chaldaïques et les dieux procliens, voir les tableaux établis par Lewy 2011, Brisson 2000 et Hoffmann 2016, pp. 895–913.

[8]. Tripartition rituelle déjà formulée par Syr./Herm. *In Phdr* 178.9–20 Couvreur (= 186.8–20 Lucarini–Moreschini) [commentant *Phdr.* 250b8 καὶ ἐτελοῦντο]. La séquence néoplatonicienne τελετή, μύησις, ἐποπτεία est critiquée, du point de vue de l'histoire des religions, par Dowden 1980, pp. 416–7 note 12). Le schéma triadique est remplacé par

Après les plans intelligible et intelligible-intellectif, qui s'organisent selon une même structure (deux triades dont chacun des termes est lui-même triadique), une multiplicité croissante apparaît au troisième plan divin, celui des intellectifs (*Theol. Plat.* V 2.9.13–15) :

> Car le domaine intellectif n'est pas un et indivisible, mais il a reçu des processions plus diversifiées que les classes de dieux plus élevées[9].

On observe alors deux processions successives, organisées chacune par le nombre 7, dont on rappellera le prestige polymorphe dans la pensée pythagoricienne (et néoplatonicienne), illustré par un chapitre des *Theologoumena Arithmeticae*[10].

Une première procession aboutit à une hebdomade intellective « primaire », dont le noyau primitif est encore triadique, car analogue à l'organisation des ordres supérieurs, intelligible et intelligible-intellectif. Une triade divine, constituée de Cronos, Rhéa et Zeus, correspond à la triade de l'intelligible, de la vie et de l'intellect, et est analogue aux trois termes de la triade intelligible – les trois « pères » intelligibles : l'un, l'éternité, le Vivant-en-soi – (*Theol. Plat.* V 2.9.16–24) :

> Il y aura donc, ici aussi (κἀνταῦθα), trois pères qui divisent (διελόντες) l'être intellectif dans son entier ; le premier [Cronos] correspond à l'intelligible (κατὰ τὸν νοητόν), le deuxième [Rhéa] à la vie (κατὰ τὴν ζωήν), le troisième [Zeus le Démiurge] à l'intellect (κατὰ τὸν νοῦν). Et attendu qu'ils imitent (μιμούμενοι) les pères intelligibles qui divisent en trois le plan intelligible (οἳ τὸ νοητὸν πλάτος τριχῇ διεῖλον), ils auront les uns par rapport aux autres une différence (διαφοράν) de même sorte que celle des pères intelligibles : et donc le premier procède d'une manière analogue au premier père (ὁ μὲν ἀνάλογον προελθὼν τῷ πρώτῳ πατρί) et est intelligible [Cronos] ; le deuxième d'une manière analogue au deuxième père, et a rattaché à lui-même toute la vie intellective [Rhéa] ; et le troisième [Zeus], d'une manière analogue au troisième père, et referme (συγκλείων) tout le domaine intellectif, comme le troisième père intelligible referme tout le domaine intelligible.

À chacun de ces trois « pères » intellectifs qui sont en rapport d'analogie avec les trois « pères » intelligibles, sont « co-unifiés » trois autres dieux, anonymes mais désignés par l'épithète d'« immaculés » (ἄχραντοι) – ou d'« implacables » (ἀμείλικτοι) dans le vocabulaire chaldaïque. Leur existence et leur liaison, respectivement, avec Cronos, Rhéa et Zeus, sont *nécessaires*. Pour chacun des trois « pères », chacun des « immaculés » est cause de son caractère propre : Cronos *demeure* dans une transcendance et une « pureté *stable* » ; Rhéa, qui représente la vie, réalise une

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Damascius, dans son Commentaire au *Phédon* (I §167 100–101), par un schéma à cinq degrés qui établit un parallèle entre les étapes de l'initiation mystérique et l'échelle néoplatonicienne des vertus. Sur tout cela, voir Hoffmann 2021, pp. 196–8.

[9]. Οὐ γὰρ εἷς ἐστι καὶ ἄτομος ὁ νοερὸς διάκοσμος, ἀλλὰ ποικιλωτέρας ἔλαχε προόδους τῶν ὑψηλοτέρων γενῶν.

[10]. Voir De Falco 1922, 54.10–71.21.

procession qualifiée d'immaculée en raison de sa transcendance maintenue par rapport au monde sensible ; et Zeus est l'intellect démiurgique qui transcende lui aussi le Monde, tout en étant principe de sa production. Chacun des « immaculés », de façon différenciée, est *cause* de chacune de ces propriétés des dieux pères, dont il garantit la transcendance. Il y a certes aussi des « immaculés » dans les dieux qui appartiennent aux ordres supérieurs aux intellectifs (c'est-à-dire dans les dieux intelligibles-intellectifs), mais l'état d'union qui là-bas caractérise les dieux fait que les « immaculés » de là-bas ne se distinguent pas alors des « pères » qui leur correspondent, sinon sous le mode *causal* (κατ᾽ αἰτίαν), mais non selon une distinction *réelle* (qui se dirait, dans le lexique de Proclus, καθ᾽ ὕπαρξιν) (*Theol. Plat.* V 2.10.1–18) :

> Étant donné qu'il y a ces trois pères, et que le premier demeure en lui-même (μένοντος ἐν ἑαυτῷ), le deuxième procède et donne la vie à toutes choses (προϊόντος καὶ τὰ πάντα ζωοποιοῦντος), et le troisième resplendit des actes créateurs de sa démiurgie, *il est nécessaire* (ἀνάγκη), je pense, que leur soient co-unifiés (συνηνῶσθαι) trois autres dieux : le premier sera avec le premier père [Cronos] une cause de pureté stable (μονίμου καθαρότητος), le deuxième, avec le deuxième père [Rhéa], une cause de procession immaculée à travers toutes choses (τῆς ἀχράντου […] διὰ πάντων προόδου), le troisième, avec le troisième père [Zeus], une cause de démiurgie transcendante (τῆς ἐξηρημένης […] δημιουργίας). En effet, dans les dieux supérieurs aux dieux intellectifs, il y a, comme on le sait (cf. *Theol. Plat.* IV 22.67.17–68.2)[11], des divinités immaculées (ἄχραντοι θεότητες), mais seulement sous le mode de la cause (κατ᾽ αἰτίαν) parce que, grâce à l'unité sans distinction et à l'identité qui rassemble les puissances (διὰ τὴν ἕνωσιν τὴν ἀδιάκριτον καὶ τὴν ταὐτότητα τὴν συναγωγὸν τῶν δυνάμεων), ces dieux-là [*scil*. les dieux supérieurs aux dieux intellectifs] n'ont nul besoin de communion (κοινωνίας) avec ces divinités [immaculées] ; en revanche, dans les dieux intellectifs où non seulement la distinction est parfaite (διάκρισις παντελής) pour autant que ce soit possible dans les mondes universels, mais encore où il y a davantage de relation avec les inférieurs et de communion des touts avec <les> êtres plus particuliers, il faut aussi une divinité ou une puissance immaculée (δεῖ δὴ καὶ τῆς ἀχράντου θεότητος ἢ δυνάμεως) qui corresponde à la cause paternelle en étant à la tête de l'identité et de l'existence invariable (ταὐτότητος καὶ ἀκλινοῦς ὑποστάσεως ἐξηγουμένη), tout en se divisant comme les pères (συνδιελοῦσα τοῖς πατράσιν αὐτήν), de sorte que chacun des dieux immaculés forme un couple (συνεζεῦχθαι) avec le père qui lui correspond.

À ces deux triades qui forment des couples (3 x 2 [un dieu « père » + un dieu « immaculé »] soit 6 dieux intellectifs), s'ajoute une monade séparative et triadique, qui est – avec les deux triades déjà mentionnées – cause pour les intellectifs de leur distinction (τῆς διακρίσεως αὐτῶν αἰτία τοῖς νοεροῖς) (*Theol. Plat.* V 2, 10.19–23), on obtient donc 3 x 2 + 1= 7, et si cette monade séparative est dite aussi « triadique », c'est qu'elle est non seulement cause de la division interne du plan intellectif mais aussi de la séparation de celui-ci par rapport aux ordres divins supérieurs comme

[11]. Voir Saffrey – Westerink 1981, pp. 164–5 note 1 *ad Theol. Plat*. IV 22.68.2 : le plan intelligible-intellectif anticipe les triades intellectives (la triade des « pères » et la triade des « immaculés »).

aux plans inférieurs de la réalité : elle a donc *trois* fonctions diacritiques (*Theol. Plat.* V 2.10.23–11.6). Proclus est pris d'un mouvement d'admiration devant cette merveilleuse organisation du premier plan intellectif, qui manifeste un renversement paradoxal puisque la monade séparative est certes triadique, mais davantage monadique, tandis que la cause paternelle et la cause immaculée (les trois « pères » sont alors unifiés en une seule cause paternelle, monadique, et les trois « immaculés » en une seule cause immaculée, elle aussi monadique, en raison de la puissante intégration qui est leur loi) sont des monades qui sont davantage triadiques (car ce sont trois « pères » *plus* trois « immaculés »). D'un côté, donc, les deux causes monadiques sont davantage triadiques, de l'autre et inversement, la cause séparative triadique est davantage monadique. Dans une conclusion partielle stylistiquement très raffinée (*Theol. Plat.* V 2.11.7–23), Proclus déclare notamment (*Theol. Plat.* V 2.11.7–11) :

> la cause de la distinction [est] une monade une et triple (μία […] καὶ τριπλῆ μονάς), tandis que les triades paternelle et immaculée [ont] un caractère monadique (τριὰς δὲ μονοειδής ἥ τε πατρικὴ καὶ ἡ ἄχραντος)[12] ; et, chose plus extraordinaire que tout (τὸ πάντων παραδοξότατον), la cause qui distingue est davantage monadique, tandis que la cause paternelle et la cause immaculée sont davantage triadiques.

Le monde intellectif manifeste ainsi une « communion admirable » (κοινωνία θαυμαστή), il est profondément unifié et intégré, « *imitant* l'unité des êtres intelligibles (τὴν τῶν νοητῶν ἕνωσιν) par le moyen de leur présence mutuelle et de leur fusion (διὰ τῆς ἐν ἀλλήλοις παρουσίας καὶ συγκράσεως) » ; il est le Sphairos de là-bas (selon Empédocle)[13], il est monade (comme *image* de la monade intelligible, τῆς νοητῆς μονάδος εἰκών) et hebdomade parfaite (ἑβδομὰς παντελής). La fin du chapitre (*Theol. Plat.* V 2.14.4–17) développera le thème de la parenté étroite de l'hebdomade et de la monade.

Après avoir envisagé les trois « pères » et les trois « immaculés » comme, respectivement, *une* cause paternelle et *une* cause immaculée, auxquelles s'ajoute la monade séparative – ce qui fait réapparaître une structure triadique ! – Proclus conçoit une seconde procession des intellectifs, qui s'organise comme suit. Après cette « toute première procession des dieux intellectifs », qui en a révélé « l'heptade », une autre structure apparaît. Chaque élément de l'heptade première, c'est-à-dire chacun des termes des triades paternelle et immaculée, ainsi que la monade séparative, peut être considérée comme une « monade » dont procèdent, une par une, sept hebdomades de rang second (*Theol. Plat.* V 2.11.25–27). Chaque monade de l'heptade première est considérée comme

[12]. Car elles sont puissamment intégrées et unifiées.
[13]. Sur l'usage néoplatonicien de l'image présocratique de la Sphère pour désigner l'unité des plans divins transcendants (cf. Parm. DK28B8 [vv. 43–44] ; et Emped. DK31B27 [v. 4], Diels-Kranz), voir la note 2 de Saffrey – Westerink 1987, p. 156, *ad Theol. Plat.* V 2.11.18–19.

« transcendante » (ἐξῃρημένη) et elle fait venir à l'existence – elle produit –, une autre monade qui est le principe d'une nouvelle série hebdomadique à laquelle, comme sa cause et son principe, elle est « coordonnée » (συντεταγμένη). On doit donc distinguer entre ces deux niveaux de monades (transcendantes *vs.* coordonnées), chacune des monades « coordonnées » étant principe d'une hebdomade de second rang dont elle est aussi le terme premier, antérieur à deux niveaux triadiques. On observe un effet de multiplication interne (7 x 7), tout comme dans les deux ordres antérieurs – intelligible et intellectif – on observait au niveau de chaque plan une triade dont chaque terme est triadique.

Chacune des monades de l'heptade de premier rang produit donc une hebdomade avec laquelle elle est couplée, et qui présente le même caractère propre (paternel/cronien, vital, démiurgique, immaculé, séparatif), conformément à la loi des *séries* procliennes. Le modèle structurant l'hebdomade de premier rang (une triade de « pères », une triade d'« immaculés » et une monade séparative) est reproduit *de façon spéculaire et inversée* dans *chacune* des hebdomades de second rang, c'est-à-dire que dans chacune de ces hebdomades on observe, au terme d'une inversion comme en miroir : une monade (qui est en tête de l'hebdomade) suivie de deux triades. Une fois apparue « la toute première procession des dieux intellectifs », organisée en une heptade, surgissent ainsi sept autres hebdomades intellectives (*Theol. Plat.* V 2.11.25–12.12) :

> (…) en second lieu il faut concevoir sept autres hebdomades inférieures (δεύτεραι) à celle-ci, qui font procéder (προάγουσαι) les monades de cette heptade jusqu'aux toutes dernières. Chaque monade [*de l'heptade première*] est à la tête d'une hebdomade intellective avec laquelle elle forme un couple (συζύγου), et elle fait s'étendre cette hebdomade depuis le haut, c'est-à-dire depuis le sommet de l'Olympe, jusqu'aux mondes de dieux derniers et terrestres (μέχρι τῶν τελευταίων καὶ χθονίων διακόσμων). Je veux dire ceci : la première monade paternelle [Cronos] en fait venir à l'existence sept du même genre (ἑπτὰ τοιαύτας[14] ὑφίστησιν), la deuxième [Rhéa] à son tour sept monades qui donnent la vie (ζωοποιούς), et la troisième [Zeus] sept monades démiurgiques ; et chacune des divinités immaculées produit un nombre de monades égal aux pères [*donc chacune des trois en produit 7*][15], et la monade qui distingue, sept monades. De fait, toutes ces causes-là [= *chacune des monades de l'heptade première*] procèdent ensemble les unes avec les autres (συμπρόεισιν [...] ἀλλήλοις) ; et de même que la première triade des pères coexiste avec la triade immaculée et la monade qui distingue, de la même façon aussi les triades [*scil*. paternelles] inférieures reçoivent en lot (ἔλαχον) sept triades immaculées qui leur correspondent (συστοίχους) et sept monades qui distinguent[16].

[14]. Τοιαύτας : les sept monades constituant cette hebdomade seconde ont en commun le caractère propre (ἰδιότης) paternel qui est celui de Cronos, et pareillement chaque monade de rang 1 communique son ἰδιότης (vitale, démiurgique, immaculée […]) à chacune des monades secondaires qu'elle régit.

[15]. Comprendre : chaque divinité immaculée produit un nombre de monades égal *à celui que produisent les pères*, c'est-à-dire à chaque fois sept monades constituant une hebdomade de rang 2.

[16]. Proclus explique ici que les hebdomades de rang 2 reproduisent en miroir la structure de l'heptade. Sur la fin de ce texte, voir la note 1 de Saffrey – Westerink 1987, pp. 156–7 *ad Theol. Plat*. V 2.12.10–12 : ces hebdomades de second rang pourraient correspondre à la descente au degré de l'âme. Mais Proclus précise que l'hebdomade intellective de

Cette complexe organisation hebdomadique de l'ordre intellectif est soigneusement décrite dans une dépendance mimétique par rapport aux ordres divins antérieurs : par ce moyen, Proclus assure la cohérence et la continuité de la Procession, tout ordre de réalité reproduisant solidement l'organisation des ordres supérieurs, avec un accroissement de la pluralité. L'hebdomade de premier rang est comme une *monade* par rapport aux hebdomades inférieures, elle est nommée σφαῖρα νοερά (cf. *Theol. Plat.* V 2.11.18–19) et « elle est venue à l'existence sur le modèle du plan intelligible » (κατὰ τὸ νοητὸν ὑπέστη πλάτος, *Theol. Plat.* V 2.12.17–18). En effet, « elle imite (μιμουμένη) le caractère paternel de l'intelligible [*scil. la première triade intelligible, l'un*] par le moyen de la triade paternelle, le caractère éternel de la puissance [τὸ τῆς δυνάμεως αἰώνιον, *soit la deuxième triade intelligible, l'*αἰών] par le moyen de l'identité immaculée (διὰ τῆς ἀχράντου ταὐτότητος) [*scil. la triade des* ἄχραντοι], et la multiplicité apparaissant à l'extrémité inférieure [*de l'intelligible = le Vivant-en-soi*] par le moyen de la monade qui divise le tout (διὰ τῆς τῶν ὅλων διαιρετικῆς μονάδος) » (*Theol. Plat.* V 2.12.14–21). Autrement dit : l'hebdomade intellective est ici décrite *triadiquement* en tant qu'elle imite la structure *triadique* du plan intelligible, la triade paternelle et la triade conjointe des immaculés étant considérées comme deux unités monadiques. En tant qu'elle imite le plan intelligible, l'hebdomade intellective tout entière est comme résorbée en une triade.

Alors que l'heptade première imite l'ordre de l'intelligible, les hebdomades de second rang s'organisent sur le modèle de l'ordre intelligible-intellectif (*Theol. Plat.* V 2.12.21–23, κατὰ τὰ νοητὰ καὶ νοερὰ γένη προεληλύθασιν) : on observe donc dans chaque procession hebdomadique de second rang tout d'abord une monade suspendue à une monade productrice (*transcendante*) appartenant à l'heptade première : « chaque monade [de l'heptade première = monade *transcendante*] fait venir à l'existence (ὑφίστησι) au sommet de ces classes [qui procèdent] (κατὰ τὰς ἀκρότητας ἐκείνων) une monade *coordonnée* (μονάδα συντεταγμένην) à la multiplicité qui procède à partir d'elle-même » et elle est analogue (de façon lointaine) à l'Un, puisque toute ἀκρότης est apparentée à l'Un (ἑνοειδής :[17] *Theol. Plat.* V 2.12.23–26 ; cf. *Theol. Plat.* III 4.14.11–15). Cette monade principielle située au sommet de l'hebdomade produit une multiplicité qui lui est apparentée (et à laquelle elle communique l'ἰδιότης d'un dieu de l'heptade première) et elle engendre (ἀπογεννᾷ) ainsi deux triades sur le modèle de l'organisation caractéristique des ordres intermédiaire et troisième (κατὰ [...] τὰς μέσας καὶ τρίτας προόδους), c'est-à-dire de l'ordre intelligible-intellectif et de l'ordre intellectif (*Theol. Plat.* V 2.12.26–13.3). Donc chaque

rang 2 s'étend « depuis le sommet de l'Olympe jusqu'aux mondes de dieux derniers et terrestres (μέχρι τῶν τελευταίων καὶ χθονίων διακόσμων) » (*Theol. Plat.* V 2.12.2–3), ce qui désigne les dieux encosmiques.

[17]. Ce type de composé en -ειδής signifie une *participation* (ici : ἑνοειδής = apparenté à l'unité). Le mode d'unité des intelligibles, plus radical, est exprimé par l'adjectif ἑνιαῖον, « unitaire » (*Theol. Plat.* V 2.13.4).

hebdomade de rang second se laisse analyser en : une monade + deux triades (1 + 2 x 3), ce qui constitue un ordre inversé par rapport à celui de l'hebdomade première, où la monade vient en dernier, *après* les triades : il y a comme un renversement spéculaire. Cette organisation secondaire *imite* le plan intelligible-intellectif, en même temps que l'organisation triadique (3 x 3) du plan intelligible-intellectif – qui est dans un état déjà dégradé par rapport à la perfection de la triade intelligible – est « invité » à se déployer en hebdomades. Proclus situe ainsi chacun des deux niveaux intellectifs hebdomadiques (premier et second) dans une dépendance par rapport, respectivement, au plan intelligible et au plan intelligible-intellectif, et l'on remarque le caractère imagé du vocabulaire employé :

– Les monades constituant l'heptade primaire « *font passer* le plan intelligible, qui est unitaire, à une multiplicité triadique » ([...] τὸ νοητὸν πλάτος ἑνιαῖον ὑπάρχον εἰς τριαδικὸν πλῆθος προήγαγον), ce qui fait allusion à la structure *triadique* de l'hebdomade (ou heptade) intellective primaire, constituée de *triades* monadiques et d'une monade *triadique*) [*Theol. Plat.* V 2.13.3–5].

– Parallèlement, les monades intellectives (coordonnées ?) « elles aussi *invitent* les triades intelligibles-intellectives à se transformer en hebdomades intellectives » (καὶ αἱ νοεραὶ μονάδες τὰς νοητὰς καὶ νοερὰς τριάδας εἰς ἑβδομάδας *προκαλοῦνται* νοεράς), c'est-à-dire que les intelligibles-intellectifs servent de modèles structurels aux hebdomades intellectives *de rang second* [*Theol. Plat.* V 2.13.5–7 ; cf. 12.21–23].

Dans ce qui suit (*Theol. Plat.* V 2.13.7) le texte de Proclus présente une lacune embarrassante, signalée par Saffrey et Westerink et par Michele Abbate, dont l'étendue est impossible à mesurer (une perte de texte par saut du même au même, la lacune étant précédée de νοεράς ?) mais qui peut être au moins en partie restaurée à partir du contexte, lequel nous a appris que ce qui produit les hebdomades de seconde procession, et notamment les monades-ἀκρότητες coordonnées, ce sont les monades productrices transcendantes appartenant à l'hebdomade première (cf. *Theol. Plat.* V 2.12.23-25 : ἑκάστη γὰρ μονὰς κατὰ τὰς ἀκρότητας ἐκείνων ὑφίστησι μονάδα συντεταγμένην τῷ ἀφ᾽ ἑαυτῆς προϊόντι πλήθει). Le problème de texte posé par la lacune que détectent les éditeurs se résout partiellement en considérant que le sujet au singulier de ὑπέστησε doit logiquement être l'heptade première constituée des sept monades intellectives transcendantes qui produisent les monades « coordonnées », et l'on peut restituer *ad sensum* : <ἡ γὰρ πρωτίστη ἑβδομὰς ou νοερὰ ἑπτὰς ?> τὰς μὲν συντεταγμένας ταῖς ἑβδομάσι μονάδας ὑπέστησε κατὰ τὰς ἀκρότητας τῶν τριάδων, τὰς δὲ διττὰς τριάδας κατὰ τὰς δευτέρας ἐκείνων καὶ τρίτας ὑποβάσεις, « <la toute première hebdomade ou l'heptade intellective> fait venir à l'existence les monades coordonnées aux hebdomades [secondes] au sommet des triades [de ces hebdomades] et les deux triades dans leurs abaissements deuxième et troisième », ce qui décrit précisément l'ensemble

[monade^2-ἀκρότης + les deux triades^2], c'est-à-dire les éléments constitutifs des hebdomades de seconde procession.

La structure de l'hebdomade (de rang second) reflète en elle-même, dans le même ordre hiérarchique dégressif, l'ensemble des trois ordres (intelligible, intelligible-intellectif et intellectif) (*Theol. Plat.* V 2.13.10–15) :

> C'est la raison pour laquelle toute hebdomade [intellective secondaire] a comme première monade [la monade-ἀκρότης] une monade intelligible, et comme deuxième classe triadique à la suite de cette monade, une classe intelligible-intellective, et enfin comme troisième triade à la suite, une triade intellective, et tout cela sous le mode où cela se produit *dans les intellectifs* (καὶ ταῦτα ὡς ἐν νοεροῖς πάντα *scil.* νοερῶς sous le mode intellectif) ; car tout cela est caractérisé par la propriété (ἰδιότητα) de la monade [*intellective*] qui a fait venir à l'existence cette hebdomade [*intellective secondaire*].

La 2^e triade de cette hebdomade est proprement intellective, et se comporte, *mutatis mutandis*, comme le Vivant-en-soi qui, au 3^e rang de la triade intelligible, est l'intelligible au sens propre, dont l'appellation (νοητόν) s'applique à la totalité de la triade *intelligible*.

SCHEMA RECAPITULATIF DES HEBDOMADES INTELLECTIVES

heptade primaire (σφαῖρα νοερά, imite l'ordre *intelligible*) :

- Triade des Pères // triade des Immaculés (3 x 2)
- Puis la monade séparative (+ 1)

- Cette **heptade** = une série de sept **monades (**transcendantes), qui produisent (ὑφίστησι) chacune une à une sept monades « coordonnées » régissant sept hebdomades de rang 2, lesquelles sont la multiplicité procédant de chacune de ces monades « coordonnées » et ayant en commun son ἰδιότης

Sept hebdomades secondaires (**imitent l'ordre *intelligible-intellectif***) sont organisées selon un schéma inversé par rapport à l'heptade primaire.

L'ensemble des plans divins est reflété en chacune :

• Une monade-ἀκρότης (1) : elle est ἑνοειδής <et **intelligible** : *Theol. Plat.*V 2.13.10-11>

(la monade « coordonnée » à la multiplicité qui en procède, selon la loi de la « série »)

• Une triade^1 (3) : de rang analogue à l'**intelligible-intellectif**

• Une triade^2 (3) : proprement **intellective**

Table 1

Proclus s'attache à justifier la dépendance des hebdomades intellectives par rapport aux plans divins supérieurs (intelligible et intelligible-intellectif) dans un texte conclusif (*Theol. Plat.* V 2.13.16–14.17), et rappelle que les puissances intellectives (αἱ νοεραὶ δυνάμεις : c'est-à-dire les triades ou monades intellectives constituant l'heptade première) « ont procédé sur le modèle des classes intelligibles (κατὰ τὰς νοητὰς τάξεις) et ont fait venir à l'existence (ὑπέστησαν) les sept hebdomades [secondaires] que voilà selon les [monades] intellectives premières (κατὰ τὰς πρώτας νοεράς [μονάδας ?]) » (*Theol. Plat.* V 2.13.16–18). Ces monades intellectives premières sont des « causes transcendantes » (ἐξῃρημένας αἰτίας) qui ressemblent aux dieux intelligibles (ὁμοιοῦσθαι τοῖς νοητοῖς θεοῖς), tandis que les causes « coordonnées et procédant complètement » (συντεταγμένας καὶ προϊούσας πανταχῇ), c'est-à-dire les monades dérivées qui président aux hebdomades secondaires, ressemblent aux dieux intelligibles et intellectifs ([ὁμοιοῦσθαι] τοῖς νοητοῖς καὶ νοεροῖς) qui les tout-premiers divisent et organisent les mondes τριαδικῶς (*Theol. Plat.* V 2.13.18–23).

La causalité différenciée des dieux intelligibles, des dieux intelligibles-intellectifs et des dieux intellectifs peut en effet se décrire de la façon suivante, selon une séquence 1-3-7, qui explique en particulier, au plan intellectif, l'entrelacs des schémas triadique et hebdomadique. Les dieux intelligibles « contiennent les causes de l'univers d'une manière uniforme et cachée (μονοειδῶς καὶ κρυφίως) », ils produisent tout μονοειδῶς et les nombres eux-mêmes sont en eux μοναδικῶς ; l'on comprend qu'à leur ressemblance les premières puissances intellectives soient des monades. Les dieux intelligibles-intellectifs produisent tout τριαδικῶς : « ce qui est *monade* dans les intelligibles est *nombre* dans les intelligibles-intellectifs », et en eux « les monades sont distinguées sous le mode numérique ». Les dieux intellectifs quant à eux produisent tout ἑβδομαδικῶς « car ils déroulent (ἀνελίσσουσι) les triades intelligibles-intellectives en hebdomades intellectives », et ils « déploient les puissances concentrées de ces triades [intelligibles-intellectives] en diversité intellective (τὰς συνῃρημένας ἐκείνων δυνάμεις εἰς ποικιλίαν ἐξαπλοῦσι νοεράν) » (*Theol. Plat.* V 2.13.23–14.4). Le verbe ἐξαπλοῦν exprime le fascinant dynamisme de la Procession.

L'accroissement de la multiplicité par la dynamique de l'hebdomade demande à être résorbé par un rappel à la loi de l'unité, qui est un principe néoplatonicien : divisés, les intellectifs sont néanmoins encore dans un certain état d'unité, en raison de l'affinité entre l'heptade et la monade, que confirme l'autorité des Pythagoriciens. Les dieux intellectifs « définissent (διώρισαν) la multiplicité (πλῆθος) et la diversité (ποικιλία) des intellectifs par les nombres les plus proches de la monade (τοῖς ἐγγυτάτω τῆς μονάδος ἀριθμοῖς) ». En effet, même si la série intellective (ὁ νοερὸς ἀριθμός) « est plus déployée (ἀνήπλωται [...] μᾶλλον) que les séries qui sont supérieures à elle, et [même si] elle est divisée en des processions plus variées [que les séries supérieures] (διῄρηται

ποικιλωτέραις προόδοις) », toutefois elle conserve « sa parenté avec la nature de la monade (τὴν πρὸς τὴν μονάδα συγγένειαν) ». En effet, la pluralité hebdomadique « est mesurée par la monade et vient à l'existence en premier à partir d'elle (ὑφίσταται πρώτως ὑπ᾽ αὐτῆς) ». La monade et l'heptade ont une *affinité* confirmée par le fait que les Pythagoriciens « appellent l'heptade la lumière de l'intellect (τὸ κατὰ νόον φῶς)[18] », et admettent sans doute que son ὕπαρξις est intellective et que l'heptade se rattache ainsi à la monade. Un argument ressortissant à la métaphysique néoplatonicienne de la lumière vient couronner ce développement final : la lumière (τὸ φῶς) exprime « le caractère unitaire » (τὸ ἑνιαῖον), qui est originellement celui des hénades divines, et à partir de l'heptade (intellective) il appartient à tous les « nombres divins[19] » (*Theol. Plat*. V 2.14.4–17).

Les argumentations diverses par lesquelles Proclus situe l'ordre intellectif dans une étroite dépendance mimétique par rapport aux ordres divins supérieurs, et notamment par rapport au plan intelligible-intellectif, sont renforcées par la réorganisation proclienne de la triade hésiodique d'Ouranos, Cronos et Zeus, par laquelle Plotin résumait – contre le complexité des systèmes gnostiques de son époque – la simplicité du système des trois niveaux de la Réalité[20], tandis que Proclus, réinterprétant le mythe dans sa théologie, fait d'Ouranos le centre absolu de la hiérarchie théologique, entité médiane de l'ordre intelligible-intellectif qui occupe lui-même une position médiane[21]. La généalogie hésiodique permet d'ancrer fermement l'ordre intellectif dans l'ordre immédiatement supérieur, comme le résume le schéma suivant :

Un-Bien et hénades

Ordre intelligible

Ordre intelligible-intellectif

- Dieux συναγωγοί : Iynges (lieu supracéleste)
- Dieux συνεκτικοί : Mainteneurs/συνοχεῖς — **OURANOS**
- Dieux τελεσιουργοί : Télétarques (voûte subcéleste)

Ordre intellectif

2 triades (monadiques)

CRONOS : intelligible dans l'ordre intellectif (Être) + un ἄχραντος

Rhéa-Hécate : Puissance (Vie) + un ἄχραντος

ZEUS (le Démiurge) : pleinement intellectif (Pensée) + un ἄχραντος

+ une monade séparative (triadique)

Table 2

[18]. Sur cette expression, voir la note 3 de Saffrey – Westerink 1987, pp. 157–8, *ad Theol. Plat.* V 2.14.13.
[19]. Sur la notion de « nombre divin », voir infra, p. 000 et note 34.
[20]. Hadot 1981a, pp. 124–37 ; Hadot 1981b, pp. 205–14 ; Oliveira 2008, pp. 109–33, Oliveira 2013, pp. 199–226 ; Soares Santoprete 2017, pp. 829–58 (avec une bibliographie complète) ; Soares Santoprete 2016, p. 120 et 122–3 ; Jurasz 2016 ; Darras-Worms 2018, pp. 101–3 et 241–56.
[21]. Voir Hoffmann 2017, pp. 757–64.

Après les structures hebdomadiques, apparaît au VI^e^ livre de la *Théologie Platonicienne* une loi d'organisation selon la dodécade [4 x 3] qui laisse apparaître une persistance du noyau triadique.

Après l'ordre intellectif et en dépendance par rapport à lui, Proclus propose en effet une distinction encore triadique entre trois niveaux divins : les dieux hypercosmiques (ὑπερκόσμιοι)[22], ou dieux-chefs « assimilateurs » (ἀφομοιωτικοί) [*Theol. Plat.* VI 1–14] qui procèdent immédiatement du Démiurge intellectif et sont en continuité directe avec le plan intellectif (*Theol. Plat.* VI 1–2), les dieux « séparés du monde » (ἀπόλυτοι) qui sont hypercosmiques-encosmiques (*Theol. Plat.* VI 15–24), et les dieux encosmiques, dont le livre VI de la *Theol. Plat.*, ne traite pas. Cette organisation en trois niveaux répond à la distinction entre les dieux intelligibles, les dieux intelligibles-intellectifs et les dieux intellectifs, étudiés dans les livres III–V de la *Theol. Plat.* : l'introduction des niveaux intermédiaires des dieux intelligibles-intellectifs et des dieux « séparés du monde » (ἀπόλυτοι) correspond à la volonté d'introduire des médiations pour assurer la complétude et la continuité du système processif – ainsi que des correspondances –, et d'organiser les plans divins selon des triades « platoniciennes » du type Être-Vie-Pensée ou Permanence-Procession-Conversion[23]. Le texte de référence fondamental est le cortège des dieux dans le mythe du *Phèdre* (246e4–248c2) dont l'exégèse a une longue histoire dans la tradition platonicienne : ce texte a déjà été mobilisé comme référence de la théologie des intelligibles-intellectifs[24], et Proclus hérite de l'interprétation de Syrianus, qui lui-même avait marché sur les traces de Jamblique[25]. Ces ordres sont construits selon la dodécade, qui est organisée d'une manière telle que les divisions triadiques sont encore structurantes, et que le 12 se décompose en 4 *triades*. Hermias avait décrit en ces termes la perfection de la dodécade, nombre produit par la multiplication du nombre impair 3 qui est « parfait »[26] et du nombre pair 4 qui est « fécond », ces deux nombres ayant respectivement comme « principes » (ἀρχαί) la monade et la dyade (Syr./Herm. *In Phdr.* 137.3–6 Couvreur = 143.2–6 Lucarini – Moreschini et *In Phdr.* 138.11–14 Couvreur [= 144.12–15 Lucarini – Moreschini]) :

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[22]. Sur ce terme et l'origine de la doctrine, voir Saffrey – Westerink 1997, pp. IX–XX.

[23]. Voir Saffrey – Westerink 1981, pp. XXXVI–XXXVII sur les 6 degrés hiérarchiques, et Saffrey – Westerink 1997, pp. XVIII–XX. Voir *Theol. Plat.* VI 2.10.29–11.30.

[24]. Voir supra, 000–000. Le passage qui intéresse la théorie des dieux hypercosmiques et des dieux « séparés » est précisément *Phdr.* 246e4–247a4, que je cite pour mémoire : « Voici donc le grand roi des régions célestes, Zeus, qui, conduisant son char ailé, s'avance le premier, ordonnant et réglant toutes choses. Après lui vient l'armée des dieux et des démons, rangée en onze groupes, car Hestia reste dans la maison des dieux, toute seule. Les autres dieux qui, dans le nombre des douze, sont placés au commandement des groupes, dirigent ceux-ci, chacun à la place qui lui fut assignée » ; tr. Vicaire 1998.

[25]. Saffrey – Westerink 1997, pp. XX–XXVIII, avec traduction du texte de Syr./Herm. *In Phdr.* II 136.17–140.16 Couvreur (= 142.16–146.18 Lucarini – Moreschini).

[26]. Sur la perfection du nombre 3, voir Saffrey – Westerink 1997, pp. 160–1 note 1 *ad Theol. Plat.* VI 67.3–4.

si bien que l'on obtient 3 x 4 = 12, qui est une mesure parfaite et une multitude totale de dieux ; de fait, ce nombre 12 est engendré à partir du nombre 3 qui est parfait, et du nombre 4 qui est fécond, multipliés l'un par l'autre et mélangés[27].

(…) à partir <du> nombre parfait 3 et du nombre générateur 4, a été engendré par mélange le nombre 12, nombre qui embrasse la totalité de l'ordre divin des dieux, et (…) monade et dyade sont principes du 3 et du 4 (…)[28].

Proclus suit fidèlement Hermias dans son explication de la dodécade, lorsqu'il déclare, au cours de l'étude des dieux « séparés du monde », qui sont intermédiaires entre les hypercosmiques et les encosmiques, que la dodécade hypercosmique-encosmique, produit du 4 (issu de la dyade) et de la triade, achève de manière parfaite la procession des dieux hypercosmiques et organise en le « gardant » (φρουρεῖν) l'ordre des dieux célestes, c'est-à-dire encosmiques (*Theol. Plat.* VI 18.86.22–87.5) :

De même en effet que la dyade, chez les dieux, préside à la puissance féconde (τῆς γονίμου δυνάμεως), et la triade, à la perfection toute première (τῆς πρωτίστης τελειότητος), de même aussi la dodécade est le symbole de la procession complète (τῆς παντελοῦς [...] προόδου σύμβολον). En effet, puisque ces dieux-là (*scil*. les dieux ἀπόλυτοι), non seulement enferment la limite inférieure (πέρας) des puissances invisibles et transcendantes au monde (*scil*. les dieux hypercosmiques), mais encore surmontent (ἐπιβεβήκασι) les dieux célestes (*scil*. les dieux encosmiques), pour ces deux raisons, la dodécade leur convient, et en tant qu'ils mènent à son achèvement la procession des dieux hypercosmiques (ὡς τὸ παντελὲς ἐν τῇ προόδῳ τῶν ὑπερκοσμίων συμπεραίνουσι), et en tant qu'ils sont à la tête (προεστηκόσι) des dieux célestes. De fait, ils fournissent à partir d'eux-mêmes à ces dieux célestes la répartition (διανομήν) en douze et ils les gardent dans ce nombre-là d'une manière spéciale (φρουροῦσιν αὐτοὺς ἐν τῷδε τῷ ἀριθμῷ διαφερόντως).

Une autre analyse de la dodécade est donnée parallèlement, par Proclus lui-même, dans sa dissertation sur le mythe d'Er, lorsqu'il énonce en ces termes les propriétés de la dodécade (3 x 4), dont le premier terme (3) est perfecteur et agent de conversion, tandis que le second terme (4), est facteur de stabilité pour les réalités engendrées, ce qui correspond aux caractères propres de leurs principes respectifs rappelés par Hermias, la monade et la dyade (Procl. *In R.* II 120.24–28) :

[27]. (…) ὡς γίνεσθαι τρ[ε]ὶς τέσσαρας δώδεκα, τέλειόν τι μέτρον καὶ πᾶν πλῆθος θεῶν· καὶ γὰρ ὁ ἀριθμὸς οὗτος ὁ δωδέκατος ἐκ τῶν ἀριθμῶν τοῦ τε τρίτου ὄντος τελείου καὶ τοῦ τετάρτου ὄντος γονίμου πολλαπλασιασθέντων καὶ συγκερασθέντων ἀποκυΐσκεται. Cf. Saffrey – Westerink (1997) p. XXIII.

[28]. (…) ὁ δωδέκατος ἀριθμὸς ἐκ <τοῦ> τελείου ἀριθμοῦ τοῦ τρίτου καὶ τοῦ γενεσιουργοῦ τοῦ τετάρτου κατὰ σύγκρασιν ἀπεγεννήθη, ὅλον τὸν τῶν θεῶν περιέχων θεῖον διάκοσμον, τοῦ δὲ τρίτου καὶ τετάρτου ἀρχαὶ μονὰς καὶ δυάς (…). Cf. Saffrey – Westerink 1997, pp. XXIV–XXV. Cf. 138.18–20 Couvreur (= 144.18–20 Lucarini – Moreschini). Le texte se poursuit avec une citation d'Orphée. Voir encore 139.21–25 Couvreur (= 145.24–28 Lucarini – Moreschini) : le nombre 12, symbole de perfection (τελειότητος σύμβολον).

> [Le nombre douze] résulte de la multiplication l'un par l'autre des tout premiers nombres (συνέστηκεν [...] ἐκ τῶν πρωτίστων ἀριθμῶν πολλαπλασιασάντων ἀλλήλους), je veux dire 3 et 4. De ces deux, l'un (3) mène à la perfection (τελεσιουργός) et ramène aux principes (ἐπιστρεπτικὴ πρὸς τὰς ἀρχάς), l'autre (4) est fécondant (γόνιμος) et tout ensemble il donne une assiette aux engendrés et il est enharmonique (ἑδραστικὴ τῶν γεννωμένων ἅμα καὶ ἐναρμόνιος)[29].

Les classes des dieux-chefs sont engendrées à partir des dieux intellectifs mais « sont divisées d'une manière analogue (μεριζόμεναι ἀνάλογον […]) à la totalité des dieux intelligibles et de ceux qui précèdent les intellectifs, appelés à la fois intelligibles et intellectifs », c'est-à-dire par triades, leur existence vient de la « première démiurgie » (ἐκ τῆς μιᾶς δημιουργίας), c'est-à-dire de l'activité du démiurge Zeus, et leur « engendrement unifié » (τὴν ἡνωμένην ἀπογέννησιν) vient de la troisième triade des intelligibles qui est le Vivant-en-soi (*Theol. Plat.* VI 2.9.10–16).

Proclus opère une habile réduction de la structure hebdomadique caractéristique du plan intellectif à une nouvelle organisation, tétradique, selon un schéma 3 +1. Les dieux-chefs se divisent sur le modèle de la triade des dieux « pères » intellectifs et revêtent les trois puissances « paternelle » et « hégémonique », « vivifiante et génératrice », « élévatrice et convertissante » (*Theol. Plat.* VI 5.26.13 [24]–26). À ce noyau triadique qui reproduit les structures transcendantes s'ajoute une quatrième puissance qui rassemble en une seule fonction les fonctions des autres dieux intellectifs, les immaculés (ἄχραντοι) qui sont des gardiens (*Theol. Plat.* VI 5.26.27–27.16)[30]. On obtient donc une tétrade formée sur la base d'une triade : 3 + 1. La dodécade des dieux hypercosmiques est ainsi constituée de *quatre* triades qui sont décrites en *Theol. Plat.* VI 6–13 (avec un long développement sur la doctrine du démiurge : *Theol. Plat.* VI 6–9), et qui assument quatre fonctions : démiurgique, vivificatrice, élévatrice et « gardienne ». Chaque terme de la tétrade se divise triadiquement, mais de façon telle qu'à chaque fois la triade peut être dite monadique – afin de conjurer le danger d'une pluralisation aggravée.

Au premier rang des dieux-chefs se trouvent les trois « pères », qui « [dépendent] tous de la monade démiurgique et [viennent] au second rang après elle » (πάντες τῆς δημιουργικῆς εἰσι μονάδος ἐξηρτημένοι καὶ δεύτεροι μετ᾽ἐκείνην ἡμῖν πεφήνασιν ὄντες) [*Theol. Plat.* VI 10.45.9–13]. « Ils sont produits à partir des pères intellectifs [*scil. la triade Cronos/Rhéa/Zeus*] et divisés selon eux » (προβέβληνται μὲν ἀπὸ τῶν νοερῶν πατέρων καὶ διῄρηνται κατ᾽ ἐκείνους) (*Theol. Plat.* VI

[29]. Tr. Festugière 1967, p. 65. Voir aussi Saffrey – Westerink 1997, pp. 170–1, les notes 1 et 2 *ad Theol. Plat*. VI 18.85.12 et 15.

[30]. Voir Saffrey – Westerink 1997, p. 138 note 1 *ad Theol. Plat.* VI 5.27.7. Proclus suit Syr./Herm. *In Phdr.* 136.29–32 Couvreur (= 142.29–32 Lucarini – Moreschini) qui définit dans les mêmes termes les quatre fonctions divines : « être un dieu qui donne l'être (τὸν μέν τινα τὸ εἶναι παρέχοντα) », « un dieu qui donne la vie (τὸν δὲ τὸ ζῆν) », « être un dieu gardien cause de conservation et de permanence immutable (τὸν δὲ φρουρᾶς καὶ τῆς ἀτρέπτου μονῆς αἴτιον φρουρητικόν τινα θεόν) », et « être un dieu cause de la conversion et du retour des produits vers leurs propres principes (τὸν δὲ τῆς ἐπιστροφῆς καὶ τοῦ τὰ προιόντα ἐπὶ τὰς οἰκείας ἀρχὰς ἀνατείνεσθαι αἴτιον) ».

10.45.15–16), ce qui signifie que chacun de ces dieux hypercosmiques est dans un rapport d'analogie avec, respectivement, Cronos, Héra et Zeus. En même temps, ils sont en rapport avec le plan intelligible-intellectif, et cette triade « par rapport à la série assimilatrice tout entière, [correspond] aux pères intelligibles-intellectifs » (*Theol. Plat.* VI 10.45.18–20 : εἰσὶν ἀνάλογον τοῖς νοητοῖς τε καὶ νοεροῖς πατράσι πρὸς ἅπασαν τὴν ἀφομοιωτικὴν σειράν)[31]. Leurs fonctions hégémoniques sont présentes aussi aux niveaux des dieux séparés du monde et des dieux encosmiques, et s'exercent sur l'univers entier, sur les parties de l'univers, sur les éléments, sur les régions de la terre et même sur les points cardinaux. Le noyau triadique du premier ensemble de dieux-chefs imite la triade des « pères » intellectifs, et réalise la triade universelle de l'être ou de la permanence, de la procession et de la conversion, ce que Proclus exprime ainsi (*Theol. Plat.* VI 10.46.2–8) :

> Quant au lot (λῆξις […] καὶ διανομή) qu'ils ont reçu en partage, c'est, en premier lieu, eu égard, si tu le veux, au Tout dans son entier : pour le premier, produire les êtres (τοῦ μὲν τὰς οὐσίας παράγοντος), pour le deuxième, produire les vies et les êtres engendrés (τοῦ δὲ τὰς ζωὰς καὶ γενέσεις), pour le troisième, administrer les divisions en espèces (τοῦ δὲ τὰς εἰδητικὰς διαιρέσεις ἐπιτροπεύοντος) ; c'est encore, pour le premier, solidement établir dans l'unique démiurge tout ce qui procède à partir de là-bas (τοῦ μὲν ἑδράζοντος ἐν τῷ ἑνὶ δημιουργῷ πάντα τὰ ἐκεῖθεν προϊόντα), pour le deuxième, inciter à la procession (τοῦ δὲ εἰς τὴν πρόοδον ἐκκαλουμένου), pour le troisième, convertir vers lui-même (τοῦ δὲ ἐπιστρέφοντος εἰς αὐτόν).

L'examen des diverses actions de ces dieux-chefs permet de reconnaître en eux les Cronides : Zeus, Poséidon et Pluton (*Theol. Plat.* VI 10.47.20–23), qui constituent ainsi la triade démiurgique. La deuxième triade est vivificatrice (*Theol. Plat.* VI 11.48.3–5 : διακόσμησιν [...] γόνιμον καὶ ζωοποιόν). Elle dépend, dans la triade des « pères » intellectifs, du principe de vie, occupant le rang médian, qui est Rhéa (*Theol. Plat.* VI 11.48.14–19) :

> De même donc que, à partir de la monade paternelle, est venue à l'existence la triade des dieux-chefs qui sont démiurges, de même aussi à partir de la source vivifiante qui, dans ces démiurges [intellectifs] a reçu en lot *le centre intermédiaire* [*Or*. *Chald*. 50 dP], procède le monde vivificateur des dieux assimilateurs ([…] οὕτω δὴ καὶ ἀπὸ τῆς ζωογόνου πηγῆς, τὸ μέσον κέντρον ἐν ἐκείνοις κληρωσαμένης, ὁ ζωογονικὸς διάκοσμος προβέβληται τῶν ἀφομοιωτικῶν θεῶν) ; et là encore on trouve une triade maintenue dans l'être par une unique monade.

De même que la triade paternelle (*i.e*. ici « démiurgique ») est *monadique* car subsistant « selon un unique intellect parfait » (καθ' ἕνα νοῦν ὑφεστήκει τέλειον : Zeus) [*Theol. Plat.* VI

[31]. Voir Saffrey – Westerink 1997, p. 146 note 1 *ad Theol. Plat.* VI 10.45.18–20.

11.48.19–20], de même la triade dispensatrice de vie (ἡ τῆς ζωῆς χορηγὸς τριάς) est *monadique* parce qu'elle est unifiée par sa fécondité généreuse (*Theol. Plat.* VI 11.48.21–49.1) :

> (...) la triade dispensatrice de la vie est aussi monadique : elle est pleine de puissance féconde (πλήρης μὲν γονίμου δυνάμεως), pleine de perfection immaculée (πλήρης δὲ ἀχράντου τελειότητος), elle participe à la vivification totale (μετέχουσα μὲν τῆς ὅλης ζωογονίας) et, *grâce aux canaux de la vie* (τοῖς τῆς ζωῆς ὀχετοῖς : *Or*. *Chald*. 65 et 110 dP), elle remplit tous les êtres inférieurs des biens de la génération (τῶν γεννητικῶν ἀγαθῶν), et fait procéder la lumière vivificatrice (τὸ ζωογονικὸν φῶς) pour assurer une participation généreuse aux êtres inférieurs.

Dénommée « corique », cette triade voit converger les enseignements de Platon, des « Théologiens » (Orphée) et des *Oracles Chaldaïques*, et au cours de son développement (*Theol. Plat.* VI 11.48.2–55.27), Proclus dégage la triade divine suivante, où l'on reconnait encore la triade universelle Être-Vie-Pensée (*Theol. Plat.* VI 11.52.19–24) :

> Voici donc les trois monades vivificatrices : Artémis, Perséphone et Athéna notre souveraine ; la première est le sommet (ἀκρότης) de toute la triade et elle convertit vers elle-même la troisième monade, la deuxième est la puissance qui vivifie l'univers (ζωοποιὸς τῶν ὅλων), la troisième est un intellect divin et immaculé (νοῦς θεῖος καὶ ἄχραντος) qui, à la manière d'un dieu-chef (ἡγεμονικῶς), enveloppe dans l'unité les vertus totales.

La troisième triade des dieux-chefs est celle des dieux « élévateurs » (*Theol. Plat.* VI 12.56.2–65.3) et « elle convertit tous les êtres vers leur propre principe » (*Theol. Plat.* VI 12.56.6–8). Platon appelle cette triade « apolloniaque », Apollon et le Soleil étant identifiés, et elle est enveloppée d'un seul nom, comme dans le cas de la triade « corique » (*Theol. Plat.* VI 12.57.21–27) :

> (…) il embrasse la triade tout entière dans ce cas encore au moyen d'un nom unique (τὴν μὲν οὖν ὅλην τριάδα κἀνταῦθα δι' ἑνὸς ὀνόματος περιλαμβάνει), tout comme il l'a fait pour la triade précédente ; et de même que, plus haut, par le nom de Corè il désigne, comme on l'a vu, le genre entier des principes vivificateurs (τὸ σύμπαν ἐδήλου γένος τῶν ζωογονικῶν ἀρχῶν), de même, dans le cas de ces dieux aussi, appelle-t-il Apolloniaque la triade tout entière, tandis que, par la multiplicité des puissances de ce dieu, il *indique* la multiplicité qui est dans cette triade (ταῖς δὲ πολλαῖς τοῦ θεοῦ τούτου δυνάμεσι τὸ ἐν ταύτῃ πλῆθος ἐνδείνκυται).

Le nom unique souligne, une fois encore, le caractère monadique de la triade, tandis que le verbe ἐνδείνκυται signifie le mode d'expression indirect et allusif (ἔνδειξις) utilisé par Platon pour signifier la pluralité interne à la triade.

La tétrade des dieux-chefs est analogue à l'organisation des dieux intellectifs qui comportent, outre les trois « pères » (Cronos, Rhéa et Zeus), des dieux « immaculés » identifiés aux Courètes. Les trois premières triades hypercosmiques (démiurgique, corique et apolloniaque) *imitent* les puissances des trois « pères » intellectifs et leurs fonctions, et il convient que le principe de la pureté transcendante maintenue par les ἄχραντοι soit lui aussi reproduit. Aussi après la triade apolloniaque trouvons-nous une dernière triade, qui est celle des dieux immaculés et gardiens (*Theol. Plat.* VI 13) dont l'existence est *nécessaire* (*Theol. Plat.* VI 13.65.6–16) :

> Ajoutons (…) la doctrine relative aux dieux immaculés considérés dans les principes de leur existence (τὴν περὶ τῶν ἀχράντων θεῶν ἐν ταῖς ἀρχικαῖς ὑποστάσεσι θεωρίαν) (…). En effet, *il est nécessaire* que les dieux-chefs de l'univers, étant venus à l'existence d'une manière analogue aux rois intellectifs (ἀνάγκη τοῖς νοεροῖς βασιλεῦσιν ἀνάλογον ὑφεστηκότας τοὺς ἡγεμόνας τῶν ὅλων) bien que leur procession s'accompagne de division et de fragmentation, de même qu'ils *imitent* les puissances paternelles, génératives et convertissantes des dieux-chefs (καθάπερ τὰς πατρικὰς αὐτῶν καὶ γεννητικὰς καὶ ἐπιστρεπτικὰς μεμίμηνται δυνάμεις), de même aussi, en vertu de leur caractère propre de dieux-chefs (κατὰ τὴν ἡγεμονικὴν ἰδιότητα), [il est nécessaire qu'ils] reçoivent parmi eux les monades immuables (τὰς ἀτρέπτους ἐν αὐτοῖς μονάδας [...] παραδέξασθαι) et mettent, en tête de leurs propres processions, des causes gardiennes inférieures à eux (προστήσασθαι φρουρητικὰς αἰτίας δευτέρας τῶν οἰκείων προόδων).

Aux Courètes qui sont apparus dans la classe des dieux intellectifs, et qui ont entouré et protégé Zeus après qu'il a été mis au monde à partir de Rhéa (*Theol. Plat.* VI 13.66.1–3), correspondent analogiquement les Corybantes, constitués en triade au niveau hypercosmique, et ainsi les dieux-chefs reproduisent mimétiquement l'organisation des dieux-intellectifs. Le nom même des Corybantes signifie la protection qu'ils accordent à Coré et la pureté dont ils sont les gardiens. Ils sont garants de pureté (κορός) et leur fonction est donc strictement analogue à celle des Courètes (les ἄχραντοι) au plan intellectif (*Theol. Plat.* VI 13.66.3–16) :

> C'est donc dans les dieux intellectifs que la toute première classe des Courètes est venue à l'existence, tandis que la classe des Corybantes, d'une manière analogue aux Courètes de là-bas (ἀνάλογον [...] τοῖς ἐκεῖ Κούρησιν), marche devant Corè et la garde de tous côtés, comme le dit la théologie [Orphée], et c'est pourquoi ils ont reçu cette dénomination ; mais si tu veux parler comme Platon en a l'habitude, c'est parce qu'ils président sur la pureté (προΐστανται τῆς καθαρότητος), qu'ils conservent la classe de Corè immaculée, immuable dans les générations et stable dans les processions vers les astres, que, pour cette raison, ils ont été appelés Corybantes. En effet, le mot *coros* (κορός) partout indique la pureté, comme Socrate le dit dans le *Cratyle* [396b6-7]; et d'ailleurs notre souveraine Corè elle-même ne doit, semble-t-il, son nom qu'à sa pureté et à sa vie immaculée (ἐκ τῆς καθαρότητος καὶ τῆς ἀχράντου ζωῆς).

Parmi diverses autres justifications, Proclus remarque que le nombre trois convient particulièrement aux Corybantes, en raison de sa perfection, et leur fonction de *garde protectrice* (φρουρεῖν, διαφυλάττειν) est bien exprimée par le *trois* (*Theol. Plat.* VI 13.67.2–8) :

> Ce nombre-là, le nombre trois, convient à ces puissances gardiennes (ὁ ἀριθμὸς οὗτος, ἡ τριάς, προσήκει ταῖς φρουρητικαῖς ταύταις δυνάμεσιν) en tant qu'il est parfait (τέλειος) et qu'il embrasse d'une manière uniforme (μονοειδῶς) commencement, milieu et fin des êtres inférieurs ; car tout ce qui garde de tous côtés cherche à embrasser ce qui est gardé et à conserver dans l'immutabilité l'être de ce qui est ainsi gardé, sa puissance et son opération (πᾶν γὰρ τὸ φρουροῦν πανταχόθεν σπεύδει περιλαμβάνειν τὸ φρουρούμενον καὶ τάς τε οὐσίας αὐτῶν καὶ τὰς δυνάμεις καὶ τὰς ἐνεργείας ἀκλινεῖς διαφυλάττειν).

Les dieux hypercosmiques sont ainsi formés de quatre triades de dieux, et leur organisation imite à un niveau inférieur celle des dieux intellectifs, dont l'organisation hebdomadique se trouve résorbée en tétrade.

La dodécade des dieux hypercosmiques-encosmiques (*Theol. Plat.* VI 15–24) reproduit à son niveau propre cette ordonnance. Cette classe est célébrée comme « séparée du monde » (ἀπόλυτον τὸ γένος τοῦτο τῶν θεῶν ἀνυμνεῖν εἰώθαμεν), « supracéleste » (ὑπερουράνιον), « immaculée » (ἄχραντον), « élévatrice » (ἀναγωγόν), « parfaite » (τέλειον) (*Theol. Plat.* VI 15.74.21–75.2), et Proclus précise qu'elle résulte de deux causes : les processions (paternelles, fécondes, élévatrices et gardiennes) des dieux assimilateurs (les dieux-chefs hypercosmiques) et « la monade démiurgique [qui] divise tout ce qui procède en premiers, intermédiaires et derniers » (*Theol. Plat.* VI 20.93.8–13). Les classes de dieux séparés du monde qui, « pour l'esprit de l'homme, constituent une multiplicité insaisissable et innombrable (ἀπερίληπτον [...] πλῆθος καὶ ταῖς ἀνθρωπίναις ἐπιβολαῖς ἀναρίθμητον) » sont déterminées dans le *Phèdre* « selon la mesure de la dodécade » (κατὰ τὸ τῆς δωδεκάδος μέτρον) (*Theol. Plat.* VI 18.85.6–9). Le nombre douze en effet « convient aux dieux séparés du monde, en tant qu'il est complet, formé à partir des nombres tout premiers et constitué de nombres parfaits (ὡς παντελῆ καὶ ἐκ τῶν πρωτίστων ἀριθμῶν ἀποτελεσθέντα καὶ ἐκ τῶν τελείων συμπληρούμενον), et [Platon] a embrassé toutes leurs processions au moyen de cette mesure (ἐν τούτῳ τῷ μέτρῳ) », la dodécade étant un principe de détermination (ἀφορίζει) de toutes ces classes de dieux et de leurs ἰδιότητες (*Theol. Plat.* VI 18.85.12–18).

Les dieux séparés du monde (ἀπόλυτοι) ont procédé à partir des dieux assimilateurs (ἀφομοιωτικοί) – qui eux-mêmes procèdent des dieux intellectifs, intelligibles-intellectifs et intelligibles (*Theol. Plat.* VI 20.92.22–25) – et les processions des dieux assimilateurs se divisent en quatre (elles sont paternelles, fécondes, élévatrices et gardiennes), tandis que la cause démiurgique – la triade des « pères » intellectifs –, qui est au-dessus des dieux assimilateurs, est à l'origine de la

division triadique. Pour cette raison, la procession des dieux séparés du monde – les dieux du *Phèdre* – s'organise selon le nombre douze, fruit de la multiplication 4 x 3, et la division triadique dérive ultimement de l'intelligible (*Theol. Plat.* VI 20.92.17–93.25, spéc. 93.8–25) :

> Or si nous nous souvenons de ce qui a été dit plus haut (*Theol. Plat.* VI 5.26.13–27.16), nous avons divisé en quatre (τετραχῇ […] διῃρήμεθα) les processions intermédiaires des dieux assimilateurs ; et nous avons dit que, parmi ces processions, les unes sont paternelles (πατρικάς), les autres, fécondes (γονίμους), les autres, élévatrices (ἀναγωγούς), les autres enfin, gardiennes (φρουρητικάς). Et puisque la monade démiurgique divise tout ce qui procède en premiers, intermédiaires et derniers, tout comme aussi le père intelligible qui lui est supérieur, tandis que les dieux qui la suivent (*scil*. les dieux hypercosmiques) font procéder d'une manière tétradique sur les êtres inférieurs leurs propres *canaux* (ὀχετούς) [cf. *Theol. Plat.* V 12.41.1–18] voilà que nous apparaît cette dodécade des dieux séparés du monde (ἡ δυωδεκὰς [...] τῶν ἀπολύτων [...] θεῶν), laquelle en haut procède selon la triade (ἄνωθεν μὲν κατὰ τὴν τριάδα προϊοῦσα), et en bas se multiplie tétradiquement (κάτωθεν δὲ τετραδικῶς πολλαπλασιαζομένη). C'est pourquoi, parmi les chefs qui constituent cette dodécade, les uns ont reçu triadiquement (τριαδικῶς) le caractère démiurgique et paternel (τὸ δημιουργικὸν καὶ πατρικόν), les autres triadiquement le caractère génératif et vivifiant (τὸ γεννητικὸν καὶ ζωογονικόν), les autres triadiquement le caractère élévateur (τὸ ἀναγωγόν), les autres enfin, triadiquement, le caractère immaculé et gardien (τὸ ἄχραντον καὶ φρουρητικόν). En effet toutes ces propriétés caractéristiques (ἰδιότητες) leur viennent de la multiplicité des dieux assimilateurs, tandis que la division en premiers, intermédiaires et derniers vient de la cause démiurgique [*i.e. Zeus, qui caractérise proprement la triade intellective*].

Un chapitre entier est alors consacré à l'étude des propriétés de la dodécade (*Theol. Plat.* VI 21.93.27–96.29). Proclus prolonge alors une description déjà proposée en *Theol. Plat.* VI 18.85.19–28, qui consiste, à partir du texte de Platon, *Phdr.* 246e4–247a4, à diviser la dodécade des dieux en deux monades principales (Zeus et Hestia, qui « demeure seule dans la maison des dieux ») *plus* dix autres monades correspondant à des « commandements plus particuliers » (ἄλλων ἡγεμονιῶν μερικωτέρων), comme le commandement d'Apollon qui régit « le mode de vie divinatoire » (τὸ μαντικὸν τῆς ζωῆς εἶδος), le commandement d'Aphrodite qui régit « le mode de vie amoureux » (τὸ ἐρωτικόν), ou encore le commandement d'Arès qui régit « le mode de vie qui divise » (τὸ διαιρετικόν)[32]. Le nombre douze n'est d'ailleurs pas un nombre numérique, quantitatif (ce type de nombre n'existe pas chez les dieux), mais « il consiste en une propriété de l'existence » (ἐν ἰδιότητι τῆς ὑπάρξεως) (*Theol. Plat.* VI 18.86.20–22)[33]. Proclus ajoute des précisions dans le second exposé. La dodécade est à nouveau structurée en 2 +10 parce que, parmi les dieux séparés du monde Zeus et

[32]. Le texte du *Phèdre* ne mentionne pas le nom des dieux qui composent le cortège de Zeus, mais ce sont les dieux traditionnellement gouvernés par Zeus dans la religion grecque.
[33]. Il s'agit d'un nombre intelligible et divin, et Proclus fait allusion à une doctrine d'origine plotinienne. Voir Saffrey – Westerink 1997, p. 172 note 4 *ad Theol. Plat.* VI 18.86.22.

Hestia se distinguent des dix autres par une fonction rectrice, « un rang plus hégémonique » (ἡγεμονικωτέραν τάξιν, *Theol. Plat.* VI 21.94.4), avec des particularités propres : Zeus, « étant cause de mouvement pour tous [les dieux], est le chef de leur voyage vers l'intelligible (τῆς εἰς τὸ νοητὸν πορείας [...] ἡγεμών) », conformément au récit du mythe platonicien, « (…) tandis que Hestia fait briller sur tous la puissance stable et inflexible (τὴν μόνιμον καὶ ἀκλινῆ δύναμιν) » [*Theol. Plat.* VI 21.94.21–24)] On remarque que Hestia appartient à la série « immaculée » (τῆς ἀχράντου σειρᾶς) et Zeus à la série « paternelle » (τῆς πατρικῆς), ce qui les rattache aux deux triades, « immaculée » et « paternelle », de l'ordre intellectif. Hestia et Zeus sont causes, l'une de toute stabilité, l'autre, de tout mouvement vers les êtres supérieurs : ainsi, « le caractère de stabilité, d'immutabilité et d'uniformité (τὸ μόνιμον καὶ ἄτρεπτον καὶ ἀεὶ ὡσαύτως ἔχον) vient à tous les dieux encosmiques de l'Hestia supracéleste » (*Theol. Plat.* VI 21.95.15–17), tandis que « tous les mouvements, aussi bien les opérations séparées (τὰς ἐνεργείας τὰς χωριστάς) que <les> conversions des inférieurs vers les supérieurs (<τὰς> τῶν δευτέρων ἐπὶ τὰ πρώτα στροφάς), adviennent à tous les êtres à partir de Zeus » (*Theol. Plat.* VI 21.95.22–25). Quant aux dix autres chefs, ils communiquent chacun le caractère (ἰδιότης) qui leur est propre à chacun des termes de la série multiple qu'ils régissent, jusqu'aux tout derniers termes (μέχρι τῶν ἐσχάτων) (*Theol. Plat.* VI 21.96.4–9). Dans un texte fondamental, Proclus énonce ensuite que la dodécade platonicienne révèle l'organisation analogue des dieux hypercosmiques, des dieux « séparés » et des dieux encosmiques (*Theol. Plat.* VI 21.96.10–29), que distingue seulement le fait d'être commandant ou commandé : « (…) le caractère de commandant et de chef (τὸ ἀρχικὸν καὶ τὸ ἡγεμονικόν) convient seulement aux dieux hypercosmiques, le caractère *d'être rangé en bon ordre* (τὸ τετάχθαι) et *le bon ordre* par lui-même (τὸ τεταγμένον) [cf. *Phdr.* 247a1–4], aux dieux encosmiques (car ce sont ces dieux-là qui ont participé au bon ordre et qui ont reçu le bon ordre par participation), tandis que ces deux caractères ensemble conviennent aux dieux séparés du monde » (*Theol. Plat.* VI 21.96.15–20). Et la structure dodécadique, dont on a vu à propos des dieux hypercosmiques qu'elle reproduit de façon pluralisée l'ordonnance du plan intellectif, ordonne ces trois niveaux, qui dépendent tous de ce diacosme intellectif pour leur organisation.

Après ce développement sur la dodécade décomposée en 2 + 10, l'exposé de Proclus consacré aux dieux « séparés du monde » redevient conforme au schéma tétradique : ces dieux s'organisent en une tétrade dont chaque terme est triadique, soit quatre triades.

Les deux dodécades hypercosmique et « séparée » sont composées chacune de quatre triades. Nous découvrons en premier lieu, au niveau des dieux « séparés », une triade démiurgique composée de Zeus, Poséidon et Héphaïstos, qui reproduisent chacun à son niveau propre les trois fonctions de la triade paternelle intellective, et constituent la première triade de ces dieux

hypercosmiques-encosmiques : dans cette triade démiurgique Zeus « a reçu le rang le plus élevé (τὴν ὑψηλοτάτην τάξιν) parce que d'en haut à partir du degré intellectif (ἄνωθεν ἀπὸ νοῦ) il dirige les âmes et les corps (ψυχὰς καὶ σώματα κατευθύνων) et *prend soin de toutes choses* (πάντων ἐπιμελούμενος) comme le dit Socrate [cf. *Phdr.* 246b6] » ; Poséidon a en charge le monde psychique et « il est le dieu cause du mouvement et de toute génération » (κινήσεως […] αἴτιος καὶ γενέσεως πάσης) ; et Héphaïstos agit comme un démiurge en direction du plan inférieur et encosmique puisque « il insuffle la nature dans les corps et *fabrique* tous les sièges encosmiques des dieux (τὴν φύσιν ἐμπνεῖ τῶν σωμάτων καὶ πάσας τὰς ἐγκοσμίους ἕδρας τῶν θεῶν δημιουργεῖ) » (*Theol. Plat.* VI 22.97.7–17).

L'ordre des triades est modifié par rapport à celui des triades hypercosmiques parce que en deuxième lieu vient immédiatement la triade « gardienne et immuable » (φρουρητικὴ καὶ ἄτρεπτος) composée de Hestia, Athéna et Arès (*Theol. Plat.* VI 22.97.18–98.2), qui est donc rapprochée de la triade démiurgique, de même que l'analyse de la dodécade en 2 + 10 avait mis en évidence un couple constitué de Zeus (qui appartient à la série « paternelle ») et de Hestia (qui appartient à la série « immaculée ») [*Theol. Plat.* VI 18.85.19–23 et 21.94.4–96.4]. Dans la triade « vivifiante » (ζωογονική), composée de Démétèr, Héra et Artémis (*Theol. Plat.* VI 22.98.3–13), Proclus rappelle les fonctions de l'Héra intellective, qui occupe elle aussi une position médiane, dans la triade intellective des « pères » : « Héra constitue le rang intermédiaire (τὴν μεσότητα συνέχει), parce qu'elle fait procéder l'engendrement de l'âme (τὴν τῆς ψυχῆς ἀπογέννησιν προϊεμένη) : de fait, c'est la déesse intellective qui fait jaillir à partir d'elle-même, on le sait, toutes les processions des classes d'âmes (καὶ γὰρ ἡ νοερὰ τῶν [ἄλλων] ψυχικῶν γενῶν ἀφ' ἑαυτῆς προὐβάλλετο πάσας τὰς προόδους) », (*Theol. Plat.* VI 22.98.5–8) semblable en cela à l'Hécate chaldaïque.

Cette organisation des quatre triades de dieux « séparés du monde » semble, en filigrane, être en un certain sens triadique, et se conformer à la triadologie néoplatonicienne habituelle (être, vie pensée ~ essence, puissance, activité ~ permanence, procession, conversion) parce qu'elle conjoint habilement les deux triades démiurgique et gardienne (on retrouve le couple Zeus-Hestia des chapitres 18 et 21), ménage une place médiane à la triade vivifiante qui préside à toute production, et mentionne en dernier, en un texte superbe, la triade élévatrice (Hermès, Aphrodite, Apollon) qui préside à l'amour et aux choses de l'esprit. On remarque que ces trois dieux convertissent, respectivement et de façon hiérarchisée, vers le Bien, le Beau et le Vrai (τὴν νοερὰν… ἀλήθειαν), ce qui évoque dans un ordre différent la triade platonicienne du beau, du sage et du bon (καλόν, σοφόν, ἀγαθόν, *Phdr.* 246e1) à laquelle répond la triade anagogique ἔρως, ἀλήθεια, πίστις mentionnée ailleurs par Proclus[34] (*Theol. Plat.* VI 22.98.14–24) :

[34]. Hoffmann 2000, pp. 459–89.

> Dans la dernière triade, la triade élévatrice (τῆς ἀναγώγου τριάδος), Hermès est le pourvoyeur de la philosophie (φιλοσοφίας […] χορηγός) et, par la philosophie, élève les âmes (ἀνάγει τὰς ψυχὰς), et, par les puissances dialectiques, ramène les âmes, tant universelles que particulières, vers le Bien lui-même (ταῖς διαλεκτικαῖς δυνάμεσιν ἐπ᾽αὐτὸ τὸ ἀγαθὸν ἀναπέμπει τάς τε ὅλας καὶ τὰς μερικάς) ; Aphrodite est la cause primordiale de l'inspiration amoureuse répandue dans tout l'univers (τῆς δι᾽ ὅλων διηκούσης ἐρωτικῆς ἐπιπνοίας[35] ἐστὶν αἰτία πρωτουργός) et familiarise avec le Beau les vies qu'elle élève par elle-même (καὶ πρὸς τὸ καλὸν οἰκειοῖ τὰς ἀναγομένας ὑφ᾽ ἑαυτῆς ζωάς) ; Apollon enfin, par l'art des Muses[36], perfectionne et convertit tous les êtres (τὰ πάντα τελειοῖ καὶ ἐπιστρέφει), *dirigeant à la fois toutes choses* (πάντα ὁμοπολῶν), comme le dit Socrate[37], et, par l'accord et le rythme, les attire vers la vérité intellective et vers la lumière de là-bas (δι᾽ ἁρμονίας καὶ ῥυθμοῦ πρὸς τὴν νοερὰν ἀνέλκων ἀλήθειαν καὶ τὸ ἐκεῖ φῶς).

Le tableau suivant permet de voir à la fois la parenté de structure et les différences d'ordre entre la dodécade hypercosmique et la dodécade hypercosmique-encosmique, qui classe différemment les triades, rapproche au second niveau la triade démiurgique et la triade gardienne, réserve la dernière place à la triade élévatrice qui convertit vers le Bien, le Beau et le Vrai, et semble suggérer une structure de type [1,2]-3-4 qui ressemble fort à une triade typiquement néoplatonicienne du type (1) Être - (2) Procession/Puissance/Vie - (3) Conversion/Pensée :

Tétrade = quatre triades	**Dieux hypercosmiques**	**Dieux ἀπόλυτοι**
Triade démiurgique (masculine)	Zeus, Poséidon, Pluton	1.Zeus, Poséidon, Héphaïstos
Triade vivifiante (féminine)	Artémis, Perséphone, Athéna	3. Déméter, Héra, Artémis
Triade élévatrice	Apollon = Soleil	4. Hermès, Aphrodite, Apollon
Triade gardienne	Corybantes	2. Hestia, Athéna, Arès

Table 3

Ce parcours cavalier de la *Théologie Platonicienne* fait ainsi apparaître une progression arithmétique qui, si l'on part du Premier principe, l'Un-Bien, pour descendre à travers les ordres divins (intelligibles, intelligibles-intellectifs et intellectifs) et atteindre enfin les ordres hypercosmique, hypercosmique-encosmique (et encosmique), constitue une macrostructure numérique : 1–3–7–12. Le tableau ci-contre permet d'avoir une vue d'ensemble des progrès de la pluralité au niveau de la macrostructure de la *Theol. Plat.*[38] Cette progression numérique ne semble

[35]. Cf. Pl. *Phdr.* 265b4–5.
[36]. Sur la théologie d'Apollon et des Muses, voir Hoffmann 2024b, pp. 154–66.
[37]. Pl. *Cra*. 405d4.
[38]. On se reportera aussi à Van den Berg 2001, spéc. p. 40 (fig. 1 : tableau de la hiérarchie des dieux), qui montre que plusieurs de ces dieux sont destinataires des hymnes de Proclus.

pas se retrouver dans les ouvrages anciens d'arithmétique[39], mais l'on observe que cette suite arithmétique progresse de manière croissante : pour passer de 3 à 7 [= 3 x 2 +1] on ajoute 4, puis pour passer de 7 à 12 [= 4 x 3] on ajoute 5. Ces structures numériques suggérées à Proclus par les dialogues de Platon eux-mêmes – nous avons vu l'importance des notations numériques dans le *Phèdre* – expriment clairement les progrès de la διάκρισις au cours de la Procession, et symbolisent l'aggravation de la multiplicité. L'on m'a fait remarquer de façon très suggestive[40] un rapprochement possible avec le début de la « série de Hofstadter »[41]. Quoi qu'il en soit d'une concordance peut-être fortuite avec une donnée de l'arithmétique moderne, à tout le moins pouvons-nous admirer la virtuosité avec laquelle Proclus, en véritable « artiste de la raison » (selon l'expression de Kant dans la *Critique de la Raison Pure*[42]), a su organiser les hiérarchies divines et décrire scientifiquement – au sens de la théologie comme science – la richesse du polythéisme grec qu'il souhaitait illustrer face à la pauvreté du monothéisme[43].

Niveau de réalité	**Structure numérique**	**Nombres**
L'Un-Bien et les hénades	1	**1**
Plans divins transcendants (dieux νοητοί, dieux νοητοὶ καὶ νοεροί, dieux νοεροί)	3 niveaux triadiques	**3** **—> 9**
Triade intelligible	3 x 3 = trois triades (*monadiques*) avec un autre schéma dyade-tétrade : Un / Éternité (dyade) / Vivant-en-soi (tétrade)	**3** **—> 9**
Triade intelligible-intellective	3 x 3 = trois triades Lieu supracéleste / Ciel / Voûte subcéleste	**3** **—> 9**
Hebdomade intellective de rang 1 Peut être décrite aussi comme une série de 7 monades, dont chacune produit une hebdomade	3 + 3 + 1 Triade des « pères » (Cronos, Héra, Zeus) Triade conjointe des « immaculés » + Monade séparative	**7**
7 hebdomades intellectives de rang 2	1 + 3+ 3 Inversion de l'ordre par rapport à l'ordre de l'hebdomade de rang 1	**7** **Nombre total de monades 7 x 7 = 49**

[39]. Je remercie vivement Carole Hofstetter pour cette information et pour sa remarque sur la structure de la progression arithmétique.
[40]. Je remercie vivement Gerd Van Riel pour cette observation faite lors de ma communication.
[41]. Sloane 1973, p. 100 ; Hofstadter 1985, p. 83.
[42]. Voir Tremesaygues-Pacaud 1971, p. 562. La formule est reprise par Hadot 1995, p. 387 ; Davidson – Worms 2010, p. 24.
[43]. Voir Hoffmann 2012, pp. 161–97.

Plans divins inférieurs Trois tétrades : Dieux ὑπερκόσμιοι (4), ἀπόλυτοι (4), ἐγκόσμιοι (4)	3 tétrades x 4 triades : chaque tétrade (hypercosmique, « séparée », encosmique) est composée de 4 triades	**12**
Plans divins inférieurs Trois tétrades : Dieux ὑπερκόσμιοι (4), ἀπόλυτοι (4), ἐγκόσμιοι (4)	la *tétrade* est organisée sur le modèle de l'*hebdomade* (réduite à une tétrade), soit : une triade sur le modèle de la triade des « pères » intellectifs + la puissance « gardienne » —> **Réduction de 7 à 4**	**3 tétrades** **3 x (4=3 +1)** **donc 12**
La dodécade organise les trois ordres hypercosmique, hypercosmique-encosmique, encosmique (ce dernier, non traité dans la *TP*)	Deux analyses possibles du 12 : (1) Zeus/Hestia (2) et les 10 autres dieux (décade) (2) Quatre triades à chaque niveau	Ou bien **2 + 10 = 12** (*Theol. Plat*. VI 21) Ou bien **4 x 3 = 12**
Dodécade hypercosmique Dépend de l'ordre intellectif	4 triades : démiurgique, vivifiante, élévatrice, gardienne	**4 x 3 = 12**
Dodécade hypercosmique-encosmique (dieux « séparés du monde »)	4 triades : démiurgique, gardienne, vivifiante, élévatrice	**4 x 3 = 12**

Table 4

Bibliographie

M. Abbate (ed.), *Proclo. Teologia Platonica*, Milano, Bompiani, 2019.

L. Brisson, *La place des Oracles Chaldaïques dans la* Théologie Platonicienne, in A.-Ph. Segonds – C. Steel (eds.), *Proclus et la Théologie Platonicienne*, Leuven, University Press - Paris, Les Belles Lettres, 2000, pp. 109–62.

A. I. Davidson – F. Worms (eds.), *P. Hadot, l'enseignement des antiques, l'enseignement des modernes*, Paris, Éditions rue d'Ulm, 2010 (2013[2]).

K. Dowden, *Grades in the Eleusinian Mysteries*, in «Revue de l'Histoire des Religions», 197.4 (1980), pp. 409–27.

A.-J. Festugière, *Proclus, Commentaire sur le* Timée, III vol., Paris, Vrin, 1967.

A.-J. Festugière, *Proclus, Commentaire sur le* Timée, IV vol., Paris, Vrin, 1967.

P. Hadot, *Ouranos, Kronos and Zeus in Plotinus' Treatise Against the Gnostics*, in H. J. Blumenthal – R. A. Markus (eds.), *Neoplatonism and Early Christian Thought. Essays in Honour of A. H. Armstrong*, London, Variorum Publications, 1981a, pp. 124–37.

P. Hadot, *Images mythiques et thèmes mystiques dans un passage de Plotin (V, 8, 10–13),* in *Néoplatonisme. Mélanges offerts à Jean Trouillard*, Fontenay-aux-Roses, École Normale Supérieure, 1981b, pp. 205–14.

P.Hadot, *Qu'est-ce que la philosophie antique ?*, Paris, Gallimard, 1995.

Ph. Hoffmann, *La triade chaldaïque* érôs, alètheia, pistis *de Proclus à Simplicius*, in A.-Ph. Segonds – C. Steel (eds.), *Proclus et la Théologie Platonicienne*, Leuven, University Press - Paris, Les Belles Lettres, 2000, pp. 459–89.

Ph. Hoffmann, *Un grief antichrétien chez Proclus : l'ignorance en théologie*, in A. Perrot (ed.), *Les Chrétiens et l'hellénisme. Identités religieuses et culture grecque dans l'Antiquité tardive*, Paris, Éditions Rue d'Ulm, 2012, pp. 161–97.

Ph. Hoffmann, *Le rituel théurgique de l'ensevelissement et le Phèdre de Platon. À propos de Proclus,* Théologie Platonicienne, *IV, 9*, in A. Van den Kerchove – L. G. Soares Santoprete (eds.), *Gnose et Manichéisme. Entre les oasis d'Égypte et la Route de la Soie. Hommage à Jean-Daniel Dubois*, Turnhout, Brepols, 2016, pp. 859–914.

Ph. Hoffmann, *Les âges de l'Humanité et la critique du christianisme selon Damascius*, in «Revue de l'Histoire des Religions», 234.4 (2017), pp. 737–75.

Ph. Hoffmann, *L'herméneutique de Proclus et la constitution d'un système théologique dans la* Théologie Platonicienne, in «Comptes rendus des séances de l'Académie des Inscriptions et Belles-Lettres», 3 (2019), pp. 961–1005.

Ph. Hoffmann, *Le néoplatonisme tardif et les mystères. Quelques jalons*, in F. Massa – N. Belayche (eds.), *Les Philosophes et les mystères dans l'empire romain*, Liège, Presses Universitaires de Liège, 2021, pp. 193–203.

Ph. Hoffmann, *Proclus sur l'existence et la providence des dieux. Démonstrations scientifiques et mode de vie philosophique*, in O. Boulnois – S. Hermann De Franceschi – Ph. Hoffmann (eds.), *Connaître Dieu. Métamorphoses de la théologie comme science dans les religions monothéistes*,Turnhout, Brepols, 2024a, pp. 41–80.

Ph. Hoffmann, *ζάθεοι βίβλοι. Livres et religion savante dans le néoplatonisme tardif (V^e^-VI^e^ siècles)*, in I. Tanaseanu-Döbler (ed.), *Libraries, Handbooks, Encyclopedias. Ancient and Early Medieval Repositories of Knowledge and Their Religious Aspects*, Tübingen, Mohr Siebeck, 2024b, pp. 135–76.

Ph. Hoffmann – M.-A. Gavray, *Modes d'exposition de la théologie dans le néoplatonisme athénien (Proclus, Damascius, Simplicius)*, in O. Boulnois – Ph. Hoffmann – C. Lafleur – J.-M. Narbonne (eds.), *L'Esprit critique dans l'Antiquité*, II. *La Naissance de la théologie comme science*, Paris, Les Belles Lettres, 2024, pp. 219–58.

D. Hofstadter, *Gödel, Escher, Bach. Les Brins d'une guirlande éternelle*, Paris, Dunod (New York, Basic Books, 1979).

I. Jurasz, *L'Intellect-Kronos chez Plotin. La place du mythe dans la noétique plotinienne*, in «Methodos. Savoirs et textes», 16 (2016), URL : https://methodos.revues.org/4401 ; DOI : 10.4000/methodos.4401 (Seen August 15 2025).

A.-L. Darras-Worms, *Plotin. Traité 31 (V, 8) Sur la beauté intelligible*, Paris, Vrin, 2018.

A. Tremesaygues – B. Pacaud (eds.), *Kant,* Critique de la Raison Pure, Paris, PUF 1971[7] (Paris, PUF, 1944).

W. Kroll, *De Oraculis Chaldaicis*, Breslau, Koebner, 1894.

H. Lewy, *Chaldaean Oracles and Theurgy. Mysticism Magic and Platonism in the Later Roman Empire*, Paris, Études Augustiniennes, 2011 (Le Caire, Institut Français d'Archéologie Orientale, 1956[1]; Paris, Études Augustiniennes, 1978[2]).

L. Oliveira, *A genealogia mítica: Urano, Cronos e Zeus em Plotino*, in «Revista de Estudios Filosóficos e Históricos da Antiguidade», 25 (2008), pp. 109–33.

Loraine Oliveira, *Plotino escultor de mitos*, São Paulo, Annablume, 2013.

H. D. Saffrey, *Wilhelm Kroll. Discours sur les oracles chaldaïques*, Paris, Vrin, 2016.

H. D. Saffrey – L. G. Westerink (eds.), *Proclus.* Théologie Platonicienne, 6 vols., Paris, Les Belles Lettres, 1968–1997.

H. D. Saffrey – L. G. Westerink (eds.), *Proclus.* Théologie Platonicienne. *Livre I*, Paris, Les Belles Lettres, 1968.

H. D. Saffrey – L. G. Westerink (eds.), *Proclus.* Théologie Platonicienne. *Livre IV*, Paris, Les Belles Lettres, 1981.

H. D. Saffrey – L. G. Westerink (eds.), *Proclus.* Théologie Platonicienne. *Livre V*, Paris, Les Belles Lettres, 1987.

H. D. Saffrey – L. G. Westerink (eds.), *Proclus.* Théologie Platonicienne. *Livre VI. Index général*, Paris, Les Belles Lettres.

H. Seng, *Un Livre sacré de l'Antiquité tardive : les* Oracles Chaldaïques, Turnhout, Brepols, 2016.

N. J. A. Sloane, *A Handbook of Integer Sequences*, London / New York, Academic Press, 1973.

L. G. Soares Santoprete, *New Perspectives on the Structure of Plotinus' Treatise 32 (V 5) and his Anti-Gnostic Polemic*, in H. Seng – L. G. Soares Santoprete – C. O. Tommasi (eds.), *Formen und Nebenformen des Platonismus in der Spätantike*, Heidelberg, Universitätsverlag Winter, 2016, pp. 109–62.

L. G. Soares Santoprete, *Le mythe d'Ouranos, Kronos et Zeus comme argument antignostique chez Plotin*, in L. G. Soares Santoprete – A. van den Kerchove (eds.), *Gnose et Manichéisme. Entre les oasis d'Égypte et la Route de la Soie. Hommage à Jean-Daniel Dubois*, Turnhout, Brepols, 2017, pp. 829–58.

R. M. Van den Berg, *Proclus' Hymns. Essays, Translations, Commentary*, Leiden / Boston / Köln, Brill, 2001.

P. Vicaire *et al.* (ed.), *Platon,* Phèdre, Paris, Les Belles Lettres 1998.

ABSTRACT

Philippe Hoffmann, *Les structures numériques des ordres divins dans la* Théologie Platonicienne *de Proclus. De la triade à l'hebdomade et à la dodécade*

To examine the relationship between "science" and "theology" in Proclus's thought, this chapter proposes to study the arithmological structures at work in the organisation of scientific theology as set out in a veritable "summa" by the Platonic Theology, which brings together all the dogmas extracted from a comprehensive reading of Plato's dialogues. We thus study the way in which the divine orders are structured, those of the intelligible gods, the intelligible-intellectual gods and the intellectual gods. The first two orders are organised into triads, each term of the triad being itself triadic. The third order, that of the intellective gods, is more complex due to an increase in multiplicity: it is organised according to a hebdomadic structure. The intellective hebdomad is constructed as: two triads augmented by a monad: the triad of the "fathers" (Cronos/Rhea/Zeus), flanked by the anonymous triad of "immaculate" gods plus a "separative" monad, the triadic core of this hebdomad depending on the previous triads to ensure the "mimetic" continuity of the processive system. This order occupies, structurally, the middle position in the set of three divine orders. Below, the order of hypercosmic gods and that of hypercosmic-encosmic gods are organised according to a dodecad. The procession of gods is thus structured, starting from the One-Good, according to an arithmetic series: 1-3-7-12, which resembles the beginning of Hofstadter's series described by modern mathematics.

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III

OTHER SCIENCES AND THE DIVINE

Gerd Van Riel

PROCLUS' POTAMOLOGY: THE ROLE OF RIVERS IN ESCHATOLOGY

Plato's Underworld

In the ancient Greek descriptions of Hades, the underworld was always associated with rivers. Hesiod and Homer are still rather vague about the topology of the underworld, and in general, they seem to know only one river: the Styx, which starts from Oceanus and flows into the cavities of Tartarus [1]. Yet in the tenth book of the *Odyssey*, the number of rivers in the underworld is suddenly extended: in this passage (X 513−515), Hades is located where three rivers meet, one of which is said to be a branch of Styx:

> There into Acheron flow Pyriphlegethon and Cocytus, which is a branch of the water of Styx, and there is a rock, and the junction of the two resounding rivers[2].

So according to this text, there are four rivers in Hades, which stands in contrast with Homer's and Hesiod's account elsewhere. The question has been raised whether this is not a later addition, stemming from a time (probably before the standardization of the Homeric corpus by Pisistratus in the late 6th c. BCE) when the topography of Hades had become more sophisticated[3].

At any rate, by the time of Plato, a standard view of the rivers in Hades had been laid down.

There are four of them: Styx, Acheron, Pyriphlegethon, and Cocytus.

An obvious question then is, how about the river Lethe? In fact, the earliest extant reference to the "river of forgetfulness" is Plato's *Republic*, and even there it is not so clear that we are dealing with a river called Lethe. It is true that before Plato, the poet Simonides of Ceos (6th−5th c. BCE) mentioned the δόμοι Λήθης, the Halls of Lethe, but this seems to be a reference to the goddess of forgetfulness rather than

1. Mackie 1999, p. 486.
2. ἔνθα μὲν ἰς Ἀχέροντα Πυριφλεγέθων τε ῥέουσι / Κώκυτός θ', ὃς δὴ Στυγὸς ὕδατός ἐστιν ἀπορρώξ, / πέτρη τε ξύνεσίς τε δύω ποταμῶν ἐριδούπων. See Mackie 1999, p. 485.
3. Mackie 1999, pp. 486−7.

to a river. The next occurrence is found in *The Frogs* of Aristophanes (dated 405 BCE), where explicit reference is made (v. 186) to τὸ Λήθης πεδίον, the plain of Forgetfulness, located in the underworld. And this is exactly the formula used by Plato in the myth of Er in the tenth book of the *Republic*. In this text, Plato mentions only one river, called Ameles, "the river of Unheeding", situated in the plain of Lethe (Pl. *Resp.* X 621a−b):

> They travelled to the plain of Forgetfulness in burning, choking, terrible heat, for it was empty of trees and earthly vegetation. And there, beside the river of Unheeding, whose water no vessel can hold, they camped, for night was coming on. All of them had to drink a certain measure of this water, but those who weren't saved by reason drank more than that, and as each of them drank, he forgot everything and went to sleep[4].

The souls drank a certain measure of this water, meaning that they might eventually have some memory of the lessons they learnt in the underworld[5], except for the ones whose reason was not strong enough to save them: they forgot everything. When they finally fell asleep, an earthquake opened the way to this world again, and the souls left the underworld. Interestingly, in the conclusion to this myth, Socrates wraps up this narrated event using the term "river of Forgetfulness" (τὸν τῆς Λήθης ποταμόν, *Resp.* X 621c). So it seems to be Plato himself who introduced this fifth river, and he probably did not even mean to call it Lethe – its name is Ameles, and it is only in retrospect that it is described as the river of forgetfulness (without capital letters). At any rate, the plain of Forgetfulness did exist before Plato, but it is interesting to see that through Plato's account, the river Lethe came to join the other four rivers in later tradition.

Of these other four rivers, Plato's *Phaedo* offers not just a description, but they become part of a complete geography of the underworld. In the first place, according to the *Phaedo* narrative (*Phd.* 111e–112a), Tartarus is an immensely wide hole in the earth, similar to all the different cavities that constitute different biospheres like ours, but deeper than all others, and spreading unto the centre of the sphere of the earth. There, in the place called Tartarus, all water that flows under the surface of the earth comes together and is pumped up again.

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4. πορεύεσθαι ἅπαντας εἰς τὸ τῆς Λήθης πεδίον διὰ καύματός τε καὶ πνίγους δεινοῦ· καὶ γὰρ εἶναι αὐτὸ κενὸν δένδρων τε καὶ ὅσα γῆ φύει. σκηνᾶσθαι οὖν σφᾶς ἤδη ἑσπέρας γιγνομένης παρὰ τὸν Ἀμέλητα ποταμόν, οὗ τὸ ὕδωρ ἀγγεῖον οὐδὲν στέγειν. μέτρον μὲν οὖν τι τοῦ ὕδατος πᾶσιν ἀναγκαῖον εἶναι πιεῖν, τοὺς δὲ φρονήσει μὴ σῳζομένους πλέον πίνειν τοῦ μέτρου· τὸν δὲ ἀεὶ πιόντα πάντων ἐπιλανθάνεσθαι; tr. Grube 1992.

5. See Morgan 2000, p. 209.

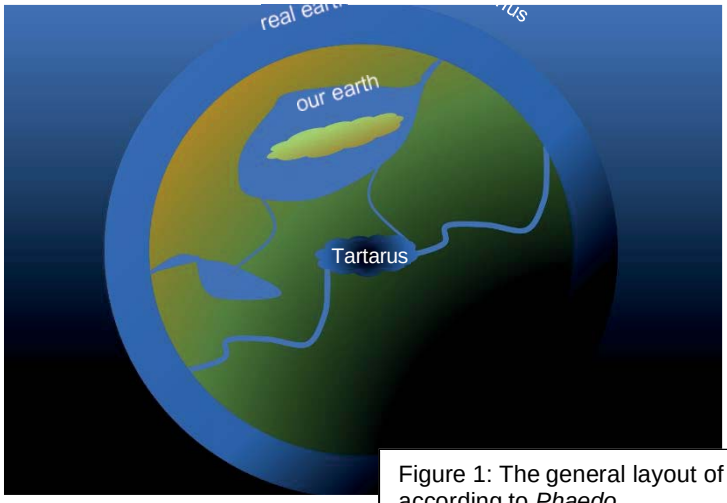

Figure 1: The general layout of the world according to *Phaedo*

Furthermore, in this account of the *Phaedo*, Styx is not referred to as a river, but as a lake formed by the river Cocytus. And Oceanus is considered to be a river, a stream that flows around the earth. According to this description, the Acheron flows away from Oceanus into a cavity of the earth, to debouch into the Acherousian lake. Cocytus and Pyriphlegethon also originate in Oceanus, and they flow through the depths of the earth into Tartarus, each of the two passing along the Acherousian lake, though without mingling with its waters (*Phd.* 112e–113a). Pyriphlegethon dives into the earth through hot and fiery regions, and Cocytus winds through a wild region with a blue-gray color, then forming the Stygian lake, before descending back into Tartarus (*Phd.* 113b−c). At the same time, Pyriphlegethon and Cocytus are said to bring the souls out of Tartarus in a cycle that passes again via the Acherousian lake (*Phd.* 114b−c), which is no doubt due to the pumping movement of the entire water household of the system[6].

6. See Pender 2012.

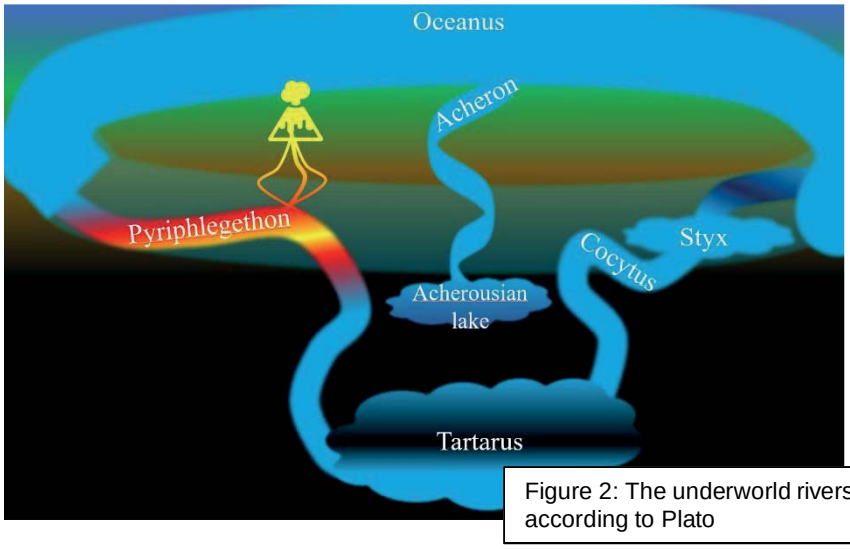

Figure 2: The underworld rivers according to Plato

The Later Neoplatonists' Reading: μυθικῶς and φυσικῶς

In the interpretation of the later Neoplatonists, the topography of the underworld is not just an allegorical feature, but also a real description of a truly existing part of the cosmos. This interpretive starting point is then bound to face Aristotle's criticisms as laid down in the *Meteorologica*, where Aristotle contends that Plato's description cannot be correct, as it implies rivers' flowing upwards, thus challenging the laws of physics[7].

It would have been easy for interpreters of Plato to dismiss Aristotle's critique by stating that this is just imagery, and that Aristotle lacked the phantasy to see that this should be taken metaphorically. And obviously, on the one hand, that is what the Neoplatonists will do: according to them, the myth of the *Phaedo* provides us with a metaphorical description of the place of punishment below the earth, where the souls are gathered in the afterlife. It is to be taken as an exhortation to become godlike, and to avoid punishment in the afterlife[8]. Hence, in many respects, Plato's language needs to be taken μυθικῶς[9].

Yet, at the same time, in the Neoplatonic interpretation, this mythical meaning can only make sense if it relies on a straightforward description of the real nature of this place[10]. Thus, the topography depicted by the *Phaedo* has to be taken φυσικῶς as well

7. See Arist. *Mete.* B 2.355b32−34; 356a14−25.
8. Procl. *In R.* I 168.3−23; II 128.12−23; Dam. *In Phd.* I 471; Olymp. *In Grg.* 241.11−28; *In Mete.* 144.21−35.
9. Olymp. *In Mete.* 144.7−15; see Steel 2012, p. 176.
10. See Procl. *In R.* II 132.5−13; II 93.18−25; II 129.4−130.4 (referring to Numenius); II 132.14−17. See also Proclus' literal reading of the geography of *Phaedo* at *In Ti.* I 270.8−271.18 Van Riel (= I 180.25−181.28 Diehl).

as μυθικῶς, that is to say, the allegorical or metaphorical meaning relies on a description of a physical reality. It should be added, though, that this reference to a "physical reality" does not necessarily mean that the text needs to be taken literally. A physical reading may also be allegorizing. Olympiodorus, for instance, complemented his statement that Tartarus is about morality and punishment, by adding a φυσικῶς explanation that took the description of Tartarus as a reference to matter (ὕλη), with its typical turmoil and chaotic motion[11]. The variation on the scale between strictly literal and entirely allegorical readings is also a matter of applying different readings to different parts of the *Phaedo* myth. The description of the earth, for instance, which is the first part of the myth, would notably be taken literally by late Neoplatonists, as Carlos Steel has shown[12]. Yet for other parts of the narrative, as we shall see in the case of the account of the rivers, the interpretation will be more allegorizing, even though it will remain altogether physical.

The Physics of Psychic Motion

Why was it important to maintain such a physical reading? Wouldn't it be enough to point out that the eschatological myths are conveying a moral lesson in the guise of a fantastic mythical setting, as present-day readers would have it? To Plato's Neoplatonic readers, that would indeed be an insufficient reading, which renders the message less powerful than it should be. For the vicissitudes of the soul's moral life are not just metaphors: they correspond to real movements that require a genuinely spatial environment. If the world of generation is a real world, then the realms above and beyond it should also be localized. And that is exactly what Plato's eschatological myths are doing. To be sure, they are not literal renderings of the realms beside ours, but, as Damascius echoes Plato himself, they describe what it *should* look like, given what we know about the soul[13].

So, in the Neoplatonists' view, the topography of the underworld is subservient to explaining the soul's ongoing transmigration between the levels of reality in a physical way. The first question is, however, why does the soul have descend

11. Olymp. *In Mete.* 146.25−27; Gertz 2011, p. 186.
12. See Steel 2012, pp. 179−83.
13. See Procl. *In R.* II 128.23−132.19; II 140.14−21; II 156.12−157.8. Dam. *In Phd.* I 466.2−4 clearly expresses this: οὐ πᾶν δὲ μῦθός ἐστιν, ἀλλ'ὅσον συμπεραίνεται λέγων ὡς "ταῦτα ἢ τὰ τοιαῦτα" (*Phd.* 114d2−3) χρὴ τὰ ἐν Ἅιδου ἡγεῖσθαι. τοῦτο γὰρ ἦν καὶ τὸ εἶδος τῶν Πλατωνικῶν μύθων ἅτε καλῶς τὴν ἀλήθειαν μιμουμένων, ὡς ἐν Πολιτείᾳ (*Resp.* II 277d9) φησίν.

altogether, and face eventual punishment if things go wrong? Why can't the soul just remain in the intelligible world? Proclus treats the question explicitly in his *Commentary on the* Timaeus, and replies that the soul abandons contemplation for two reasons. First, the soul's life reflects the life of the gods, which is characterized by a twofold activity: contemplation and providence. In so far as they contemplate, the gods are static, but their providential activity is in motion, directed towards the lower realms. So if the soul reflects this life, it cannot just remain contemplative, but has to be performing a providential activity as well, which, in the individual soul's case, involves the care for an individual body. Secondly, the perfection of the cosmos requires that there be not only immortal intellective beings (i.e., gods), nor only mortal irrational animals, but also a class in between: mortal animals that partake in reason and intellect – and the latter are to be animated by souls that descend and ascend throughout the world of generation[14]. Thus, the descent of the soul into the world of generation is not a matter of punishment. It is a normal part of its existence, also because at the top of the system, souls that contemplate all the time get tired, and long for an active life to counterbalance their contemplative life, just as much as the souls that have been living in punishment are longing for a change[15].

The scientific description of the soul's migration also entails another question: how does an immortal and intelligible entity such as the soul become mobile? As such a thing would in principle be immobile, we need a theoretical tool to explain how locomotion of the soul can happen. So again, we are faced with the fact that mythical language about the soul needs to be embedded in some kind of topological or physical description. Moreover, as always, the Proclean system requires that there be an intermediary between two opposites, meaning that an intelligible being such as the soul cannot be connected with the corporeal realm without any mediating instance. In this case, the solution comes from a specific Platonic concept, mentioned in *Timaeus*, *Phaedo*, and *Phaedrus*, where Plato makes reference to an ὄχημα, a vessel or vehicle, by which the soul is vehiculated through the cosmos[16].

[14]. Procl. *In Ti.* V 214.3−23 Van Riel (= III 324.4−24 Diehl); V 152.2−4 (= III 278.25−26 Diehl). See also Fortier 2018, p. 311.
[15]. Procl. *In R.* II 159.19−160.11.
[16]. Pl. *Ti.* 44d8–e2: ἵν'οὖν μὴ κυλινδούμενον ἐπὶ γῆς ὕψη τε καὶ βάθη παντοδαπὰ ἐχούσης ἀποροῖ τὰ μὲν ὑπερβαίνειν, ἔνθεν δὲ ἐκβαίνειν, ὄχημα αὐτῷ τοῦτο καὶ εὐπορίαν ἔδοσαν. *Phd.* 113d4−6: καὶ οἱ μὲν ἂν δόξωσι μέσως βεβιωκέναι, πορευθέντες ἐπὶ τὸν Ἀχέροντα, ἀναβάντες ἃ δὴ αὐτοῖς ὀχήματά ἐστιν, ἐπὶ τούτων ἀφικνοῦνται εἰς τὴν λίμνην. *Phdr.* 247a8–b3: ὅταν δὲ δὴ πρὸς δαῖτα καὶ ἐπὶ θοίνην ἴωσιν, ἄκραν ἐπὶ τὴν ὑπουράνιον ἀψῖδα πορεύονται πρὸς ἄναντες, ᾗ δὴ τὰ μὲν θεῶν ὀχήματα ἰσορρόπως εὐήνια ὄντα ῥαδίως πορεύεται, τὰ δὲ ἄλλα μόγις.

Someone like Proclus does not need to be told twice that intermediary levels are necessary. Even more so, according to him, the difference between the eternal, unchangeable and noetic being that is the soul and the corporeal mass of matter is too big to be bridged in one jump, by introducing merely one ὄχημα[17]. Hence he argues for the existence of a cumulative system of different vehicles of the soul. The first one, he says, is the astral or luminous vehicle[18]. It is eternal (ἀίδιον) and make the soul encosmic. Thus, for encosmic souls, this vehicle is part of their nature (συμφυές)[19], which means that it cannot be taken away from them. The second vehicle is called pneumatic[20]. It makes the soul belong to the world of γένεσις, as it is added at the moment when an encosmic soul enters the realm of generation, and is taken away again when the soul leaves this realm[21]. It is what Timaeus calls "a troublesome mass of fire, water, air, and earth that had afterwards adhered to the soul, confusing and unreasoning"[22]. It is, thus, made out of the entirety of simple elements and is not indissoluble[23]. But it is anterior to the body and survives it after death. Finally, at a third stage, the soul enters the shell-like vehicle[24], i.e., the body (which Timaeus calls a third "vehicle", *Ti.* 69c7), the existence of which is characterized as chthonic[25]. It owes its unity to the pneumatic vehicle onto which it is added[26].

Based on this triple division between vehicles of the soul, Proclus explains the different levels of psychic existence. The system is cumulative, in that sense, that a lower soul always also includes the mode of existence (the vehicle) of the higher one: the astral souls (the souls of the gods and the planets) only have the astral vehicle, whereas the souls of elemental bodies are carried by an astral plus a pneumatic

[17]. Procl. *In Ti.* V 176.23−24 Van Riel and 177.9−13 (= III 297.24−25 and 298.5−10 Diehl).
[18]. ὄχημα αὐγοειδές: Procl. *In R.* II 154.25−26; II 195.11; II 196.26−27; II 199.21; *Theol. Plat.* III 5.19.10−11; *In Ti.* III
112.21 Van Riel (= II 81.21 Diehl); III 117.14−15 Van Riel (= II 85.3 Diehl); V 257.6 Van Riel (= III 355.16 Diehl).
[19]. Procl. *In Ti.* V 178.5 Van Riel (= III 298.28 Diehl).
[20]. ὄχημα πνευματικόν: Procl. *In R.* II 349.4; *In Ti.* V 94.8 Van Riel (= III 234.11 Diehl); V 98.21−22 Van Riel (=III 237.25 Diehl); V 99.24−25 Van Riel (= III 238.20 Diehl); V 223.19−20 Van Riel (= III 331.7 Diehl).
[21]. Procl. *In Ti.* V 177.19−22 Van Riel (= III 298.15−16 Diehl).
[22]. Pl. *Ti.* 42c−d: τὸν πολὺν ὄχλον καὶ ὕστερον προσφύντα ἐκ πυρὸς καὶ ὕδατος καὶ ἀέρος καὶ γῆς, θορυβώδη καὶ ἄλογον ὄντα.
[23]. Procl. *In Ti.* V 208.9−11 Van Riel (= III 320.14−16 Diehl); V 176.24−177.5 Van Riel (= III 297.26−298.2 Diehl), quoted below (see note 42).
[24]. ὄχημα ὀστρεῶδες: Procl. *In Ti.* V 178.7 Van Riel (= III 298.28 Diehl); V 208.12 Van Riel (= III 320.17 Diehl); ὀστρεῶδες σῶμα: Procl. *In R.* II 126.12; 187.11; *In Ti.* V 99.1−2 Van Riel (= III 237.26−29 Diehl); ὀστρεῶδες ὄργανον: *In R.* I 172.4−5; *In Ti.* V 177.19−20 Van Riel (= III 298.16 Diehl); V 179.12−13 Van Riel (= III 299.25 Diehl).
[25]. Procl. *In Ti.* V 177.19−20 Van Riel (= III 298.15−16 Diehl) and V 178.7 Van Riel (= III 298.28 Diehl).
[26]. Procl. *In Ti.* V 208.9−19 Van Riel (= III 320.14−20 Diehl).

vehicle, and the individually embodied souls are always riding an astral plus pneumatic plus shell-like vehicle[27].

In this same vein, Proclus explains the faculties of the soul at the different levels of its existence. To be sure, in each vehicle, the soul is equally tripartite, displaying the functions of reason, anger and desire. It is thus clear that the rational and the irrational parts of the soul are structurally present at every level, even though, at each stage, their relative status will differ[28]. The highest soul's life is noeric, devoted to the contemplation of true forms, and thus mainly determined by its rational faculty. Yet at the same time, Proclus emphasizes that the highest soul is a chariot with two horses, along the lines of the *Phaedrus* myth[29]. At this level, the irrational functions of the soul are present as "the highest pinnacles of non-rational life" (τὰς ἀκρότητας τῆς ἀλόγου ζωῆς)[30], i.e., that these irrational forces are present there in a causal mode (κατ'αἰτίαν), and that "the non-rational life exists in a rational way in the souls", just like "body exists in an incorporeal way in the intelligible causes"[31].

If a human soul succeeds in actualizing its highest functioning, led by reason, it will partake in the chorus of the divine souls, as explained by the *Phaedrus* (246e). Thus, in its purely astral vehicle, the soul lives in the supracelestial place (ἐν τῷ ὑπερουρανίῳ τόπῳ), in the chorus of its leader- god[32]. Yet this is hard to attain, and when a soul gets there, it will at any rate have to descend again. It then takes on the second vehicle, of an elemental nature and bound to the world of generation, in which irrationality becomes an active part of the soul's existence (Procl. *In Ti.* V 176.20−177.5 Van Riel = III 297.21−298.2 Diehl):

> For souls as they descend to earth take on from the elements garments of this type or that (airy, watery, earthy), and then, so equipped, finally enter into this dense mass here. For how could they immediately pass from immaterial *pneumata* to this body? So even before they descend into this, they possess the non-rational life and the vehicle belonging to that life, made ready from the simple elements, and they put on from these the "troublesome mass", so called

[27]. Procl. *Theol. Plat.* III 5.19.3−15; See *In Ti.* V 136.6−138.19 Van Riel (= III 266.24−268.20 Diehl); V 178.5−13 Van Riel (= III 298.27−299.4 Diehl).
[28]. See Fortier 2018, p. 309. Also Opsomer 2006, pp. 152−61.
[29]. Procl. *In Ti.* V 138.2−7 Van Riel (= III 268.5−8 Diehl); Fortier 2018, p. 309.
[30]. Procl. *In Ti.* V 97.23 Van Riel (= III 236.32 Diehl); See *In Ti.* V 100.9−10 Van Riel (= III 238.30−31 Diehl).
[31]. Procl. *In Ti.* V 160.1−4 Van Riel (= III 284.27−30 Diehl) and V 161.9−12 Van Riel (= III 285.30−286.1 Diehl).
[32]. Procl. *In Ti.* V 132.22−133.11 Van Riel (= III 264.3−13 Diehl).

> because it is of a different type from the vehicle born along with the souls, since it is made from a variety of garments and weighs the soul down[33].

Thus, irrationality in the soul's life is not the effect of the body (the third vehicle), but of the second one, which explains how it can play a role in the soul's life apart from the body, and in the punishment in Hades[34]. In this way, the life of the human soul is embedded in the different modes of existence one finds throughout the entire cosmos. This was to be expected, as a human being is a μικρὸς κόσμος, because, Proclus says, the human being also "has an intellect and reason and a divine body and a mortal one, just like the universe, and he has been divided up in a way that corresponds to the divisions of the universe"[35]. Interestingly, Proclus explicitly refers to "cosmic physics" here (*In Ti.* V 257.7−8 Van Riel = III 355.18−19 Diehl):

> Accordingly, in order to make you see the totality under the aspect of the parts, the explanation of human existence has been made to correspond to cosmic physics as a whole[36].

Thus, indeed, the different life choices, though depicted in mythical terms, are in fact a matter of physics, of locating the individual encosmic souls in specific areas of the cosmos. And the point at which the souls become physical is their connection with one or more of the vehicles. The vehicles are to be seen as the physical locations of the soul, making it possible for the soul to migrate from one *place* in the universe to another.

The Role of Rivers in the Soul's Locomotion

In his *Phaedo* commentary, Damascius raises the question of why judgment in the afterlife (*Phd.* 108b) has to take place in a specific location, given that the divine judges are omnipresent, and the souls themselves could undergo the judgment

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[33]. εἰς γῆν κατιοῦσαι γὰρ αἱ ψυχαὶ προσλαμβάνουσιν ἀπὸ τῶν στοιχείων ἄλλους καὶ ἄλλους χιτῶνας, ἀερίους ἐνυδρίους χθονίους, ἔπειθ'οὕτω τελευταῖον εἰς τὸν ὄγκον τὸν παχὺν τοῦτον εἰσκρίνονται· καὶ πῶς γὰρ ἔμελλον ἀμέσως ἀπὸ τῶν ἀύλων πνευμάτων εἰς τόδε τὸ σῶμα χωρεῖν; καὶ πρὶν οὖν εἰς τοῦτο κατέλθωσιν, ἔχουσι τὴν ἄλογον ζωὴν καὶ τὸ ἐκείνης ὄχημα κατεσκευασμένον ἀπὸ τῶν ἁπλῶν στοιχείων, καὶ ἐνεδύσαντο ἀπὸ τούτων "ὄχλον", οὑτωσὶ καλούμενον ὡς ἀλλότριον μὲν τοῦ συμφύτου τῶν ψυχῶν ὀχήματος, ἐκ παντοδαπῶν δὲ χιτώνων συγκείμενον, βαρύνοντα δὲ τὰς ψυχάς; tr. Tarrant 2017.
[34]. Procl. *In Ti.* V 99.10−18 Van Riel (= III 238.5−13 Diehl).
[35]. Procl. *In Ti.* V 256.20−22 Van Riel (= III 355.7−11 Diehl): δεῖ δὲ ὡς τὸν ὅλον κόσμον, οὕτω καὶ τὸν ἄνθρωπον ἐπισκέψασθαι τελείως, διότι μικρός ἐστι καὶ οὗτος κόσμος· ἔχει γὰρ καὶ νοῦν καὶ λόγον καὶ θεῖον σῶμα καὶ θνητόν, ὥσπερ τὸ πᾶν, καὶ διῄρηται ἀνάλογον τῷ παντί; tr. Tarrant 2017.
[36]. ἵν'οὖν καὶ μερικῶς ἴδῃς τὸ ὅλον, ὁ περὶ ἀνθρώπου λόγος τῇ πάσῃ φυσιολογίᾳ συντέτακται (my tr.).

anywhere[37]. The context of this section is entirely Proclean[38], which means that question and answer most probably stem from Proclus' own commentary. The answer is threefold (Dam. *In Phd.* I 499.3−6):

> In the first place, souls become more amenable to treatment when removed from the influence of the lower regions; secondly, some places are more suited than others to a particular person or a particular form of illumination; thirdly, we prefer to appear before the gods in holy places, even though they are everywhere[39].

Again, this requires the physics of locomotion, and Proclus does not fail to stress that in this context of the *Phaedo* also, Plato indicates that the souls make use of their vehicles (*In Ti.* V 96.6−9 Van Riel = III 235.22−25 Diehl):

> Even in Hades he depicts the souls making use of their vehicles. He says, "Getting up into what serve for them as vehicles" (*Phd.* 113d) they cross the river, according to Socrates in the *Phaedo*[40].

In the eschatological myths, these ὀχήματα are primarily the vessels with which the souls cross rivers, as in the text quoted just now, or enter into the rivers to be brought to different places in the underworld. Indeed, in Plato's *Phaedo*, but also in the myth of Er, the role of the rivers is mainly that of conveyor belts through Hades, leading the souls to their places of judgment, punishment, or elevation.

About the exact role of the rivers, Proclus refers us to his "commentary on the *Nekyia* of the *Phaedo*", the part of his (lost) *Phaedo commentary* that dealt with the eschatological myth[41]. As this text is lost, we do not have Proclus' general explanation, but enough is preserved in the extant works, in particular in his commentary on the myth of Er (the 16th essay of *In R.*) and his remarks on the creation of the soul in *Timaeus* (the 5th book of *In Ti.*), to reconstruct the larger part of Proclus' analysis.

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[37]. Dam. *In Phd.* I 499.1−3: διὰ τί εἰς τὸν αἰθέριον συλλέγονται τόπον; οἵ τε γὰρ δικασταὶ πάντα ὁρῶσι πανταχοῦ καὶ αἱ ψυχαὶ πανταχοῦ οὖσαι δύνανται τυγχάνειν τῆς ψήφου.

[38]. As indicated by Westerink (footnote *ad loc.*), the view that the judgment takes place in the αἰθέριος τόπος is Proclus'.

[39]. ἢ πρῶτον μὲν αἱ ψυχαὶ ἀπαλλαγεῖσαι τῶν χειρόνων τόπων ἐπιτηδειότεραι γίγνονται· δεύτερον ἄλλοι {δι'} ἄλλων τόπων πρὸς τόνδε ἢ τήνδε τὴν ἔλλαμψιν ἐπιτηδειότεροι· τρίτον ὅτι καὶ μᾶλλον ἐν ἱεροῖς τόποις βουλόμεθα προσιέναι τοῖς θεοῖς, καίτοι πανταχοῦ εἰσιν; tr. Westerink 1977.

[40]. καὶ γὰρ ἐν Ἅιδου τὰς ψυχὰς ποιεῖ τοῖς ἑαυτῶν ὀχήμασι χρωμένας —"ἀναβάντες γάρ", φησίν, "ἃ δὴ αὐτοῖς ὀχήματά ἐστιν", ἐπεραιοῦντο τὸν ποταμόν, ὥς φησιν ὁ ἐν Φαίδωνι Σωκράτης; tr. Tarrant 2006.

[41]. Procl. *In R.* II 179.9−17.

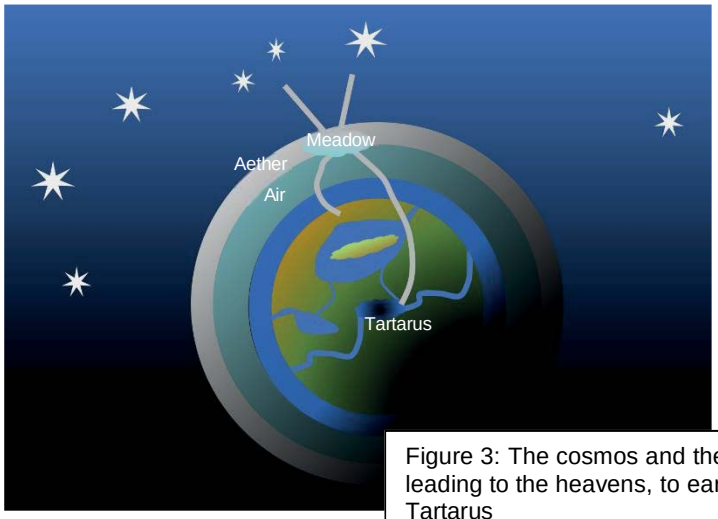

Figure 3: The cosmos and the chasms leading to the heavens, to earth and to Tartarus

In Proclus' interpretation, the myth of Er provides the higher strata of the geography, while the *Phaedo* myth describes the earth and the subterranean areas.

The myth of Er starts off with the gathering of the souls in a "wondrous place" (δαιμόνιος τόπος, *Resp.* X 614c), which Proclus identifies with the "meadow" (λειμών) where the souls arrive from all directions to camp for seven days, and where the souls exchange what they experienced in the areas they came from (*Resp.* X 614e and 616b). Proclus locates this meadow between heaven and earth, i.e. in the ether[42], and as Plato describes it, there are two pairs of chasms opening up into that place, two into the earth, and opposite to them, two into the heavens. Between them, there are judges who send the souls up to the heavens or down into the earth[43]. So the four chasms that according to the myth of Er open up in the wondrous place (*Resp.* X 614c) are pathways, one pair leading the souls into heaven and bringing them down from heaven, while the second pair makes them descend to earth and into the underworld[44]. In this sense, two pathways lead to what Plato calls λήξεις, i.e., allotted dwelling places (namely the heaven and the underworld), while two others lead toward γένεσις (one descending from heaven, one ascending from the subterranean world)[45].

In this context we encounter the first river, the one that leads from the wondrous place into genesis (i.e., from ether to earth), in the plain of Lethe. Proclus

[42]. Procl. *In R.* II 131.27−132.2; II 135.12−14.
[43]. Proclus also connect this description with the myth of the *Gorgias*: Procl. *In R.* II 132.20−133.27; at II 132.5−13, he indicates that the *Gorgias* narrative implies a triple pathway: one into the Isles of the Blessed, one into Tartarus, and a third into generation. As he explains at II 139.18−140.25, however, he considers the myth of the *Gorgias* to be a bit more focusing on particular situations, while the narrative of the myth of Er is more general (καθολικώτερον and περιληπτικώτερον). This explains why in one case, Plato mentions only three pathways, and in the other he distinguishes four chasms.
[44]. Procl. *In R.* II 138.24−139.18; II 140.26−141.13.
[45]. Procl. *In R.* II 132.14−19; II 155.23−157.8

locates this plain immediately below the wondrous place (or the meadow), indicating that as the meadow is identical to the ether, the plain of Lethe is the zone of dense air just below it, which our souls enter "in their descent towards the lifelessness of matter"[46]. More in general, as the air is the highest stage of the world of generation, the arrival at the plain of Lethe stands for the soul's entering γένεσις altogether, and the river of Lethe represents the flow of materiality that surrounds us in this realm[47].

When commenting on the river to which the souls are bound at *Timaeus* 43a, Proclus provides the same explanation (*In Ti.* V 216.8−16 Van Riel = III 325.25−326.2 Diehl):

> Certainly the term "river" does not signify the human body alone, but also all the coming-to-be that engulfs us from the outside on account of its unstable and unbalanced flow. So too in the *Republic* what he called the River of Lethe meant the entire generation-working nature[48].

Proclus seems to have recognized that the other river mentioned here by Plato, the Ameles, is essentially the same as the river of Lethe. For he attributes the exact same characteristics to Ameles as to Lethe[49]. As we saw, the event of entering into γένεσις implies the souls' taking on a second vehicle, the pneumatic one, on top of their luminous ὄχημα.

In this context, even the forgetfulness that fills us when drinking from this river is explained in a physical way: Proclus explains the loss of memory by the fact that by this drinking, our pneumatic vehicle becomes wet, and the humidity causes the imprints of the ideas in our soul to fade[50].

Moreover, this humidification does not only obliterate the imprints of the ideas, it also makes the pneumatic vehicle much heavier, and tears the soul further down[51].

Thus, the imagery of streams and rivers is also applicable to the higher spheres, where the chasms provide the passage ways to and from the meadow. The

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[46]. Procl. *In R.* II 346.25−347.16.
[47]. Procl. *In R.* II 95.9−12: ὁ δὲ τῆς Λήθης ποταμὸς πᾶσαν τὴν ῥύσιν τῶν ἐνύλων καὶ τὸ ῥόθιον κύτος (*Or. Chald.* 48) ἡμῶν, ἀεὶ λήθης ἀναπιμπλὰς τὰς ψυχὰς τῶν ἀεὶ ἑστώτων λόγων. See *In R.* II 354.10−23.
[48]. See also the references to ποταμὸς τῆς γενέσεως at *In Ti.* V 217.18 Van Riel (= III 326.28 Diehl); V 223.3 Van Riel (= III 330.20 Diehl) and 226.12 Van Riel (= III 333.1 Diehl).
[49]. See Procl. *In R.* II 348.2−349.3, where the river Ameles is likened to the thick moisty air that surrounds the earth, and identified as the flux of γένεσις. Moreover, Proclus here repeats the identification of this river with the one mentioned at *Ti.* 43a.
[50]. Procl. *In R.* II 349.3−8.
[51]. Procl. *In R.* II 349.8−12. See also *In Ti.* V 223.19−24 Van Riel (= III 331.6−12 Diehl), where Proclus says that the "inundation" mentioned at *Ti.* 43b hits the pneumatic vehicle first, before making the soul wet.

driving force of this imagery is the idea that the highest levels of reality are overflowing with their power, and are always filling the lower beings with life, layer by layer[52].

While this analysis is resting on a more metaphorical interpretation of the rivers, the description of the river of Lethe is physical in a more literal sense, as we saw, since it is the place where the soul enters generation. And the same is true for the rivers in the underworld, which according to Proclus correspond to the composition of the pneumatic vehicle out of the basic elements. For indeed, Proclus says, we find in the underworld (i.e., in the lowest ranges of the universe) the dregs (ὑποστάθμαι) of everything, and the four rivers over there correspond with the four elements in their lowest mode of existence[53].

This view, which Proclus took over from the Orphic tradition[54], meets Damascius' criticism. For, says Damascius, the position of the rivers does not correspond to the natural place of the elements, and, moreover, if they were "rivers", the elements should share a characteristic that allows for this common denominator, which the elements don't have. For that reason, Damascius rejects the purely physical reading and stresses the moral and theological significance of the rivers[55]. But Damascius is too dismissive of Proclus here. In fact, the latter position is also Proclus'. In his commentary on the myth of Er, Proclus indicates the following (Procl. *In R.* II 183.11−23):

> Above all, the pinnacle of badness, the fact that the souls "were guided" (…) "to Tartarus", the jail of punishment, is a clear definition of the things that this type of life deserves, and that this person, as he was recalcitrant and fled all laws, was forced into the most disordered place of the entire world-order, and as he ran away from intellect, he was forced into the darkest possible place. For that is how Tartarus is: the place of all disorderly and obscure matter, into which the lowest ends of the cosmic elements flow together. It is the opposite of the Olympus, for the latter is all light and envelopes all things, as their highest pinnacle, while the former is obscure, the deepest trench that is enveloped by all things. That is why the poets call it "the subterranean abyss" (Hom. *Il.* VIII 13−14)[56].

[52]. Procl. *In Ti.* V 105.21−106.3 Van Riel (= III 242.29−243.4 Diehl).
[53]. Procl. *In Ti.* III 67.21−68.4 Van Riel (= II 49.17−19 Diehl); see IV 182.17−183.4 Van Riel (= III 141.25−30 Diehl); Dam. *In Phd.* II 145.1−6.
[54]. See *Orph. fr.* 341 Bernabé (= frr. 123 and 125 Kern); also fr. 154 Bernabé (= fr. 222 Kern), quoted by Procl. *In R.* II 340.11−23.
[55]. See Gertz 2011, p. 184.
[56]. ἔκτον ἐπὶ πᾶσιν τὸ τῶν κακῶν τέλος, ὅτι ἄγοιντο παρ'αὐτῶν μετὰ τὰς ἀσπαλάθους ἐπὶ τὸν Τάρταρον, τὸ τῆς τίσεως δεσμωτήριον, ἐναργῶς ὥρισεν, οἵων ἐστὶν ἄξιον τὸ τοιόνδε τῆς ζωῆς εἶδος, καὶ ὅτι ἄτακτον ὂν καὶ φυγὸν ἅπαντας θεσμοὺς εἰς τὸ ἀκοσμότατον συνέωσται τοῦ κόσμου, καὶ ἀποδρᾶν νοῦν εἰς τὸ ἀλαμπέστατον. τοιοῦτος γὰρ ὁ Τάρταρος, χῶρος ὢν πάσης ἀτάκτου καὶ σκοτεινῆς ὕλης, εἰς ὃν συρρεῖ τὰ

The rivers thus have a double function: they are the streams that carry the dregs of the elements, and they are the streams that drag down the souls as a punishment. In his *Commentary on the* Republic, Proclus renders this even clearer (Procl. *In R.* I 121.27−122.9):

> Just as for souls which are ascending into the heavens a wide variety of places have been assigned for their allocation up there, so one must also believe that places have been dedicated under the earth for souls which are still in need of punishment and purification. These places are made up, on the one hand, from diverse effluences of the elements above the earth, which effluences [Plato and Homer] have called "rivers" and "streams". While on the other hand, different orders of daemones have been appointed there, some as avengers, some to inflict punishment, some for purification, and some as judges. If the poetry has spoken of "places dank and terrible to look upon which the gods despise" (*Il.* XX 65), it is not right to find fault with this[57].

In this way, the description of the rivers as the dregs of the world of genesis, each river following its own element, is paralleled by the moral value of the soul's descent into its lowest mode of existence, where its proper motion ends and its powers are shared with the body. And this close connection to corporeality has its deleterious influence on the soul, which sinks into darkness and disorder, away from the life of intellect and light[58].

The Proper Characteristics of the Rivers

With the identification of the rivers with the elements comes a specific characterization of each of them: Pyriphlegethon, given its etymology, is associated with

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ἔσχατα τῶν κοσμικῶν στοιχείων, πρὸς δὲ τὸν Ὄλυμπον ἀντίθετος. ὁ μὲν γὰρ ὁλολαμπής, πάντα περιέχων, ὑψηλότατος. ὁ δὲ σκοτεινός, κοιλότατος τόπος ὑπὸ πάντων περιεχόμενος· διὸ καὶ ἡ ποίησις βαθύτατον αὐτὸν εἶναί φησιν βέρεθρον ὑπὸ χθονός. (my tr.).

[57]. ἀλλ'ὥσπερ ταῖς εἰς οὐρανὸν ἰούσαις ψυχαῖς πολλοὶ τόποι καὶ παντοδαποὶ τῆς ἐκεῖ λήξεως ἀφωρίσθησαν, οὕτω δεῖ νομίζειν καὶ ταῖς κολάσεως ἔτι καὶ καθάρσεως δεομέναις τοὺς ὑπὸ γῆς τόπους ἀνεῖσθαι, ποικίλας μὲν ἀπορροίας ἔχοντας τῶν ὑπὲρ γῆς στοιχείων, ἃς δὴ ποταμοὺς καὶ ῥεύματα κεκλήκασιν, δαιμόνων δὲ τάξεις διαφόρους ἐφεστώσας, τὰς μὲν τιμωρούς, τὰς δὲ κολαστικάς, τὰς δὲ καθαρτικάς, τὰς δὲ κριτικάς. εἰ δὲ ἡ ποίησις "σμερδαλέ" εὐρώεντα τά τε στυγέουσι θεοί περ'ἐκεῖνα προσείρηκεν, οὐδὲ τοῦτο αἰτιᾶσθαι προσήκει; tr. Baltzly *et al*. 2018.

[58]. See Procl. *In R.* II 126.8−18. This combination of a physical and a symbolic reading is the case also of Olympiodorus (*In Mete.* 148.7−13), who follows Proclus in identifying the underworld with matter and the instability of the world of becoming, while at the same time reading the rivers and Tartarus as symbolic references to the soul's wickedness and punishment (because of its getting too closely attached to the world of becoming). See Gertz 2011, pp. 186−7.

fire, Styx or Cocytus (part of which is the Stygian lake, in Plato's account)[59] with earth, Acheron with air, and Oceanus with water[60]. Plato's narrative only partially supports this identification (e.g., the Pyriphlegethon is fiery, but it is also characterized as muddy)[61], but as said, the evidence from the *Orphic hymns* (obviously also based on Plato's account) comes to the rescue. At any rate, the rivers conduct the souls' vehicles through Hades, and the river by which a soul ends up being dragged along corresponds to the sort of misbehavior the soul displayed in its previous life[62]. As the *Phaedo* has it, average people embark on the Acheron, and are brought to the Acherousian lake, where they are rewarded for their good deeds and punished for their wrongdoings. Incurable perpetrators are thrown in the deepest hole of Tartarus, never to emerge from it. Curable perpetrators who did violence to their parents or committed murder but repented it, are thrown into Tartarus, but the violators are left out of it through Pyriphlegethon, and the murderers through Cocytus. Both of those rivers bring the souls along the Acherousian lake, where, as we saw, the perpetrators cry out to the souls dwelling there to release them, and the cycle is repeated (apparently through the pumping system of the waters in the underworld) until they are eventually rescued by the souls in the Acherousian lake (*Phd.* 113d–114b).

In many ancient accounts, the visions of Tartarus and Hades are about retribution, or pay-back time (or rather pay-back eternity). Others, among whom Plato takes pride of place, have always complemented this goal of retribution with the aim of purification of the soul[63]. For indeed, as we saw earlier, this purification only makes sense if the judgment is not the final one, i.e., that one believes in reincarnation, and that the punishment in Tartarus is not the end of the story. This aim of purifying the souls, instead of just imposing a retributive punishment on them, obviously becomes even more important in the Neoplatonic interpretation: Plato was still referring to the possibility of "eternal punishments", whereas Proclus *cum suis* considered all judgment and punishment to last for one cosmic revolution at the longest, after which the souls are at any rate released and make a new life choice[64].

[59]. Pl. *Phd.* 113b−c.
[60]. Dam. *In Phd.* II 145.12−6.
[61]. Pl. *Phd.* 113a−b.
[62]. See Pender 2012, p. 214, who stresses the active force and energy expressed in verbs such as ἐκβάλλει, ἐκπίπτει, etc. in the description of the rivers (*Phd.* 113a−b).
[63]. See, e.g., Procl. *In R.* II 184.14−28.
[64]. On Plato's upholding the possibility of eternal punishment, see Annas 1982, p. 131, who points out that by accepting that the punishment in Tartarus is not eternal, the myth of Er differs from the account in the *Gorgias*, and also from the myth in the *Phaedo*. Annas 1982, pp. 127−8 interprets this as leading to a contradiction between the supposed finality of the judgements in Hades and the theory of reincarnation. However, Pender 2012, pp. 229−32 argues, against Annas, that "the geography of the true earth, with its

The punishment is proportional, not only in its grades of severity, but also in so far as it is structurally identical to the misconduct that elicited the punishment. The idea seems to be similar to the one behind exposure therapy, as a means to enact some kind of katharsis. As Proclus explains (Procl. *In R.* II 147.13−14 and 20−23):

> For indeed, by suffering under these passions we get to hate them, not by acting in accordance with them […] whereby Dike all but shouts that the souls that indulge in these sorts of life must of necessity be joined with similar agents, and in this conjunction they must suffer from them what is befitting for those agents to do[65].

The interesting point to note here is that in the geography of the underworld, not only the punishment is structurally identical to the misbehavior, but also the gateway to the place of punishment is specific, depending on the character of the misbehavior of the souls. So the rivers themselves are also playing their part in the punishment and katharsis.

This is also the case of the higher realms, notably of the four chasms that open up to the "meadow" (Proclus' ether), where the connection between heaven and genesis is made. Plato already indicated that some souls who wanted to get out of the underworld before their time was up, were held back by the chasm itself (*Resp.* X 615e:):

> They thought that they were ready to go up, but the chasm wouldn't let them through, for it roared whenever one of those incurably wicked people or anyone else who hadn't paid a sufficient penalty tried to go up[66].

Commenting on this passage, Proclus recognizes the consciousness Plato ascribes to the chasm, as it has an awareness of the progress the souls are making during the time of their punishment. Proclus explains this by locating demons in

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scale of experiences of embodiment, is precisely the way that Plato reconciles afterlife judgement and reincarnation". See also the clear indication that punishment is temporal in *Phdr.* 249a−b. The later Neoplatonists clearly denied the possibility of eternal punishment: see Procl. *In R.* II 178.2−8; Dam. *In Phd.* I 492 (arguing against Iamblichus, who accepted that some souls remain forever in the intelligible world); Olymp. *In Mete.* 146.9−25 (stressing the difference between ἀίδιον, timeless eternity, and αἰώνιον, the word used by Plato, which refers to everlastingness for the duration of an αἰών). See also Fortier 2018, p. 307; Gertz 2011, pp. 185−6.

[65]. καὶ γὰρ πάσχοντες ὑπ'αὐτῶν μισοῦμεν τὰ πάθη, ἀλλ'οὐκ ἐνεργοῦντες κατ'αὐτά. […] μόνον οὐχὶ βοώσης τῆς Δίκης ὅτι τὰς τὰ τοιαῦτα ζωῆς εἴδη φιλοφρονησαμένας ἀνάγκη συνεῖναι τοῖς ὁμοίοις, συνούσας δὲ πάσχειν ὑπ'αὐτῶν ὅσα τοῖς τοιούτοις προσήκει ποιεῖν (my tr.).

[66]. οὓς οἰομένους ἤδη ἀναβήσεσθαι οὐκ ἐδέχετο τὸ στόμιον, ἀλλ'ἐμυκᾶτο ὁπότε τις τῶν οὕτως ἀνιάτως ἐχόντων εἰς πονηρίαν ἢ μὴ ἱκανῶς δεδωκὼς δίκην ἐπιχειροῖ ἀνιέναι; tr. Grube 1992.

the chasm who render this conscious interaction possible[67]. Apparently, ascribing awareness to lifeless holes in the cosmos would be going too far. But the point is that, indeed, the conveyor belts themselves have an active role to play in the assessment of who passes through them and who doesn't.

Likewise, then, the rivers in the underworld have their own characteristic, each in accordance with one of the elements. The aim is to chastise the souls by forcing them to go through those places[68].

The meaning of this is mainly moral: every individual soul ends up in the river that is most akin to the soul's own misbehavior[69]. The river thus confirms the life choice of the soul, *and* it adds its own impetus onto that of the soul. I.e., once you end up in a particular river, your own character will be reinforced by that of the river, which makes it even harder to escape the consequences of your choice[70]. As Proclus indicates, commenting on Plato's remark (*Resp.* X 614d) that souls coming from earth are covered in dust and dirt, while those coming from heaven are pure (*In R.* II 156.25−157.6):

> No surprise that the pneuma(tic vehicle) of the souls that ascend from inside the earth still displayed a specific mark of the places of punishment down there, for instance, of Pyriphlegethon or of the Stygian river (i.e. of Cocytus), or that the ones that descended from heaven would attract some light that entered them from up there, as for instance a light often shines in stones that absorb the effluences of the sun, the moon or the stars. For indeed, every soul retains what is proper to it, the ones that ascend from inside the earth retain what is material and dark, while the ones that descend from heaven retain what is luminous and the shining that came from the rays of the cosmic light[71].

Herein lies the moral or protreptic message of the myths. The choices made by individuals set in action a series of events that render a different life choice more difficult. For the ways of attaching ourselves to the body will be reflected by the life

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[67]. Procl. *In R.* II 141.28−143.13.
[68]. Procl. *In R.* II 183.29−184.3: τὰ μὲν οὖν παντοδαπὰ δείματα δαιμόνων ὄψεις ἦσαν φοβεραὶ καὶ ὄφεσιν ἐοικότων καὶ κυσίν, καὶ τόπων καταπληκτικαὶ δυνάμεις θολεροῦ πυρὸς γεμόντων ἢ ψύξεως ἀφορήτου ὑπὸ πνίγους ἢ σήψεως (followed by a lacuna). (my tr.).
[69]. See, e.g., Procl. *In R.* II 181.8−14, on the role of fire (and Pyriphlegethon) in chastising tyrants, who themselves have destroyed all lawfulness.
[70]. See Pender 2012, pp. 217−20.
[71]. εἰ δὲ καὶ τῶν ἐκ γῆς ἀνιουσῶν ἔτι τοῖς πνεύμασιν ἐνεφαντάζετό τις ἐνοῦσα τῶν ἐκεῖ κολαστικῶν τόπων ἰδιότης, οἷον τοῦ Πυριφλεγέθοντος ἢ τοῦ Στυγίου ῥεύματος, ταῖς δὲ ἐξ οὐρανοῦ προσῆν ἐφέλκεσθαί τι φῶς ἐκεῖθεν ἐγγενόμενον, οἷον δὴ πολλάκις ἐλλάμπει καὶ λίθοις ἀπορροίας ἔχουσιν ἡλίου καὶ σελήνης καὶ ἄστρων,οὐδὲν θαυμαστόν· κατέχει γὰρ ἐν ἑαυτῇ ἑκάστη τὸ οἰκεῖον, αἱ μὲν ἐκ γῆς ἀνιοῦσαι τὸ ὑλῶδες καὶ σκοτεινόν, αἱ δὲ ἐξ οὐρανοῦ κατιοῦσαι τὸ φωτεινὸν καὶ <τὴν> ἀπὸ τῶν αὐγῶν τοῦ κοσμικοῦ φωτὸς περίλαμψιν (my tr.). A similar description is given at *In R.* I 121.10−23.

choice we make for the next incarnation, and our fate in Hades confirms this again: the rivers impose their characteristic upon us, on the basis of our soul's own inclinations (physically undergone by the souls in their vehicle), and they reinforce these inclinations by their own impetus. Thus, it becomes harder and harder to escape the vicious circle of self-confirmation, once we have been led astray by wrong choices. And as Plato indicates, the cycle will go on just as long as the better souls in the underworld deem it necessary: until we are allowed to leave the Pyriphlegethon or the Cocytus and to enter the Acherousian lake. That is to say: the chastisements do have some purificatory power, in so far as they make our souls repent their wrong choices, and fill them with the wish to leave these errors behind. But this is not an easy task, and moreover, the final decision about this escape is left over to the souls of those who were allowed to rest at the Acherousian lake in the first place[72]. So by making our first choice to perpetrate, we lose control over the plot, and are delivered to the rivers, which have the tendency of confirming our character, and to the judgment of our victims. And that cannot be an attractive perspective.

In sum, just like the abyss of Tartarus is the reverse image of mount Olympus, as we saw, one might draw a certain parallel between the rivers into which the souls are thrown and the "choruses" of the gods, in the wake of which our souls and characters are established (*Phdr.* 248a and 252c– 253c). For just as the individual gods lead their chorus of human souls into the heavens, the rivers in the underworld lead their souls through the chasms of the underworld, and the souls reflect the characters of the leaders they follow. The heart of the matter is not, however, that our souls should make a choice between either following a god or being swallowed by a river. The case is more complex, as any soul may end up in a river in the underworld, by making choices against the lofty life in heaven, and being thrown in Hades as a wrongdoer[73]. After all, the point is not which god one follows, but rather, how attached does one become to the sensible world and its lures. Once the soul allows itself to be dragged down by the body, it takes on the elemental life at the bottom of the system, where the dregs of the whole flow together. And that is the person's own responsibility: the god, after all, remains *anaitios*.

[72]. On these points, see Pender 2012, pp. 225−9.
[73]. See Herm. *In Phdr*. 197.31−198.18 Lucarini – Moreschini; see also 163.26–164.1 Couvreur = 170.27−171.8 Lucarini – Moreschini.

Bibliography

J. Annas, *Plato's Myths of Judgement*, in «Phronesis», 27.2 (1982), pp. 119−43.

D. Baltzly – J. F. Finamore – G. Miles (eds.), *Proclus. Commentary on Plato's* Republic, Vol. 1, Cambridge, Cambridge University Press, 2018.

S. Fortier, *The Far-Wanderer: Proclus on the Transmigration of the Soul*, in «The Classical Quarterly», 68.1 (2018), pp. 305−25.

S. Gertz, *Death and Immortality in Late Neoplatonism. Studies on the Ancient Commentaries on Plato's* Phaedo, Leiden / Boston, Brill, 2011.

G. M. A. Grube [revised by C. D. Reeve], *Plato,* Republic, Indianapolis / Cambridge, Hackett Publishing Company, 1992.

C. J. Mackie, *Scamander and the Rivers of Hades in Homer*, in «The American Journal of Philology»,

120.4 (1999), pp. 485−501.

K. A. Morgan, *Myth and Philosophy from the Presocratics to Plato*, Cambridge, Cambridge University Press, 2000.

J. Opsomer, *Was sind irrationale Seelen?*, in M. Perkams – R. M. Piccione (eds.), *Proklos. Methode, Seelenlehre, Metaphysik*, Leiden, Brill, 2006, pp. 136−66.

E. Pender, *The Rivers of Tartarus: Plato's Geography of Dying and Coming-Back-to-Life*, in C. Collobert – P. Destrée – F. J. Gonzalez (eds.), *Plato and Myth. Studies on the Use and Status of Platonic Myths*, Leiden, Brill, 2012, pp. 199−233.

C. Steel, *Plato's Geography: Damascius' Interpretation of the* Phaedo *Myth*, in J. Wilberding – C. Horn (eds.), *Neoplatonic Natural Philosophy*, Oxford, Oxford University Press, 2012, pp. 174−96.

H. Tarrant (ed.), *Proclus. Commentary on Plato's* Timaeus, Vol. 1, Book 1: *On the Socratic State and Atlantis*, Cambridge, Cambridge University Press, 2006.

H. Tarrant (ed.), *Proclus. Commentary on Plato's* Timaeus, Vol. 6, Book 5: *On the Gods of Generation and the Creation of Humans*, Cambridge, Cambridge University Press, 2017.

L. G. Westerink, *The Greek Commentaries on Plato's* Phaedo. Vol 2: *Damascius*, Amsterdam, North- Holland Publishing Company, 1977.

ABSTRACT

Gerd Van Riel, *Proclus' Potamology: The role of Rivers in the Underworld*

This chapter explores Proclus's interpretation of the role of rivers (potamology) in Plato's eschatological myths of the Underworld, specifically Lethe, Styx, Acheron, Pyriphlegethon, and Cocytus (as laid down in *Phaedo* and *Republic* X). The later Neoplatonists, including Proclus, rejected merely physical or solely allegorical readings. Instead, they combined a literal interpretation (at a cosmological level) with a moral message regarding the destiny of the soul. The role attributed to these rivers provides valuable insight into the interaction between body and soul, as well as the complex relationship between cosmology, theology, physics, and ethics within Proclus's comprehensive philosophical framework. By examining Proclus's "potamology", the chapter illuminates how he integrates mythological elements into his scientific and ethical account of human existence and the cosmos.

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PROCLUS' "THEOLOGICAL" SPHERICAL COSMIC WHOLE AND HIS SCIENTIFIC INNOVATIVE ASTRONOMY

Proclus was an impassioned critic of the complex and highly developed astronomy of his predecessors. Armed with an extensive scientific knowledge of physics, geometry and spherics, he offered an innovative astronomy of his own to counter what he perceived to be the deficiency of former views. He did so, however, not only on the basis of his own sophisticated knowledge of science and mathematics, but with very potent nonscientific sources. When he criticized Claudius Ptolemy's *Almagest*, written in the second century C.E., the most influential book on astronomy for more than 1,2000 years after its composition, he did so on the basis of his own views of space. He supported this critique, however, with appeals to a theology which he considered compatible with Pythagorean/ Platonic philosophy and Neoplatonic doctrine. He particularly questioned whether epicycles (circles within which a planet moves and which has a centre that is itself carried around on the circumference of a larger circle) and eccentrics (the amount by which an elliptical orbit deviates from a circle), innovations that had been deployed by former respected astronomers to account for the apparent irregular movements of the sun, moon and the planets, were anything more than mechanical contrivances. Apollonius of Perga (265 BCE−190BCE) and Hipparchus of Rhodes (190 BCE−120BCE), whose work he knew well, all had held to these constructs. He particularly opposed Ptolemy. The mechanical artificiality of Ptolemy's use of these contrivances did not meet his own demand for a causal account of motion[1]. Though his own astronomy was based on physics and mathematics, he backed up his objections with theological/Platonic ideas. 'Soul', for example, was the force which enabled motions of all kinds. In doing so, he anticipated some of the discoveries of Copernicus and Kepler as well as those of modern physics[2]. His "theological" metaphysics, surprisingly, resulted in an astronomy with mathematical and scientific veracity and a respect for physics, a more fruitful approach than any based on the ancient astronomy of the centuries that preceded him.

[1]. Segonds 1962, p. 329 points out that Festugière found the expression "artificiellement machinees" or its equivalent pertaining to Ptolemy, in all the following passages: Procl. *In Ti.* III 358.22 Van Riel (= II 264.22 Diehl); IV 84.20 Van Riel (= III 65.8 Diehl); IV 123.25 Van Riel (= III 96.19 Diehl); IV 189–190 Van Riel (= III 147 Diehl); *In R.* II 227.27−28; II 229.9−11; II 230.8−9; II 223.23; II 236.11.
[2]. See Kutash 2008, pp. 297−321.

Greek astronomy was given its uncontested first principles by Presocratic philosophers. Parmenides, for example, declared the universe (*to holon*) to be a "well-rounded sphere" (*eukuklon sphaires*)[3]. The Pythagoreans held to the notion of a central fire around which the universe revolved[4]. The conviction that the universe was spherical and that the motions of heavenly bodies must be circular, thereafter, made it imperative for future astronomers to account for the appearance of irregularity posed by the movements of the planets, the sun and the moon, as compared to the fixed stars. For Plato, perfect circular motion was displayed by the orbits of the fixed stars, a physical exemplar of "reason"[5]. Any departure from this premise was disturbing. As reported by Simplicius, it was Eudoxus, a student of Plato, who attempted a mathematical hypothesis to accomplish the goal of accounting for these anomalies[6]. Not only did they violate the presumed harmony of the celestial sphere, the figure of the perfect sphere (or dodecahedron) was the presumed shape of the universe but also a figure conceived by Theaetetus, a student of Plato, as having the ability to contain all the other so called cosmic figures (the regular solids)[7]. The enormous and sophisticated discipline of astronomy from that time forward developed around the issue of the irregular motions of the planets, an aporia which defied strong beliefs in sphericity as a visible icon of the invisible world of noetic truth. Proclus faithfully followed Plato, for example, in *Laws* (X 898a−b) where noesis is described as an activity that resembles the sphere[8].

Proclus argued repeatedly that attempts to commensurate the apparent irregular movements of the planets (the appearances), through both the use of concentric spheres and the use of epicycles and eccentrics, must be rejected. Further, notably, not only did Proclus object to Ptolemy's mechanical model and return to Plato's emphasis on the invisible world of reason, he argued that they are completely foreign to Plato's conviction that the fixed stars, engaging in perfect circular motion, are "divine living things" (*Ti.* 40b4−8). Proclus notes that Plato "[…] indicates the presence in them of both divine intellect and divine soul" (*In Ti.* IV 163.7–8 Van Riel = III 126.15−16 Diehl). Ascribing to Plato's attribution of divine origin, Proclus also held the conviction that the World Soul was the source of the presence of soul as it is manifest in celestial motion. Epicycles and eccentric, on these grounds, are "unworthy of the divine essence". Plato's in *Laws* (X 899b3−7), after all, had said:

[3]. See Parm. DK28B8.32−49, present in Pl. *Ti.* 33b1−7 as well.
[4]. Dicks 1970, p. 51.
[5]. See Pl. *Ti*. 46b5–9
[6]. See Aiton 1981, p. 79.
[7]. Kutash 2001, pp. 130−1. Kepler, in his *Mysteriun Cosmographicun* of 1597, retained this model as it fit the distances between the orbits of the six planets to the distances that would be obtained if the hypothetical spheres of the planets were inscribed in and circumscribed around, the five regular solids.
[8]. See Baltzly 2009, p. 207 note 49.

> Concerning all the stars and the moon, and concerning the years and months and all seasons, what other account shall we give than this very same, namely, that, inasmuch as it has been shown that they are all caused by one or more souls, which are good also with all goodness, we shall declare these souls to be gods[9].

Epicycles and eccentrics are pointedly criticized throughout the *Commentary on the* Timaeus. They are clearly in opposition to the primacy of a celestial world in which there is spatial unity and only one compelling centre for *all* the heavenly bodies, fixed stars and planets alike. This and only this is in accordance with the soul's own volition.

He claims that (*In Ti.* IV 190.2– 4 Van Riel = III 147.2−5 Diehl):

> It is [in fact] necessary for the very variety in their motions to be dependent upon the *soul*s [that animate the stars] so that the bodies that undergo motion do so swiftly or slowly in accordance with the *will* of these souls[10].

Clearly, Proclus relies on *Platonic Theology* to "verify" stipulations based on scientific considerations.

It is notable that Plato's views on the World Soul and the belief that Soul is a purveyor of motion, enable Proclus to apply physics to astronomy. While the views of astronomers before Proclus hold to the idea that the planets are passive bodies carried along by epicycles or eccentrics, he claims that the irregular motions of the planets are self-moving entities within dimensional space. They are intermediaries between the fixed stars and things in the sublunary regions which display rectilinear motion (*In Ti.* IV 189.7–10 Van Riel = III 147.8−10 Diehl). These independent trajectories through dimensional fields can be erratic when escaping the influence of centrific force. The universe, however, is an essentially unified whole, ultimately an eternal one being, beyond the vicissitudes of time and space. All motions will succumb to this eternal unity and deform towards circularity.

According to B. L. Van Der Waerden (1974), theological attributions are not Platonic *per se*, but characteristic of the Pythagoreans who based science on theological arguments. He raises the possibility, however, that Geminus, Proclus and Simplicius were following the widespread practice of attributing every discovery to the Pythagoreans (he cites Sir Thomas Heath). Geminus' (10 BCE-60 CE) *Introduction to Astronomy* was well known to Proclus, who quotes him twenty times in his *Commentary on Euclid's* Elements. Geminus claimed that the Pythagoreans knew about stationary points and retrogradations but still declared that the true motion of the planets must be uniform and circular, because the planets are eternal and divine. The mandate to explain the phenomena by

[9]. Tr. Bury 1926.
[10]. Tr. Baltzly 2013.

assuming circular and uniform motions, was a stipulation that was likely to have originated by the Pythagoreans, particularly Philolaos[11]. Uniform circularity, from the Pythagoreans through to Plato and to the Neoplatonists, was commended as endemic to being. In *Timaeus* the universe is deemed to be "(…) One single Whole, compounded of all wholes perfect and ageless and unnailing" (*Ti*. 33a7) and revolving in a circle. For Proclus, the unity commended in *Timaeus* demanded that Plato's view of being as one extends to the celestial sphere in the form of the circular motion exhibited by the motion of the fixed stars. Proclus, in his treatise on astronomy, *Hypotyposis Astronomicarum Positionum,* asserts that "The hypotheses of eccentrics and epicycles commended themselves also, so history tells us, to the famous Pythagoreans as being more simple than all others (…)".[12]

For Proclus, however, they are not simple at all but in fact, complicate the unity that Platonists ascribe to celestial being. The "vision of the whole", as a spherical or dodecahedral configuration of a bounded or limited universe, was a standard conceptual icon for Greek astronomers. Plato (*Ti.* 33b1−8), posited, following Pythagorean geometry, that the fitting shape to hold within itself all shapes (*schematon*) is spherical (*spheairoeides*), perfect and self-same (*homoiotaton*). This paradigmatic case of containment was thought to represent all that is and equivalent to 'being'. All change and motion had to be accounted for by reference to true being and, since the universe is a spherical whole, motion in the celestial realm must be subject to sphericity as well.

Zeno's paradoxes regarding motion brought to the attention of the ancient philosophers the fact that any kind of successive motion in the form of a purely rectilinear course, motion, in time, could potentially go on ad infinitum. In Zeno's race between the tortoise and Achilles, Achilles can never overtake the tortoise because there is a one-to-one correspondence between the infinite set of points run by the tortoise and run through by Achilles. Motion that is unidirectional and occurring across segments of space and time with no limiting parameter, could result in a potential infinite iteration. In the celestial expanse, the Platonic vision of an invisible noetic unity is actualized by the perfect circular motion of the fixed stars. This orbital sphericity, at the outermost point of the celestial space, was not a material sphere, but a spherical boundary enunciated by the orbits of the stars. The possibility of never-ending seriated infinity in rectilinear progression could never become a reality. Commandeered by the force of the "centre" a physical placeholder for noetic unity, all sorts of motions ultimately comply with circularity.

[11]. Van Der Waerden 1974, pp. 176−8.
[12]. See Heath 1981, p. 271 and note 2 citing Proclus' *Hyp.* 34.18.

A Commanding Centre

Copernicus did not ignore his debt to the long and sophisticated history of Greek astronomy. He knew that his revolutionary innovations were predicated upon the posit of a ruling "centre" and that the circular or elliptical pathways of the heavenly bodies orbited around it. Copernicus cites Aristarchos of Samos, as Sir Thomas Heath points out, and also alludes to the central fire of the Pythagoreans[13]. One of Proclus' objections against epicycles and eccentrics was that motions in circles that move opposite to the direction of the spheres on which they are located destroy the continuity of the spherical space. This "does away with the common axiom of the physicists, namely that every simple movement is either round the centre of the universe or to or from that centre"[14]. Explaining Plato's conviction that the Demiurge ordained that the planetary circles would go in opposite directions from the fixed stars (*Ti.* 39b1) Proclus claims they did so (*In Ti.* III 358.18–359.1 Van Riel = II 264.18−25 Diehl):

> (…) since they are moved by their own individual motion as well as that of the sphere of the fixed stars. Nor did Plato here or elsewhere make any mention of eccentrics or epicycles. Rather the seven circles all determine a single centre, without the addition of any other centres. (…) [further] the hypothesis of epicycles or concerns about the rising of the planets are in no way appropriate to the subject of the circles within the soul[15].

According to Helen Lang, Aristotle's account of motion, is fully teleological because it rests on a full specification of the relation of moved to mover[16]. For Proclus, the isometric circular universe is subject to a force which operates like an unmoved mover, not at the limits of the universe but at its centre. Proclus claims in his *Commentary on Euclid's* Elements, the centre "(...) represents the principle of Limit (...). The rest of the universe converging upon it, striving to be the centre and become one with it"[17]. The trajectories of celestial bodies, then deform toward the circular under the influence of a centrific position at rest. Neoplatonic doctrine regards rest as the superior state of being over motion. Immobility rules in the higher hypostases. In the centre of the celestial space, then, there is a placeholder for noetic unity in situ. Motion is commandeered centrifugally rendering all motions to conform to circularity. For Proclus, all space within the universe is constant given the fact that the outer limit is spherical and the earth, (as good as any other body), is to be at its centre. Earth, for Proclus, functions as a centrally located organizer embodying the rest/centeredness that is an analogue to noetic truth. Kepler recognized the

[13]. Heath 1981, pp. 95−7.
[14]. Lloyd 1991, p. 261 note 39 citing *In Ti.* IV 189.9ff. Van Riel (= III 146.14ff. Diehl).
[15]. Tr. Baltzly 2013.
[16]. Lang 1996, p. 259.
[17]. *In Eucl.* 154 tr. Morrow 1970.

implications of Proclus's model and speculated that Proclus actually envisaged the sun as the centre of power of the system composed of the earth and planets[18]. Unity and rest characteristic of higher hypostases is present in the celestial sphere, itself subject to both rest and motion. Notably, Proclus remarks that some commentators "(…) say that the middle is the centre of the earth, but others say that the moon is the middle (…) Still others say that the Sun is established in the place of the heart (*in Ti.* IV 134.4–9 Van Riel = III 104.17−24 Diehl)." In the following quote, theology and Pythagoreanism are co-present along with mathematical and physical considerations (*In Ti.* III 147.21–148.5 Van Riel = II 106.15−22 Diehl):

> By virtue of its guardian powers it holds together the centre. For the whole sphere is steered from thence and converges in the centre. Moreover all the troubles in the world have been corralled in its middle and it is necessary that there should be a divine guardian who is capable of marshalling them and keeping them within their proper bonds. It is for this reason that theologians and […] the Pythagoreans call the middle, "the tower of Zeus" or "the guard post of Zeus"[19].

Perfect circular movement, then, is commandeered by a centre at rest. Irregular or rectilinear motion will always succumb to its overrule. The superior ontological status of circular motion over rectilinear motion receives its imprimatur by command of the compelling centre. Anomalous motions will ultimately revert, therefore, in deference to circularity, the closest physical reflection of rest. Plato, after all, asserts that "Motion does not consent to exist in uniformity" (*Ti.* 57e3−4). The stars, then, are the exemplary case of the presence of reason *in* the universe. They are subject to the hybrid nature of the celestial sphere – rest, characteristic of *noesis*, and motion, enabled by soul. For a Neoplatonist both the celestial sphere, and the world of nature are subject to higher hypostases. Proclus' threefold hierarchy of grounding principles − One Beyond Being, One Being − and Existence is a multi-leveled hierarchy in the continuum from the One to existence. On the level of Existence (generated being) there is a variety of motions but on higher levels, these motions are subject to the One Being and grounded in a transcendent unity (One beyond Being)[20]. In the material world (existence), spatiotemporal intervals (*diastema*) are possible. In the celestial region motion is accountable to the standards of unity/rest and only circular motion achieves similarity to higher hypostases.

Proclus had a strong interest in Aristotle's physics and Stoic theories of motion and focused his commentary on Aristotle's *Physics* on motion. He rejected, however, the *Primum Mobile* as a

[18]. Siorvanes 1996, pp. 308−11 elaborates the case for Proclus' heliocentrism. See also *In Ti.* III 67.27−68.10 Van Riel (= IV 87.22–88.9 Diehl).
[19]. Tr. Baltzly 2009.
[20]. Kutash 2008, pp. 298−9.

First Cause. For Aristotle the *Primum Mobile* is connected to the outermost sphere of the heavens and is considered to impart motion to the other celestial spheres. For Proclus, rest is the transcendent eternal principle and the celestial sphere's centre is the unmoved mover, the originator of eternal circular motion. In celestial space transcendent principles and spatial and temporal unfolding are both operative. Rest, rather than motion, is the ultimate determinate of being and perfect circular motion is the material exemplar closest to rest and to noetic truth. Remarkably, this framework enabled Proclus to endorse an idea of curvilinear space, the equivalence of mass and energy, the limiting role of light, and a physics of motion.

Curvilinear space

For Twentieth Century physics, the concept of "field" is, in principle, a replacement of the idea of a particle or material point in the concept of space and presumes a space that is a continuum[21]. When light came to be regarded as a wave field; the field concept that gradually won greater independence. The cosmological theories that follow from Einstein's work such as the work of E. A. Milne, in 1932, describe all events as contained within a sphere, all intervals between two events (points in space-time) are regarded as homogeneous, with all places alike and isotropic[22]. For Proclus, a spherical limit determines the ultimate configuration of the celestial space which displays both unfolding in intervals of time and space and at the same time continuity as a simultaneous whole. Celestial space, however, is curvilinear; rectilinear unfolding anomalous. Proclus, in his *Commentary on Euclid*, and in many other of his texts, makes laudatory comments on the powerful influence of circularity (*In Eucl.* 149.9−13):

> On the heavenly bodies the circle confers their likeness to Nous, their homogeneity and uniformity, their function of enclosing the universe within limits, their fixed and measured revolutions, their eternal existence without beginning or end, and all such things[23].

The One beyond being is the ultimate source of the One Being in the Neoplatonic hierarchy and the celestial space embodies its image in its own simultaneous unity. The One is itself, "(…) without beginning and without end"[24] and sphericity is iconic. The fixed stars, then, as eternally

[21]. Einstein 1961, pp. 163−5.
[22]. See Kutash 2008, p. 302.
[23]. Tr. Morrow 1970. In *El. Theol.* 11.14 Proclus mentions that circularity forecloses the possibility of an infinite regress.
[24]. *In Eucl.* 149.12−13.

circular in motion enacts a situated reflection of the One Being while anomalous motions appear to escape its overrule. In Euclidean geometry the dodecahedron or sphere potentially contains all other figures. This is a paradigm case for containment for celestial sphericity, as well as containment marked by the orbits of the fixed stars. Physical or celestial entities assume a rectilinear trajectory in the interval structure of time and space but in the "field" which is curvilinearly structured, curvilinear dominance will have a commensurating effect on rectilinear movement and force a reverting trajectory. All displacement will cease (*In Ti*. III 17.1 Van Riel = II 12.13 Diehl):

> (...) That every simple body, when in its proper place either remains immoveable, or is moved in a circle, in order that it may by no means relinquish its proper place. For if it is moved in a different manner it will either no longer being its own place or will not yet be in it. A celestial body therefore being fiery is necessarily moved in a circle. Earth also, if it were moved without leaving the place about the middle, would be moved in a circle. For when fire is moved to the upper region it is so moved in consequence of being in a foreign place. For the same reason likewise, a clod of earth is moved downward. In short, the local motions of the elements are in a right line, (...) it is false to say that fire is moved naturally in a right line (...) when it tends to its proper place it is not yet in a condition conformable to nature (...) in its natural place it will either be immoveable or moved in a circle[25].

For Proclus, temporality and spatiality must be considered from the perspective of an eternal being and beyond being rest. "All that is eternal is a simultaneous whole (...) without diminution and without serial extension"[26]. Time in the material universe is an image of eternity but temporality (when time passes toward the future from the past it in the material or celestial space) it does so within an interval (*diastema*) sequence. Essentially, however, it is immovable, invisible and immaterial and from a panoptic perspective all time is a degraded form of an eternal Now. Proclus calls *to en* and *to estai*, "what was and what will be", "species" (*eidai*) of time generated by the Demiurge (*In Ti.* IV 48.7–9 Van Riel = III 37.15−17 Diehl). Temporal intervals like rectilinear motions are part of times progression (*In Ti.* IV 44.14–15 Van Riel = III 34.15 Diehl). Eternal unity holds all physical manifestations in the grip of eternal rest, ultimately ruling over it and grounding it in what is permanent, continuous and uniformly self-same. Time and space then, just as in modern physics, are inseparable. The life of the heavens displays its hybrid nature by these features, the heavens move according to time, but the intelligible nature (*noeta physai*) fixes these movements in the larger scheme of things. According to the whole of itself it is eternal and analogous to a breadth rather than a series[27]. The celestial sphere, is both material and can exhibit processive expansion, but it is an image in extension of the essentially extensionless whole. The simultaneous whole is

[25]. Tr. Baltzly 2013.
[26]. *El. Theol.* 52.51.
[27]. See Kutash 2009, p. 45; p. 55.

transcendent and eternal, but the celestial sphere is concomitantly eternal and subject to its regional situations. Time and space are inseparable in the sense that unfolding in space is unfolding in temporality. For Proclus, within an interval structure all motions, temporal and spatial, in material existence are potentially commensurate to an overriding whole. Time and times, are a moving image of image of eternity and interval sequences in space are a moving image of rest (*In Ti.* II 41.2–6 Van Riel = I 233.1−4 Diehl):

> Only the intelligible realm is fully eternal in virtue of itself. For this reason some of the ancients describe (a) the noetic realm in its full extent as "truly extent" (…), b) the psychic realm as "not truly existent", (c) the sense perceptible realm as a "not truly existent" and (d) matter is "truly non-existent"[28].

I would amend the phrase translated above as "noetic realm in its full extent" to correspond to the Greek *noeton platos*, "intelligible breadth". For Proclus, for whom the celestial universe is an exemplar of spherical wholeness, and also entails spatial extension, curvilinear space allows both. The celestial world can display both time (becoming) and eternity (being), motion and rest. Platonic metaphysical premises, once again, result in an astronomy of the physical world which is both scientific *and* onto-theological. Modern astronomy, of course, is based in physics and mathematics *per se*.

Proclus references to the primacy of the Circle of the Same in Plato's *Timaeus* (*Ti.* 37b6−c2), as opposed to the Circle of the Different, are a good example of his reliance on Platonic principles to advance his astronomy. Under the Circle of the Same, eternal unity dominates and will triumph over the possible aporias that stem from a potential endless dispersion and divisibility. Difference, such as is displayed by anonymous motions, participates in the One as well. The terminal point of a progression in time or space is not infinitely postponed in a curvilinear space as the spherical limit contains all the potentially infinite motion that appears as spatial progressions. Proclus celestial curvilinear space like that of modern physics, is the occupant of an unbounded but finite universe. All trajectories in space will be curvilinear or deform to a naturally curvilinear influence. In a homiletic universe, all objects in space must conform to its ruling parameters of a spherical limit. Proclus calls the circular figure "(...) the very highest of ineligibles" and the center resembles the principle of Limit. Though the lines from the centre typify Boundlessness (*In Eucl.* 155.12−19),

> (...) the line which bounds their indefinite extendedness and gathers it back to the centre is like the hidden cosmic order they constitute, which Orpheus describes as follows:

[28]. Tr. Runia – Share 2008.

(...)The Boundless in a circle
Was moving unweariedly (…)[29].

Astronomy, then recapitulates ontology. This model extends all the way to living souls which are illuminated by autonomous life and motion which enables them to revert to Nous and circle about it, enjoying self-renewal through the special periodic revolutions which unfold the partlessness of Nous, as Soul "(…) traverses a circle in her desire to embrace the Nous in herself"[30]. *All* motion and apparent motion is subject to noetic predetermination. All astronomical anomalies such as the irregular motions of the planets ultimately fall prey to its dominating 'truth and correct under the ruling symmetry.

Einstein's vision of a finite but unbounded space as Max Born pointed out, "one of the very greatest ideas about the nature of the world which ever has been conceived"[31]. When Einstein considered the problem as to whether the space and matter of the universe were finite or infinite, he followed De Sitter's suggestion that the universe might possess a finite volume but no finite boundaries[32]. Finite unboundedness would fit the case of a space that was a three-dimensional expanse with a definite area but no actual material boundary. For Proclus, the celestial region in analogue to the spherical geometrical shape that would encompass all the shapes within itself (*Ti.* 33b1−8), is a visionary construct of a finite but unbounded space. Proclus comments extensively on the gift to the universe of spherical shape and its ability to contain all shapes within its limits[33].

The Irregularity of the Planets

Dirk Baltzly points to a passage in the *Commentary on the* Timaeus which views the universe as diminishing levels *of homoitatus* in the continuum from the fixed stars to the irregular motion of the planets[34]. The planets are (*In Ti*. IV 73.13–16 Van Riel = III 56.31−57.6 Diehl):

> (…) like an intermediary between things that undergo motion that is entirely regular [*scil*. the fixed stars] and things that undergo motion that is entirely irregular [*scil*. things in the sub-lunary], for [the planets] have been allotted a motion which is regularly irregular or irregularly regular[35].

[29]. Tr. Morrow 1970.

[30]. *In Eucl*. 149.7−8 tr. Morrow 1970.
[31]. Born 1956, p. 200.
[32]. Mason 1954, pp. 567−9 discusses this.
[33]. Baltzly 2007, pp. 124−40, elaborates Proclus' views on the gift of sphericity to the universe (from III 94.8–112.10 Van Riel = II 68.6−81.10 Diehl). See also Kutash 2011, p. 115 ff.
[34]. Baltzly 2013, p. 7.

When circular motion is the true reflection of noetic principles and sphericity is the true shape of the celestial sphere, planetary irregularities present a troubling inconsistency. The seventh book of Plato's *Republic* (*Resp.* VII 529−531), after all, asserted that astronomers and scientists must conform to true reality and understood it by pure reason alone. The word planet (*planetes*), notably, is the word for "wanderer". Proclus, who had a vast knowledge of astronomy in the centuries before him, understood that the use of eccentrics and epicycles made use of superimposed circular orbits, attributing their origin to the Pythagoreans. Proclus was particularly familiar with the astronomy of Apollonius of Perga and Hipparchus of Rhodes (who developed trigonometry and accurately measured the sizes of the sun and the moon). Ptolemy however was his prime target. On Platonic terms alone, the contrivances of all three of these astronomers, were unacceptable. More sophisticated than the celestial spheres of Eudoxus, Callippus and Aristotle, they attempted to preserve uniform circle motions. Ptolemy's *Syntaxis*, argued against the idea of aethereal spheres as well. Ptolemy did consider each planet to possess a "vital force" and moved itself and this was the source of planetary motion. Still, he attributed the irregularity to eccentrics and epicycles[36]. Proclus repeatedly argued that eccentrics and epicycles were limited and too mechanical. Curvilinear space and a spherical "limit" were the true reflections of the commanding centre (a place holder for eternal unity), therefore the ultimate influence on all motions.

Proclus allowed the planets a self-moving motion (away from rest/centre). The ultimate reversion, however, would commensurate with circular wholeness. The continuity of curvilinear spacing left no room for intervening epicycles or eccentrics, which would destroy its unity. In any number of passages in his *Commentary on the* Timaeus (IV 73.20 Van Riel = III 56.28 Diehl; III 134.22 Van Riel = II 96.27 Diehl, for example), Proclus criticizes Ptolemy's account. His own explanation for the observed irregularities, emphasized the physics of motion over geometry. In his *Commentary on the* Republic (II 216.5−13), he advocated a continuous celestial expanse precluding spherical interference:

> For there is no vacuum at all between the whorls which fit into one another so that there is one continuous surface from the convex surface of the innermost (*endolatoi*) of all spheres as far as to the outermost (*exotatoi*), for the whole of this depth is called the celestial expanse and not only the surface of the greatest of them[37].

In *Hypotyposis Astronomicarum Positionum* (236.15ff) Proclus asserts that the spheres may be mere artifacts of theory:

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[35]. Tr. Baltzly 2013.
[36]. Sambursky 1962, p. 144 citing Ptol. *Syntax. Mathem*. II 7.119.21.
[37]. Tr. Siorvanes 1996, p. 281.

> For what are we to say about the eccentrics they go on about and the epicycles? (…) that they are merely contrivances [objects of thought] or that they also have existence in the spheres in which they are fixed?[38]

This would make them a result of mathematical concepts and not physical entities that exist in nature. Epicycles and eccentrics, moving in opposite directions, further, do not support the ruling principle that all celestial bodies ultimately, move around or toward the centre of the universe (*In Ti.* IV 189.10 Van Riel = III 146.17 Diehl). The perceived anomalies must be accounted for by the rule of central attraction. The result is a spiral-a mean between purely circular and rectilinear movements (*In Ti.* IV 164.5–165.20 Van Riel = III 127.31−128.16 Diehl):

> (…) when the planets make their journey through the heavens, they have their "turnings", for this is what has been said about the planets (39d8) those that make a turn. It is therefore clear that Plato means that the planets themselves [perform] these turnings, that they make their journeys − coming to be closer or further from the Earth or changing in latitude-on their own. It is not the case that they are carried by other things, (…) [counter-rotating] spheres or epicycles. Possessing a single yet varied motion thanks, as it were, to their own single nature, the planets exhibit progression or retrogradation in a helical manner after the configuration of their own orbit in all sorts of way. As a result, the motion that belongs to each one is triple. First there is the one that happens when they are turned as they undergo motion with respect to latitude (*platos*) and depth (*bathos*) in conjunction with each planet being moved around its own centre. Then there is that which take place when each one is led around its individual circle to the left [i.e., west or east]. Then there is the one [from east to west] which takes place due to the fact that the motion of the Same dominates the entire motion of the Different[39].

Proclus's analysis of motions is complex. In Proposition 14 of *Elements of Theology*, he states that "(...) everything is unmoved (*akineton*), intrinsically moved (*autokineton*), or extrinsically moved (*heterokineton)*"[40]. In *Platonic Theology* (I 14), Proclus also provides a multifaceted account as well: motions are *kinoumena monon, kinoumena* and *kinounta, autokineta* and *akineta.* The *akineton* is the prime mover and circular movement is its closest material image. The self-moved (as are the planets), though moving erratically are nevertheless under the command of an unmoved mover[41]. Siorvanes (1996) explains that "while the fixed stars traverse the heavens in one direction only, the planets do so in all six, longitude, forwards and backwards (…) in latitude

[38]. Lloyd 1991, p. 238, Lloyd translated Manutius' text *Hyp.* 236.15ff. The *Hypotyposis Astronomicarum Positionum* with Introduction, Text, Translation and Commentary by Lithari 2025 is now available.
[39]. Tr. Baltzly 2013.
[40]. See Dodds 1963, pp. 16−7.
[41]. See Procl. *El. Theol*. 201 citing *Theol. Plat.* I 14.32ff.

up and down, in depth, closer and further (from earth)" (*In Ti.* IV 190.5 –12 Van Riel = III 147.6−10 Diehl)[42]:

> And this [difference or irregularity] comes about in regular temporal periods when the stars themselves undergo motion around their own centres and also variously undertake journeys through their own spheres. [This happens]in order that they might have motion that is mixed, since they are intermediates between the fixed stars [that have only regular circular motion] and the things [in the sub-lunary realm] that undergo motion in a straight line[43].

Anomalies are due to self-volition on the part of the planetary bodies, their anomalous trajectories, therefore, need no supporting circles to explain their rectilinear motion. Copernicus subsequently employed one of his theorems of combined motions, citing Proclus.

Platonic and Neoplatonic Ontology in Scientific Astronomy

Moritz Schlick stated that for twentieth century scientists it is impossible to speak of a determinate geometry of space without taking into account physics and the behavior of natural bodies[44]. For Proclus the anomalous planar movements of the planets (their behavior) and perfect circular containment (geometry) marked by the circular motion of the fixed stars, are co-present in the celestial region, just as in modern astronomy, geometry and physics are not separate matters. When Proclus resorts to supporting these ideas with conceptual oppositions, like Being and Becoming, Sameness and Difference, however, it is clear that ontology is as powerful an explanatory tool as physics and mathematics. Proclus, at every turn, roots his astronomy in basic Platonic ontology. Plato's Circle of the Same and the Circle of Difference, Not Being and Being, as coexistent, are all brought to bear on the issues in astronomy discussed above. The basic opposition Limited (*Autoperas*) and Unlimited (*Auytoapeiron*), are present immediately beneath the One in Proclus' ontological hierarchy. This opposition is consequential for the binaries which Proclus deploys, for example, The Circles of Same and Other (*In Ti.* IV 157.25–158.1 Van Riel = III 122.11−14 Diehl):

> But since the heavens are as immaterial as possible for sensible things, the opposites are not in conflict with it, nor is there faction among them. Instead they coexist (*synyparchein*) with one another and at the same thing is moved with respect to the two revolutions[45].

[42]. Siorvanes 1996, p. 295.
[43]. Tr. Baltzly 2013.
[44]. Schlick 1929.
[45]. Tr. Baltzly 2009.

Being and Becoming, as well, coexist in the celestial realm. Becoming is operative in anomalous movements: Being is reflected in the perfect circular motion of the fixed stars. As a result of the dominance of being, rectilinear motions will revert and exhibit a spiral path (*In Ti*. IV 103.12–19 Van Riel = III 80.5−13 Diehl):

> The spiral shape is not an empty coincidence but rather fills in the intermediate status between bodies that have rectilinear motion and those that are carried around in a circle. The circle, as has been said [79.14], is only for the fixed stars, while the straight line is for Becoming. The spiral, then, is for the planets, since they have a mixture of both rotation and straightness. Their motions with respect to latitude and proximity to the Earth are proximate causes and paradigms of the motions of things down here – that is of motions upward, downward and along the diagonal[46].

Limit and Unlimited are the ultimate source of opposition that appear in lower hypostases. They are discussed at length in Proclus' texts. As Proclus contemplates the finite and/or infinite possibilities in a universe which is encircled by an immaterial spherical limit wherein beginning and end are one and a space/ time that could unfold in potentially infinite sequences, this pair of opposites are always in play. Limit, in Platonism, guarantees that the danger of potential dissipation into nonbeing through iterative seriality does not occur. Serial infinities, like incommensurable magnitudes, irrational lines, temporality or demiurgic ceaseless productivity are potentially, actual infinities: possibilities which must be brought under the rule of unity. The Circle of the Same and the Circle of the Other, Limit and Unlimited pertain to the fixed stars and the orbits of the planets, respectively[47]. Rectilinear motions of the planets, like infinities, present the prospect of limitlessness, foreign to a Platonic vision of the whole. The simultaneous whole, that is envisioned, bounded only by the fact of circularity wherein end and beginning are identical, forecloses the possibility of a bad infinity of limitless rectilinear unfolding. Sphericity provides a terminal point wherein all seriated potential infinities reach a limit. It is, in the celestial region, a mark of finitude, finite and at the same time unbounded. Limit is the sign that noetic unity is present[48]. Proclus' spherical cosmos is both unbounded and finite (limited) at the same time.

A considerable portion of Proclus' *Commentary on Plato's* Parmenides is devoted to the Unlimited Limited binary. Proclus alludes to the spherical shape in this Commentary quoting Parmenides' fragment on the well-rounded sphere (*In Prm*. VI 1129). What is *un*limited is only what is transcendent in the ultimate sense. Intellect is unlimited (*In Prm*. VI 1120) Eternity is unlimited (*In Prm*. VI 1121). Soul is unlimited. Proclus' astronomy allows that "the celestial sphere

[46]. Tr. Baltzly 2013.
[47]. Kutash 2011, pp. 100−1.
[48]. Morrow – Dillon 1987, p. 470.

(…) through the continuous infinite power of Time, (…) makes the same thing to be beginning and end (…) in respect to its unlimited circular limit. It is an image of noesis (Intellect is unlimited). When Intellect "possesses a life [which it does in the celestial sphere through the presence of soul] the danger of an unlimited dispersion in rectilinear motion, the celestial circular orbits present a remediating Limit to avert a 'bad infinity'. In the *Commentary on the* Parmenides (VI 1122.10−12) Proclus mentions the "circuits of the Soul and its cycle as uniformly completed".

At VII 1162, Proclus engages in apophatic theology (a negative theology which stipulates what the One is only properly spoken about by enumerating what it is not). The One has no shape, no centre or parts and does not undergo circular motion. The "divine Soul", however, is capable of both motion in a straight line and circular motion" (VII 1166.7–8). This is a good example of applied ontology as Proclus' astronomy, rejecting material spheres and peculiar innovations such as epicycles and eccentrics, conceives of a celestial sphere commandeered by the centre, along with a physics which relies on Soul as possessing the powers to preserve "rest" through its relation to noetic unity and to enact motion and change. The fixed stars provide Limit while honoring its source in the unlimited unity of the centre as a placeholder for Intellect.

Theology is never absent from Proclus' priorities. Proclus ability to favor physics over pure geometry implicates the involvement of Soul but Soul is present by *divine* bequest. "Soul" a hypostasis beneath Intellect in the hierarchy of being was given to the cosmos according to Plato (*Ti.* 34b5−9):

> In the centre of the body of the cosmos, the everlasting God, (…) set Soul, which He stretched throughout the whole of it, and therewith He enveloped also the exterior of its body: and as a Circle revolving in a circle established one sole and solitary Heaven […][49].

It is ultimately Soul that imbues the centre of the celestial sphere with its ability to assimilate all varieties of motion to a dominant curvilinear space. The circumnavigation of the fixed stars and ultimately the irregular motions that appear rectilinear upon observation, will fall under the Soul's sway. Divinity is ever present in Proclean Platonism. Plato asserted in *Timaeu*s that the fourth gift of the Demiurge was to give the universe the gift of spherical shape: "it's body; and as a circle revolving in a circle (…) needing no other beside (…) He generated it to be a blessed God (34b17−12)". Plato concludes *Timaeus* proclaiming, "Our cosmos is a visible Living Creature embracing the visible creatures, a perceptible God (*zoon oraton…eikon tou noetou theos aisthetos*…) made in the image of the Intelligible (*tou noetou*) (…)." (*Ti.* 92c6−9). Following Plato's vision it is clear that for Proclus the celestial sphere as a work of divine Creativity. Proclus

[49]. Tr. Bury 1968–89.

mentions Plotinus as a support for his theology by his idea that the demiurge "in some sense (…) is the same as the cosmic intellect"[50]. Aristotle addressed him with the name Zeus, citing *Ennead* III 9 [13], 1.23-27 (*In Ti.* II 147.7–12 Van Riel = I 305.23−28 Diehl)[51]:

> (…) he is also the transcendent Father and Creator, whom he posits as existing in the Intelligible, calling the entire realm between the One and the cosmos intelligible. There in his view the true Heaven and the kingdom of Kronos and the Intellect of Zeus, just as if someone would say that in heaven there is the sphere of Kromos and of Zeus and of Ares[52].

In instances such as these, theological attributions are artfully and curiously interchangeable with scientific and mathematical analyses. E. R. Dodds, commenting on Propositions 128, and 129 of Proclus' *Elements of Theology*, points out that the terms *Theois* and *Theos* were used by the Greeks in all periods in a wide and loose sense. Plato himself spoke of a *Theous Nous* of soul (*Phd.* 95c) and of the soul and the cosmos as gods (*Lg*. I 726; *Ti.* 92c)[53]. It is clear, then, that the Greek theological tradition, enabled Proclus to endorse, with divine sanction, his conviction that the universe could be both finite and unbounded, a vision of the whole that had to await Einstein's conviction that space is curvilinear and that space/time, "does not claim existence on its own but only as a structural quality of the field"[54]. The space/time in Proclus' celestial expanse, was a divine bestowal to a universe through the agency of Soul and the transcendent noetic unity of Intellect allowing the perfect circular motion which could hold the universe in both time and eternity simultaneously.

Some Consequences of Proclus 'Astronomy'

Proclus regarded light as the limit of one of the four elements, the one most prevalent in celestial space, where the four elements earth, air, fire and water, are at their "summits" (*akrotetes*). Siorvanes (1996) explains that matter has many states for Proclus and light is the limiting case of matter as it is for modern physics[55]. Thus in the celestial sphere the continuity of matter expresses itself in gyrations of energic states (*In Ti*. IV 166.12–23 Van Riel = *In Ti.* III 128.29−129.7 Diehl):

[50]. See Procl. *In Ti*. III 410.23–411.1 Van Riel (= II 305.17–20 Diehl).
[51]. Runia – Share 2008, p. 160 note 629 about *In Ti.* III 411.1 Van Riel (= II 305.18 Diehl).
[52]. Tr. Runia – Share 2008.
[53]. Dodds 1963, p. 268.
[54]. Einstein – Infeld 1967, p. 176.
[55]. See Siorvanes 1996, p. 251.

> (…) The spheres [in the heavens] have a substance that is finer and more diaphanous, while the stars are more solid. However fire predominates everywhere and the entire heaven is characterized in accordance with the power of this [element]. The fire up here is not caustic since the [region just} below the Moon (…) contains nothing caustic (…). Neither is [the fire in the heavens] something destructive or opposed to earth. Rather [the celestial gradation of fire] shines with life-generating heat, the power to illuminate and translucency (…) Therefore the fire up there is light (…)[56].

Simplicius' *Commentary on Aristotle's* Physics provides a summary of the late Neoplatonist position. Where the claim is that place is a body that is immobile, indivisible and immaterial (Place should be light), accordingly Proclus says (Simpl. *In Ph*. 612.24−613.1):

> we should envisage two spheres, the one of light, the other consisting of many bodies, equal to each other In bulk. Situate the one about the centre and put the other inside it, and you will see the whole universe within place, moving within that immobile light being itself immobile as a whole, in imitation of place, but having moving parts. In order to be in that way inferior to place [Proclus] confirms his account from Plato, who says light, akin to the rainbow, which is mentioned in the *Republic* [616b] is place[57].

For Einstein the speed of light is the crucial variable which mediates mass and energy. The more general principle under which the idea of conversion of mass to energy and the limiting role of light is the conviction that there is a unified cosmos and terrestrial and celestial regions are not isolated. For Proclus, this means that the material and celestial worlds, though in diminished states of being, still mirror the invisible paradigm. This view is contrary to Ptolemy, who had declared in *Almagest* XIII 2, following Aristotle, that the physics of the terrestrial world did not hold in the superlunary realm and visa verse**.** The principle of Oneness (*homioitaten*, i.e., self-similarity), supports the idea that rest is the truth of motion, the ruling principle of all things celestial and terrestrial**.** Anomalies that appear to depart from continuity, are manifestations of a lower state of being. They seem to depart from continuity and appear to be *anomoitaten* but ultimately the basic principle of continuity will prevail.

Another consequence for Proclus and for modern physics is the primacy of curvilinear space has consequences for planar geometry. Hans Reichenbach recognized that the criticism of Euclid always centered around the parallel postulate[58].

> While all great circles as straight lines share properties with plane straight lines, they are distinct from the latter with respect to the axiom of the parallels. All great circles of the sphere intersect and there are no parallels

[56]. Tr. Baltzly 2013.
[57]. Tr. Urmson in Sorabji 2004, p. 234.
[58]. Reichenbach 1958, pp. 2–3.

> among these straight lines. When the plane then, is meant as the surface of the sphere, Euclidean geometry holds with the exception of the formulation of the axiom of the parallels.

The parallel postulate is a defining characteristic of linear space but fails to be characteristic of a nonlinear space. Like his twentieth century counterparts, Proclus was troubled by the parallel postulate and stated clearly that it "(...) ought to be struck out of the postulates altogether"[59]. So intrigued was he by this postulate that he insisted on the need for proof and made his own attempt to prove it.

A further consequence regarding the universe as unified and all forms of material reality subject to similar underlying principles, supports the curious idea that objects can deform their geometry in accordance with limiting parameters outside themselves. Reichenbach describes rigid bodies as solid bodies which are not affected by differential forces by definition (...) *if universal forces are disregarded*[60]. In a world that bends and shifts everything with time, however, static structures are impossible.

> The concept of rigidity loses its meaning in this world (...) solid bodies (...) no longer define a unique coordinate system as internally at rest, due to differences in their elastic properties. We (...) lose the possibility of defining a geometrical state as rigid in terms of solid bodies, since this definition was based primarily on the uniform behavior of different materials. There is no longer geometry of rigid bodies[61].

Proclus ascribes to the conviction that as the tetrahedral pyramids of sublunary fire approach the heavens their faces become convex by the celestial binding of the curvature of the celestial orbs (*kyrtoutai sphiggomena*). Siorvanes (1996) discusses the fact for Proclus the elements, because they are the images of the perfect spherical figures of self-substantiated and "self-sufficient" entities in the sensible world, namely the celestial objects, "(...) seek to curve in a convex manner in order to fit and 'unite' with heaven (...)"[62]. For Proclus, deforming in order to assimilate to the contours of sphericity is possible. Just as the triangle in Riemannian space can have angles greater than 180, planar forms, deform to assimilate to the contours of sphericity. This idea applies to the rectilinear pathways of the planets. Proclus comments on 36b of *In Timaeus*, where Plato says that the right lines are bent into circles in the formation of the universe, "(...) as the continuous is simultaneous with the discrete, thus also the circular with the rectilinear (...)" (*In Ti.* III 142.3 Van Riel = II 102.5 Diehl). This oddity suggests that rectilinear motions are unnatural, a product of the disarrangement

[59]. *In Eucl.* 191.21−22 tr. Morrow 1970. See also Heath 1956, p. 202.
[60]. Reichenbach 1958, p. 22.
[61]. Reichenbach 1958, p. 264.
[62]. Siorvanes 1996, p. 221 quoting from Proclus' reply to Aristotle's Objection Six (Simpl. *In Cael*. 658.30−2 to 659.1).

which time and motion can bring into the cosmos. Since rest is the ideal and transcendent "truth" of being, geodesic conformity will prevail. Bodies in celestial space will deform in conformity to a limit which is round; rectilinear pathways will deform to the curvilinear. In modern physics, geometrical figures have been acknowledged to have different properties on different surfaces. On a sheet of paper the sum of the three angles of a triangle is equal to two right angles, on the egg, or the sphere it is larger, on a concave surface it would be smaller etc. Proclus was able, entirely on different grounds, to understand the subtleties of non-planar geometry in a way that was unprecedented for his time[63].

Conclusion

Einstein had this to say about "fanatical atheists". They were, he said, "creatures who cannot hear the music of the sphere"[64]. He could not say that about Proclus. Proclus' "heavens" are a simultaneous whole encapsulated by a spherical limit marked by the perfect circular notion of the fixed star. It is not bounded by a material celestial sphere as conceived by Eudoxus, Callippus, and Aristotle. It is not limited either by the internal presence of either eccentrics or epicycles. A noetic totality, the celestial space subsumes all anomalies under the rule of a commanding centre at rest, making them subject to a deforming reversion toward the curvilinear space to which they are ultimately commensurate. Like Einstein, Proclus, found the most elegant solution of his time to the problem of motion. Like Einstein, he wanted to know the mind of God. Both figures resorted to notions of Infinity and panoptic perspectives to solve regional mechanical problems. Both scientist philosophers maintained a vision of the whole from a panoptic view wherein rest allowed, motion, light, and matter to follow a principle of continuity. Proclus, like Einstein, rejected all previous assumptions regarding astronomy. Both eschewed the obvious logic of time and space as it was known before them. For Proclus, the physical world was ultimately ordered by the One beyond Being, manifesting diminishing reflections of its transcendent source. For the celestial world, Proclus' mission was to amend an unholy astronomy and apply his Neoplatonist "logic" in a manner that allowed for scientific innovation. Einstein had stressed that "(...) there *is* no logical path (...) between theory and observations − their concordance is to be understood extra logically"[65]. For Proclus and the Pythagorean/Platonic tradition, this "extra" was theology.

[63]. Kutash 2008, pp. 316−7.
[64]. Fox 2025, p. 45.
[65]. Kutash 2008, pp. 120−1; p. 297; Davies 1992, p. 80 note 2.

Bibliography

E. J. Aiton, *Celestial Spheres and Circles*, in «History of Science», 19 (1981), pp. 75−114.

G. Bachelard, *The New Scientific Spirit*, Boston, Beacon Press, 1984.

D. Baltzly (ed.), *Proclus. Commentary on Plato's* Timaeus, Vol. 3, Book 3, Part 1: *Proclus on the World's Body*, Cambridge, Cambridge University Press, 2007.

D. Baltzly (ed.), *Proclus. Commentary on Plato's* Timaeus, Vol. 4, Book 3, Part 2: *Proclus on the World Soul*, Cambridge, Cambridge University Press, 2009.

D. Baltzly (ed.), *Proclus. Commentary on Plato's* Timaeus, Vol. 5, Book 4: *Proclus on Time and the Stars*, Cambridge, Cambridge University Press, 2013.

M. Born, *Physics in My Generation*, London, Pergamon Press, 1956.

R. G. Bury (ed.), *Plato*, Timaeus, Cambridge, Harvard University Press, 1968–1989.

R. G. Bury (ed.), *Plato*, Laws, 2 vols., Cambridge, W. Heinemann, 1926.

P. Davies, *The Mind of God*, London, Simon & Schuster, 1992.

D. R. Dicks, *Early Greek Astronomy to Aristotle*, Ithaca, Cornell University Press, 1970.

E. R. Dodds (ed.), *Proclus.* The Elements of Theology, Oxford, Clarendon Press, 1963.

A. Einstein, *Relativity: The Special and General Theory*, New York, Crown Publishers, 1961.

A. Einstein – L. Infeld, *The Evolution of Physics*, New York, Simon & Schuster, 1967.

K. Fox, *I Am A Part Of Infinity: The Spiritual Journey of Albert Einstein*, New York, Basic Books, 2025.

Th. Heath (ed.), *Euclid. The Thirteen Books of The* Elements, New York, Dover Publication, 1956, Vol. 1.

Th. Heath, *Aristarchus of Samos*, New York, Cambridge University Press, 1981.

E. Kutash, Oikoumene, Ouranos, Ousia *and the Outside: An Analogy across three Ancient Discipline*, in «Graduate Faculty Philosophy Journal», 22.2 (2001), pp. 115−145.

E. Kutash, *Eternal Time and Temporal Expansion*, in Panayiota Vassilopoulou – Stephen R. L. Clark (eds.), *Late Antique Epistemology*, London, Palgrave Macmillan, 2009, pp. 44−66.

E. Kutash, *Ten Gifts of the Demiurge: Proclus on Plato's* Timaeus, London, Bloomsbury Publishing, 2011.

H. S. Lang, *Inclination, Impetus, and the Last Aristotelian*, in «Archives Internationales d'histoire des sciences», 46 (1996), pp. 221−60.

A. Lithari (ed.), *Proclus*, Hypotyposis Astronomicarum Positionum, Berlin / Boston, De Gruyter, 2025.

G. E. R. Lloyd, *Saving the Appearances*, in Goffrey E. R. Lloyd (ed.), *Methods and Problems in Greek Science: Selected Papers*, Cambridge, Cambridge University Press, 1991, pp. 248−77.

S. F. Mason, *A History of the Sciences*, New York, Collier Books, 1954.

G. R. Morrow (ed.), *Proclus. A Commentary on the First Book of Euclid's* Elements, Princeton, Princeton University Press, 1970.

G. R. Morrow – J. M. Dillon (eds.), *Proclus' Commentary on Plato's* Parmenides, Princeton, Princeton University Press, 1987 (1992^2).

H. Reichenbach, *The Philosophy of Space and Time*, New York, Dover Publications, 1958.

D. T. Runia – M. Share (eds.), *Proclus. Commentary on Plato's* Timaeus, Vol. 2: *Proclus on the Causes of the Cosmos and its Creation*, Cambridge, Cambridge University Press, 2008.

S. Sambursky, *The Physical World of Late Antiquity*, London, Routledge, 1962.

M. Schlick, *Espace et temps dans la Physique contemporaine*, Paris, Gauthier-Villars, 1929.

A.-Ph. Segonds, *Philosophie et Astronomie chez Proclus*, in J. Pépin – H. D. Saffrey (eds.), *Proclus : Lecteur et interprète des Anciens*, Paris, Éditions du CNRS, 1982, pp. 319−34.

L. Siorvanes, *Proclus: Neo-Platonic Philosophy and Science*, New Haven, Yale University Press, 1996.

R. Sorabji, *The Philosophy of the Commentators, 200−600 AD: A Sourcebook*, Ithaca, Cornell University Press, 2004, vol. 2.

B. L. Van Der Waerden, *The Earliest Form of the Epicycle Theory*, in «Journal of the History of Astronomy», 5 (1974), pp. 175−85.

ABSTRACT

Emilie Kutash, *Proclus' "Theological" Spherical Cosmic Whole and His Scientific Innovative Astronomy*

Proclus rejected the traditional celestial spheres of Eudoxus and Aristotle and also rejected Ptolemy's mechanical attempts to model irregular planetary movements using epicycles and eccentrics. Instead, he returned to Plato's emphasis on the invisible world, allowing his science to be governed by his Platonic theology. He envisioned a limited space that everywhere conforms to the cosmic spherical whole, thus requiring all motion to entail a circular trajectory in ultimate reversion to the One Being (a placeholder for Eternal Unity). Anomalies were due to "self-moving motion" away from the center. Crucially, Proclus viewed the planets not as passive bodies, but as self-moving entities with independent trajectories through dimensional space. Their erratic movements were complex, occurring in longitude, latitude, and depth, ultimately resulting in a spiral—a mean between purely circular and rectilinear movements. This unique theory, based on higher theological principles, accounted for the anomalies that troubled other astronomers and resulted in an innovative physical account. Copernicus later employed one of Proclus's theories of combined motions. Proclus even questioned the parallel postulate, which is characteristic of linear space, in a way that suggests analogies to Einstein's curved space and space-time continuum, highlighting his innovative scientific foresight.

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IV

A PHILOLOGICAL PROCLEAN APPENDIX

Mariella Menchelli

DIFFORMITÀ STORICO-TRADIZIONALI IN PROCLO, *COMMENTO ALL'*ALCIBIADE I E *COMMENTO AL* TIMEO TRA IL PERIODO MEDIOBIZANTINO E L'ETÀ DEI PALEOLOGI: GIORGIO PACHIMERE (NAPOLI, BN, III.E.17, *IN ALC.*), GIOVANNI CATRARIO (NAPOLI, BN, III.D.28, *IN TI.*) E LO ΥΠΑΤΟΣ ΤΩΝ ΦΙΛΟΣΟΦΩΝ DI MICHELE PSELLO (PATMOS EILETON 897)

All'interno della produzione di Proclo, il *Commento all'*Alcibiade I e il *Commento al* Timeo conoscono differenze significative nella loro trasmissione e ricezione.

Soltanto all'ultimo scorcio del XIII secolo, grazie alla raccolta libraria Farnese, imponente collezione della Biblioteca Nazionale di Napoli, si deve il più antico, ampio, testimone primario del *Commento all'*Alcibiade I nella sua parte conservata: il codice in questione, il BNN III.E.17 (N) di Giorgio Pachimere, è in parte altresì *codex unicus* per il testo dell'opera, pervenutaci mutila di circa la sua seconda metà[1].

Al contrario nel caso del commento di Proclo al *Timeo*, possediamo brevi lacerti già nella *scriptio inferior* del codice Paris, BnF, suppl. gr. 921, contenenti brani dei libri IV e V, nella mano del copista I della "collezione filosofica"[2], e alla medesima linea tradizionale appartiene il codice Città del Vaticano, BAV, Chigi R.VIII.58, della prima metà del XII secolo (libri I-V)[3], al quale risultano a loro volta legati il rotolo di Patmos, Monastero di San Giovanni il Teologo, Eileton 897, dell'XI secolo, con parti del terzo libro, e il codice di Napoli, Biblioteca Nazionale III.D.28, sottoscritto e datato all'anno 1314 da Giovanni Catrario[4], con i libri I e II del commento[5],

[1]. Sui libri della collezione Farnese cf. Formentin 2008, Formentin 2015. Su Giorgio Pachimere e i suoi autografi cf. Harlfinger 1996; sulla sua attività di notaio patriarcale, di professore e copista cf. soprattutto Golitsis 2008, Golitsis 2010, Golitsis 2020, anche in relazione al commento procliano all'*Alcibiade I*. Per il commento e la perdita cf. Segonds 1985 p. XXXIX: "Tel qu'il nous est parvenu, le *Commentaire de l'Alcibiade* est gravement mutilé : le texte de Proclus s'arrête en plein mileiu du commentaire d'*Alcibiade*, 116 A3 - B1" e p. XL: "il nous manque un peu moins de la moitié du *Commentaire* de Proclus. (…) Nous formulons l'hypothèse que le commentaire de Proclus était divisé en deux tomes et que nous n'avons conservé que le premier tome mutilé à la fin". Per il codice N come testimone cf. Segonds 1985, p. CVII, pp. CXI–CXVIII; come stabilito da Segonds 1985, esiste per le pagine 1–72.7 del commento un'altra linea tradizionale, indipendente da N, nei codici Venezia, Biblioteca Nazionale Marciana, Marc. gr. 190 (M) e Firenze, Biblioteca Medicea Laurenziana, Plut. 85.8 (R): cf. Segonds 1985, pp. CXVIII–CXX. Il capostipite comune a N-MR era in minuscola: Segonds 1985, p. CXX.

[2]. L'attribuzione alla "collezione filosofica" si deve a Diller 1954. Sul codice cf. soprattutto Mondrain 2008; per lo studio dei frammenti cf. ora Van Riel 2022, p. XV (*siglum* W). Sulla "collezione" mi limito a rimandare ad alcuni contributi recenti, con bibliografia pregressa: cf. in particolare, Marcotte 2014; Cavallo 2017; Bianconi – Ronconi 2020.

[3]. Cf. Van Riel 2022, p. XIX, per l'importante osservazione della possibile discendenza del codice H, Vat. Chis. R VIII 58, dal codice della "collezione filosofica", W nella edizione critica.

[4]. Su Catrario è possibile rimandare in primo luogo a Bianconi 2006, con bibliografia.

[5]. Su tale nodo tradizionale cf. Van Riel 2022.

proveniente dal monastero di San Giovanni a Carbonara[6]. Una linea tradizionale che si inserisce per altro in un quadro quanto mai ricco e articolato[7].

La continuità di ricezione e lettura del commento procliano al *Timeo*, che le fonti sembrano attestare, può in effetti essere posta a confronto con il recupero testuale del *Commento all'*Alcibiade I, ancora una volta di particolare significato[8]: ad una riproduzione/trascrizione che appare meno consistente nel numero dei testimoni primari, se si guarda alle testimonianze superstiti, per il commento procliano all'*Alcibiade*, corrisponde per il *Commento al* Timeo una pluralità di fonti, talora ricche del materiale esegetico scientifico-matematico presente negli *scholia vetera*, e, come vorrei mostrare, una pluralità di forme del libro e della trasmissione/ricezione, che appare legata, nel periodo mediobizantino, alla celebre figura del professore "console dei filosofi" Michele Psello[9]. Alle forme della lettura del Proclo commentatore, con Psello e nei due esemplari dell'età paleologa sopra citati, sono dedicate queste brevi note, al fine di esaminare modalità e strategie editoriali di due celebri filosofi bizantini quali Psello e Pachimere, peraltro in relazione tra loro anche sotto il profilo storico-tradizionale, tra XI secolo ed età paleologa[10].

Il Farnesianus *BNN III.E.17. "Fortune" della tradizione per il commento procliano all'*Alcibiade *I di Platone*

A più riprese commentato nell'antichità, l'*Alcibiade I* conosce, come si è visto, per il suo commento procliano una tradizione mutilata di un'ampia sezione del testo originario[11].

[6]. In questo monastero approdarono, tramite Antonio e Girolamo Seripando, i libri di Aulo Giano Parrasio. Per il codice N cf. anche Van Riel 2022: "The subscription in N is followed by a later owner's note: Antonii Seripandi ex Iani Parrhasii testamento The manuscript was thus part of the heritage of the humanist Giano Parrasio (1470-1534ca.) bequeathed to Antonio Seripandi. Via the latter's brother Girolamo Seripandi it became part of the collection of the monastery pf s. Giovanni in Carbonara". Sui libri parrasiani e la loro provenienza cf. di recente anche Vendruscolo 2018.

[7]. Cf. infra, p. 000.

[8]. Primo nell'ordine di lettura della serie dei 10 dialoghi nella scuola neoplatonica. D'altro canto, almeno il commento di Olimpiodoro figurava nella stessa "collezione filosofica" (Venezia, Biblioteca Nazionale Marciana, Marc. gr. 196). Sulla sua importanza in relazione al commento procliano cf. Segonds 1985, pp. LXIX–CIV, per quanto cf. Segonds 1985, p. LXXXVIII osservi: "Une comparaison suivie avec le commentaire de Proclus montre qu'Olympiodore n'en fournit qu'un très pâle reflet et que, par conséquent, pour les parties perdues du commentaire de Proclus, on ne peut même pas se faire une idée de ce que contenait son texte".

[9]. Cf. infra 000. La continuità è manifesta anche grazie alla copia del *Chisianus*, di età Comnena, prima della rinascita paleologa e della pluralità di testimoni in essa prodotti. Sulla figura di Psello in relazione al commento di Proclo al *Timeo* cf. di recente Menchelli 2024.

[10]. A tale proposito cf. in particolare, Golitsis 2007, per possibili "incidenti" della tradizione nei quali sono coinvolti, quali l'attribuzione di un commentario di Pachimere a Michele Psello. Per la fortuna delle opere di Psello in età paleologa cf. soprattutto Pérez Martin 2013 e 2014.

[11]. Cf. ancora Segonds 1985.

Non solo. Se per una parte del testo la tradizione può contare su una seconda linea tradizionale, per la restante il codice BNN III.E.17, che reca il commento ai ff. 1–98v[12], è, si è detto, *codex unicus*, testimone primario e solo testimone superstite indipendente[13].

Il copista principale del Neap. III.E.17, il notaio patriarcale nonché, filosofo, storiografo, professore celeberrimo Giorgio Pachimere, risulta affiancato di seguito dal suo allievo Niceforo, ora identificato da Pantelis Golitsis con Niceforo Callisto Xanthopoulos[14], e da un altro scriba anonimo, l'"anonimo x" collaboratore di Pachimere per esempio nel codice di Firenze, Biblioteca Medicea Laurenziana, Plut. 87.5 e nel codice di Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. gr. 261, sempre dunque della prima età dei Paleologi[15].

Nel *Neapolitanus*, della collezione Farnese, alla metà del secolo XVI Giovanni Onorio da Maglie ha operato alcuni interventi di restauro e apposto titolature[16].

Platone e Proclo

Il prezioso *Farnesianus* non si limita nel contenuto al commento di Proclo ma contiene di seguito alcuni dialoghi platonici, in primo luogo il *Fedone*, ai ff. 99–136, seguito dal *Carmide* (ff. 136v–148) e dal *Lachete* (ff. 149–159v)[17].

L'interesse di Pachimere per Platone è d'altro canto confermato dalla presenza della sua mano in un altro codice platonico, il Paris, BnF, grec 1810, testimone primario del commento procliano al *Parmenide* e del commento di Ermia al *Fedro*, nonché latore di alcuni dialoghi. Oltre agli interventi sui lemmi del *Commento al* Parmenide, avvenuti con l'ausilio del codice di Venezia, Biblioteca Nazionale Marciana, Marc. gr. 185 (D), il codice reca una versione "continuata" del commento procliano legata a Pachimere stesso[18].

Nel testo dei dialoghi platonici, trascritti in calce, il codice di Napoli, BN III.E.17 risale al Paris, BnF, grec 1813[19], del secolo XII, contenente Platone e Proclo, annotato da una mano

[12]. Cf. Formentin 2015.
[13]. Cf. Segonds 1985 per la trattazione.
[14]. Cf. Golitsis 2020 per la discussione.
[15]. Cf. Formentin 2015, p. 187. Sulle mani del codice cf. soprattutto Golitsis 2010.
[16]. Cf. ancora Formentin 2015, p. 187.
[17]. Il *Fedone* in particolare è corredato di una lunga nota marginale che richiederebbe studi ulteriori.
[18]. Nel caso del *Commento al* Parmenide il lavoro sul testo è stato dunque più impegnativo e articolato. Cf. Steel 2007–2009; Luna – Segonds 2007; Luna 2019. Per quanto i suoi studi platonici si inseriscano nel contesto dello studio dedicato ad Aristotele, Platone e il neoplatonismo rappresentano per Pachimere un lascito filosofico di grande significato.
[19]. Per la dipendenza del Neap. III.E.17 dal Par. gr. 1813 nei dialoghi platonici cf. in particolare, Murphy 1990 e Murphy 1994, in particolare pp. 10–1. Il *Neapolitanus* ne è una copia diretta: cf. Murphy 1990, p. 10.

altrimenti attiva nella tradizione dello scolarca ateniese di V secolo giacché responsabile di uno strato di *scholia* nel già citato BAV Chigi R VIII 58, secondo l'identificazione di Gerd Van Riel[20].

Per la sezione del commento procliano, per il quale, come si è detto, è testimone primario, un apografo del *Neapolitanus* è inoltre conservato ora presso la Biblioteca Apostolica Vaticana: il Vat. gr. 1032 (D) reca memoria della sua relazione con l'antigrafo, che si manifesta nel rapporto discepolo-maestro, grazie ad una nota a margine discussa da Segonds nella sua introduzione all'edizione[21].

La presenza della mano di Niceforo, ora Niceforo Callisto Xanthopoulos, nei codici di contenuto filosofico legati a Pachimere evoca tutta una serie di studi e relazioni, esplorati da Pantelis Golitsis tra le opere autografe di Pachimere, *in primis* la *Philosophia* (in Berlin, Hamilton 512 e Paris, BnF, grec 1930), preziosa sinossi del corpus aristotelico, e gli autografi dei suoi commenti, ancora una volta raccolti con la collaborazione di allievi e copisti[22].

Pachimere lettore del commento. Qualche osservazione supplementare

Nel Neap. III.E.17, la strategia di copia prevede che i lunghi lemmi del commento vengano trascritti da Pachimere in una grafia di modulo leggermente ingrandito, con interlineo più arioso e su area di scrittura ridotta in rapporto alla piena pagina del commento.

Il dato testuale più significativo nei lemmi è, come è stato osservato, la loro sostituzione con lemmi tratti da un manoscritto platonico, con una operazione di ri-assemblaggio di testo e commento: il manoscritto platonico al quale si attinge per i lemmi appartiene alla terza famiglia della tradizione manoscritta di Platone, le varianti ci riconducono in ultima analisi al codice Wien, Österreichische Nationalbibliothek, Suppl. gr. 7 (*siglum* W), dell'XI secolo[23].

Nella divisione della tradizione del Platone del primo tomo tra il ramo fondamentale del codice B, Oxford, Bodleian Library, Clark. 39, trascritto da Giovanni il calligrafo nell'895 per Areta di Cesarea, e i due rami TW (T = Biblioteca Nazionale Marciana, Append. Cl. IV. 1, trascritto dal copista Efrem alla metà del X secolo), sovente legati, mentre l'esegesi procliana (e dunque

[20]. Van Riel 2019.
[21]. Cf. Segonds 1985.
[22]. Cf. soprattutto Golitsis 2008, con sinossi delle opere di Pachimere, e Golitsis 2010, con ulteriore bibliografia. Per il *Quadrivium*, per esempio, il testimone di riferimento è, come è noto, un codice ora a Roma, Biblioteca Angelica, 38. I manoscritti attribuiscono a Michele Psello, almeno in parte della tradizione, un commento invece da restituire a Giorgio Pachimere, cf. Golitsis 2007.
[23]. Cf. Carlini 1961. Al contrario il commento di Proclo presuppone lezioni dell'altro ramo della tradizione, rappresentato soprattutto dal codice B di Platone, Oxford, Bodleian Library, Clark. 39; un'analisi del codice nel suo rapporto con il tardoantico si deve a Luzzatto 2010.

relativa al modello letto da Proclo nel V secolo) è legata alla linea tradizionale di B, i lemmi che ora risultano conservati appartengono ad altra linea tradizionale, in particolare, si è detto, quella di W[24].

Già se si considerano i primi lemmi del commento gli apparati critici platonici registrano per esempio una variante significativa tra testo e commento nel terzo lemma.

> 1. Ὦ παῖ Κλεινίου, οἶμαί σε θαυμάζειν ὅτι πρῶτος ἐραστής σου γενόμενος τῶν ἄλλων πεπαυμένων μόνος οὐκ ἀπαλλάττομαι (*In Alc*. 18.14);
>
> 2. καὶ ὅτι οἱ μὲν ἄλλοι δι᾽ὄχλου ἐγένοντό σοι διαλεγόμενοι, ἐγὼ δὲ τοσούτων ἐτῶν οὐδὲ προσεῖπον. (*In Alc*. 53.18);
>
> 3. Τούτου δὲ τὸ αἴτιον γέγονεν οὐκ ἀνθρώπειον, ἀλλά τι δαιμόνιον ἐναντίωμα, οὗ σὺ τὴν δύναμιν καὶ ὕστερον πεύσῃ. (*In Alc*. 59.23).

Infatti, in quest'ultimo caso in particolare per 59.23, su *Alc. I* 103a5, la tradizione platonica è divisa con divaricazione antica[25]. Nel commento procliano ricorre precisamente la lezione ἀνθρώπειον di B, comune anche a Olimpiodoro, in 73.15 Segonds (p. 59 della edizione critica), e il lemma, in 59.23, legge la lezione attestata dall'altra linea tradizionale platonica, ovvero ἀνθρώπινον TW[26].

Dunque, anche in questo passo il lemma procliano ha la lezione propria di TW[27].

L'importanza aggiuntiva del passo risiede tuttavia nel fatto che trovandoci nella parte iniziale del commento, attestata anche in MR, potremmo chiederci se tale operazione sia in realtà attestata non solo dal codice autografo di Pachimere, N (= Neap. BN III.E.17), ma anche dagli altri codici indipendenti da esso, M e R, del commento (segnalati da Segonds per i §§ 1–72, nei quali si collocano i primi tre lemmi e la loro discussione).

Al momento è stato possibile vagliare in particolare la testimonianza del Laur. Plut. 85.8: la lezione ἀνθρώπινον propria di TW compare in effetti anche nel medesimo codice Laurenziano Plut. 85.8 (che insieme al Marc. gr. 190 risulta indipendente da N). Ciò rende non solo necessario postulare un antigrafo comune alle due linee tradizionali (di N e di MR), antigrafo che di fatto Segonds ipotizza già in minuscola, ma rilevare già in tale antigrafo gli interventi sui lemmi che presuppongono il ricorso al testo platonico (di W).

Il lemma, affine a TW, risalirebbe dunque non solo al codice N di Pachimere ma ad un antigrafo comune a Pachimere e al ramo rappresentato dal codice Laurenziano (e Marciano)[28].

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[24]. La divisione della tradizione è senza dubbio già tardoantica, come attesta la tradizione indiretta. Tra le altre testimonianze per B può essere vagliata in aggiunta anche quella di Prisciano, per il quale cf. di recente Menchelli 2014.

[25]. Già l'apparato di Burnet del 1901 per esempio registrava ἀνθρώπειον B Olympiodorus: ἀνθρώπινον TW: Burnet 1901, p. 297. L'edizione di riferimento per l'*Alcibiade I* (e la quarta tetralogia) è Carlini 1964.

[26]. Cf. anche Carlini 1961, p. 181, con rimando all'edizione di Westerink 1954. Nella edizione Segonds 1985 i lemmi sono presenti soltanto con le parole iniziali e finali.

[27]. Per gli altri passi cf. la discussione in Carlini 1961.

Se è possibile che gli interventi sui lemmi si debbano a Pachimere, sembrerebbe in ogni caso necessario postulare un modello perduto del *Neapolitanus*, dunque forse di Pachimere, al quale risalirebbero anche i due codici M e R, un modello che può essere appartenuto al medesimo *entourage* del BN III.E.17 (sia il Marc. gr. 190 sia il Laur. Plut. 85.8 sono per altro posteriori, come si è visto), o comunque essere in ogni caso anteriore ad N[29].

Ri-assemblare il testo. Esempi mediobizantini tra i Philosophica minora *di Michele Psello*

L'operazione di "sostituzione" dei lemmi che è stata compiuta per esempio sul commento procliano all'*Alcibiade I* non è per altro nuova a Bisanzio.

Michele Psello, in *Philosophica minora* II 4, II 5 e II 6 O'Meara[30], nel commentare brani del *Timeo*, in tali brevi opere attingeva per il brano platonico alla tradizione di Platone, trascurando in questo modo i brevi (talora) lemmi già presenti nel commento procliano al dialogo, utilizzato invece per l'esegesi vera e propria dei brani nuovamente escerpiti da Platone[31].

Il medesimo fenomeno di ricorso diretto al testo di Platone ricorre in un'altra opera di Psello, in questo caso con protagonista il *Fedro*. Si tratta di *Philosophica minora* II 7 O'Meara, ovvero della Ἐξήγησις τῆς Πλατονικῆς ἐν τῷ Φαίδρῳ διφρείας τῶν ψυχῶν καὶ στρατείας τῶν θεῶν, una esegesi di *Phdr*. 246e–247a[32]. Le varianti delle due citazioni del dialogo platonico "incorporate nello scritto", analizzate da Carlini, rimandano per il *Fedro* alla linea tradizionale del Vind. Suppl. gr. 7 (W) di Platone: "nell'ambiente di Psello era conosciuta la tradizione di W"[33].

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[28]. Giacché resta da vagliare la testimonianza del Marc. gr. 190.

[29]. Una volta stabilita l'indipendenza dei due codici M ed R, non è possibile limitare la discussione sui lemmi al solo codice N di Pachimere. In questo senso la questione resta aperta.

[30]. Cf. O'Meara 1989.

[31]. Al tempo stesso una operazione complessa di ri-costruzione del commento attingendo al testo platonico venne compiuta da Pachimere anche per il commento di Proclo al *Parmenide* nel codice BnF Par. gr. 1810, cf. Moreschini 1964.

[32]. Come rilevato da Moreschini 2009, la ripresa da parte di Psello di Herm. *In Phdr*. 135.26-136.1 Couvreur (= 141.23–30 Lucarini – Moreschini) non è letterale: "in realtà, lo scrittore bizantino fornisce, in un primo momento, l'interpretazione del passo platonico, e poi, seguendo "i teologi dei Greci", cioè pagani (12.22), dà una parafrasi di tutta la sezione teologica del secondo libro degli *scholia* di Ermia, ricordando brevemente la sua dottrina della gerarchia degli dèi, dei principi divini, la distinzione tra sostanza e ἰδέα, le δυνάμεις dell'anima e le funzioni degli dèi, come quella di "elevare", propria di Zeus, di "collocare", propria di Hestia, di 'produrre' propria di Hera e delle ζωογόνοι θεαί (13.19–22). Ma questo non basta: dopo aver interpretato il passo platonico, Psello afferma che prima bisogna spiegare (14.4 ss.) quale sia l'idea dell'anima. Noi affermiamo, dunque, che sostanza di ogni cosa è l'uno che in esso si trova e, per così dire, l'uno più unitario di tutti, mentre la forma è la molteplicità e, per così dire, gli elementi: l'anima, infatti, è uno e molteplice, e l'idea dell'anima sono la molteplicità e gli elementi. Questo è il significato dei cavalli e dell'auriga, di cui parla Platone (14.5 ss.). Psello prosegue riprendendo fedelmente la spiegazione di Ermia (121.5–123.3 Couvreur = 126.10–128.8 Lucarini – Moreschini e 130.11–15 Couvreur = 136.10–14 Lucarini – Moreschini). Ma conclude, aderendo in modo tradizionale alla fede cristiana: "Abbiamo spiegato in modo platonico le dottrine platoniche, il che equivale a dire che abbiamo spiegato in modo ridicolo dottrine ridicole": cf. Moreschini 2009.

[33]. Carlini 1972, pp. 172–3 ("che poteva essere rappresentata anche dal capostipite di W").

Il codice W è stato variamente datato alla prima metà oppure alla seconda metà del secolo XI, ma una datazione alta, non incompatibile con Psello, appare preferibile.

Nel caso del *Timeo* sopra evocato, il codice W contiene il dialogo soltanto nella sezione trascritta dalla seconda mano, W^2, dell'ambiente del patriarca Gregorio di Cipro[34].

La linea tradizionale del *Timeo* letta da Psello, per esempio nelle brevi porzioni di testo di *Philosophica minora* II 5, II 6 O'Meara, è per noi perduta nel suo codice mediobizantino: essa corrisponde alla "fonte g" Jonkers di tre codici di età paleologa[35], il Paris, BnF, grec 2998 di Gregorio di Cipro, il Wien, Österreichische Nationalbibliothek, phil. gr. 21 di Massimo Planude e Niceforo Moscopulo, il Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. gr. 225–226 di Matteo di Efeso. Per il periodo medio-bizantino tale "fonte g" è vicina al codice Tubingensis Mb 14 (*siglum* C), in elegante *Perlschrift*, ma ne resta distinta.

Lo stesso commento procliano al *Timeo* conosce una tradizione complessa.

Il commento di Proclo al Timeo *e il Neap. BN III.D.28*

Per il commento di Proclo al *Timeo* platonico la Biblioteca Nazionale di Napoli restituisce, come si è detto, un altro testimone di rilievo con il *Neapolitanus* BN III.D.28, nuovamente un prodotto della rinascita paleologa, risalente, come si è riportato, alla fonte φ del *Chisianus* R VIII 58 e del *Patmiacus* Eileton 897.

Un aspetto rilevante è la conservazione nel codice Neapolitanus del trattato di Timeo di Locri, *De natura mundi et animae*, che nel suo proemio Proclo affermava di volere in apertura, prima del proprio *Commento al Timeo*[36].

Testo e scholia

Lo stesso corredo di scolii presente nel codice N mostra per altro la sua appartenenza, così come per il *Chisianus* e il *Patmiacus*, ad un filone tradizionale che già nell'edizione Diehl riceveva ampia trattazione, quello del codice di Venezia, Biblioteca Nazionale Marciana, Marc. gr. 195 (M), del secolo XIV, testimone primario di massimo rilievo del testo del commento, ritenuto da Diehl

[34]. Su Gregorio di Cipro e il codice W di Platone cf. soprattutto Pérez Martín 2005, con bibliografia.
[35]. Per la ricostruzione di tale nodo tradizionale cf. Jonkers 1989. Per la corrispondenza con le varianti di Psello cf. Menchelli 2016.
[36]. Su questo aspetto della storia tradizione e il suo significato cf. Menchelli 2019, Van Riel 2022.

copia di un codice di IX secolo, in minuscola antica, e che è al tempo stesso il testimone più completo degli *scholia vetera* a Proclo

Nel terzo libro il Marc. gr. 195 viene meno a 247 Diehl, così come mutilo risulta, arrivando proprio fino a gran parte del medesimo libro, l'ultimo testimone primario, il codice Paris, BnF, grec 1840 (P), di Demetrio Mosco, del tutto privo di scolii[37].

Il testo del *Commento al Timeo* resta di solida tradizione anche nei lemmi, che sono rimasti quelli originari, come ha mostrato lo stesso Diehl[38].

Giovanni Catrario, la copia congiunta di dialogo e commento, il ramo φ

Eventuali possibile interferenze con la tradizione di Platone restano pertanto nel caso del *Commento al Timeo* soltanto sporadiche, per quanto agli stessi copisti e filologi si debba in più di un caso la copia del dialogo e la copia del commento ad esso dedicato: così avviene nel caso del Neap. BN III.D.28 di Catrario, che collabora per il *Timeo* alla trascrizione del codice platonico di Firenze, Biblioteca Medicea Laurenziana, Plut. 80.19 e nel caso del manoscritto di Venezia, Biblioteca Nazionale Marciana, Marc. gr. 194 di Gregorio di Cipro, che trascrive il *Timeo* nel suo codice Paris, BnF, grec 2998; o ancora nel caso del Paris, BnF Coisl. 322, in arcaizzante, della mano del copista di Vat. gr. 225-226 di Matteo di Efeso.

Oggetto di valutazione è stata anche la attività congetturale di Catrario, valutabile soprattutto grazie al Chisianus R VIII 58, "fratello" del *Neapolitanus* nei libri I e II, mentre nel libro III il *Chisianus* risulta apparentato con Eileton 897.

L'Epitome Patmos Eileton 897: interventi d'autore e redazione autografa

La ricca circolazione del commento di Proclo al *Timeo* alla quale si è fatto riferimento, ininterrotta a Bisanzio se si considerano testimoni e copie superstiti, comporta esiti inaspettati. La biblioteca del monastero di San Giovanni il Teologo, nell'isola di Patmos, conserva, come è ormai noto, un manufatto in forma di rotolo, cartaceo, scritto *transversa charta*, dell'XI secolo, recante ampie parti del libro terzo del commento. Un testimone d'eccezione, che è stato ora utilizzato nella recente edizione di Gerd Van Riel (con il *siglum* Y = Eileton 897)[39].

[37]. Per i testimoni primari citati cf. ora Van Riel 2022.
[38]. Cf. già Diehl 1899.
[39]. Van Riel 2022.

Un rotolo lungo oltre sette metri, opistografo, dotato, come si è detto, di un apparato scoliastico analogo a quello dei manoscritti del commento medesimo di Proclo in forma di codice, ma incluso in *fenestrae* che si aprono nel corso della stesura dei brani principali[40].

Notevole naturalmente il fatto che il contenuto rimandi ad un "autore classico", quando la forma rotolo, negli esemplari conservati, se pergamenaceo ci riconduce soprattutto al rotolo liturgico, oppure, soprattutto per il cartaceo, alla sfera dei documenti[41].

Al tempo stesso il testo che il rotolo reca è di fatto un testo "nuovo", d'autore, frutto degli interventi dell'escertore: nella ricerca prosopografica si è suggerita una relazione con la personalità di Michele Psello sulla base di una variante testuale significativa, e anche del confronto con gli scritti pselliani e con le modalità di lavoro dello stesso Psello, attestate in particolare da uno scritto giuridico del *corpus psellianum*[42].

Alcuni dati

Per un breve riepilogo delle caratteristiche bibliologiche del rotolo di Patmos può bastare richiamare l'organizzazione del testo su 19 fogli, di 40 x 25, uniti da *kolleseis*, diseguale soltanto l'ultimo del testo (attuale terzultimo)[43].

Il rotolo è stato infatti in parte "ricomposto". I due fogli un tempo finali sono molto deteriorati, un deterioramento legato alla loro posizione alla fine del rotolo, posizione nella quale devono essere rimasti a lungo. Sono ora diventati il quartultimo e terzultimo foglio sul *recto* e in maniera corrispondente il terzo e quarto sul *verso*[44].

Sul *recto* del rotolo compare il lungo passo sul corpo del mondo (mutilo all'inizio e alla fine), inframmezzato di scolii copiosi e corposi, mentre sul *verso* l'Anonimo patmiaco trascrive il lungo passo con la trattazione sull'anima del mondo (anch'esso mutilo all'inizio e alla fine)[45].

[40]. Gli ampi brani del testo di Proclo includono appunto *scholia vetera* inframmezzati al testo, secondo una organizzazione del materiale che appare funzionale allo studio/lezione sul *Timeo* con il suo commento più significativo.
[41]. I dati codicologici sono incontrovertibili. Dal punto di vista del contenuto si tratterebbe del solo esemplare superstite scritto su rotolo *transversa charta* per un autore classico, come si è sottolineato più volte.
[42]. Moore 1030, cf. Moore 2005 per la situazione storico-tradizionale. Cf., inoltre, Menchelli 2024.
[43]. Per la descrizione bibliologica mi permetto di rimandare a Menchelli 2016. Cf. ora anche Van Riel 2022: "Patmos Eileton 897 chart 7.747 x 250 mm, ff. 19, in volumine. This exceptional paper scroll was discovered by A. Kominis in the 1980s during his preparation of a new catalogue of the Library of the Monastery of Saint John in Patmos (Kominis 1988). It is a primary witness, covering on its front and back side, two large parts of the third book of the Timaeus commentary".
[44]. Cf. Menchelli 2016.
[45]. Un dato "anomalo" che è forse possibile rimarcare è la caratteristica delle *kolleseis*, che talvolta presuppongono la copia su fogli sciolti perché contengono testo incluso, mentre altra volta presuppongono che il testo sia stato copiato dopo l'unione dei fogli perché la scrittura si sovrappone alla *kollesis*, un risultato legato alla modalità di lavoro di chi

La scrittura di Eileton 897 può essere assegnata alla metà del secolo XI. Il confronto con la seconda mano del Vat. gr. 65 (anno 1063), copiato dal notaio imperiale Teodoro, è parso significativo[46]. Lo stesso materiale scrittorio orienta verso figure legate alla cancelleria imperiale, nella quale l'uso della carta risulta piuttosto precoce: un esempio datato risale al 1052[47].

Appare possibile altresì chiamare a confronto alcune personalità legate al mondo dell'erudizione e dell'istruzione, come Isaia, segretario di Giovanni Mauropode, a sua volta maestro di Michele Psello, che ha sottoscritto un carme al termine della raccolta del maestro, redatta in scrittura professionale nel Vat. gr. 676 di Mauropode[48].

La scrittura dell'Anonimo di Patmos ci riconduce dunque a scritture erudite altrimenti celebri e legate a personaggi contemporanei alla attività di Psello[49].

Ancora sulle carattersitiche dell'Epitome. Che cosa è il testo dell'Anonimo Patmicaco: un testo "ricomposto" e la propensione alla congettura dell'Anonimo patmiaco

Il testo fornito dal Patmiacus è affetto da omissioni di diversa estensione, come veniva rilevato già da Linos Benakis[50]: Gerd Van Riel segnala altresì nell'introduzione la presenza di aggiunte, e nell'apparato critico sono visibili gli interventi operati nei punti di raccordo[51].

In effetti di un'opera nuova[52]; l'opera è molto aderente al testo di Proclo, ma al tempo stesso il testo procliano è stato "frantumato" ed è stato raccolto in una serie di frammenti lunghi e brevi

scrive, che sembra dunque seguire scelte estemporanee, comportandosi a seconda delle esigenze del momento, senza sistematicità.

[46]. Per il confronto con esemplari su codice e con documenti datati, cf. ancora Menchelli 2016.

[47]. Si tratta di un documento di Costantino IX Monomaco, cf. già Menchelli 2016.

[48]. Sul codice cf. Bianconi 2011. Più eleborata è la scrittura di Niceta di Serre, attivo nel codice di Venezia, Biblioteca Nazionale Marciana, Marc. gr. 476 (Arato e Licofrone).

[49]. Al quale al momento a mia conoscenza non sono stati attribuiti manufatti o interventi su codici (anche se il manoscritto più antico delle sue opere non è molto lontano nel tempo, il Laur. Plut. 57.40; posteriore, di XII secolo è il Par. gr. 1182, posto in relazione con Eustazio di Tessalonica), se si esclude il fatto che il problema si pone proprio per questo esemplare, per l'Anonimo Patmiaco. Sicura appare in ogni caso l'appartenenza al secolo XI (e non al XII e a Eustazio come era stato suggerito da Kominis e di seguito da Benakis).

[50]. Cf. Benakis 1987.

[51]. Cf. Van Riel 2022: "The manuscript displays many individual errors and shorter and longer omissions that are not shared by any other primary witness. The omissions proper to Y are often incidental but frequently enough they are the result of a deliberate and drastic shortening of the comment to a lemma, leaving out the development of a final point, and jumping straight to the new lemma".

[52]. Così Benakis 1987, di seguito a Kominis. Sotto l'aspetto del contenuto il risultato dell'operazione di raccolta è una Epitome, di fatto un "nuovo" testo: si tratta di un'operazione d'autore di epoca bizantina, giacché il testo contenuto nel rotolo è di fatto un testimone importante del Commento, e il testimone più antico per le due ampie sezioni del libro III trascritte, ma, come è stato osservato, il testo non è "continuo", e si tratta di una operazione di estrazione dal Commento di Proclo, con passi uniti da innovazioni nei raccordi testuali.

ricomposti a formare un nuovo testo, epitomato, più "snello" rispetto alla trattazione originale del Commento stesso[53].

Un testo che sembra possibile porre in relazione con Michele Psello per la modalità stessa di costruzione, comparabile con opere di Psello risultato di operazioni di raccolta e presentato naturalmente come d'autore da Psello stesso (così II 5, II 6): in questo senso soccorre anche la testimonianza di Psello in Moore 1030, in cui viene vividamente rappresentata l'attività del professore console dei filosofi[54].

In presenza di un testo "nuovo" sembra essere possibile dunque suggerire una operazione d'autore, e alcune osservazioni di carattere filologico inducono a porre il testo in relazione con Michele Psello[55].

Si è più volte discusso del termine *epistrophe* e dell'aggettivo *noeros* ad essa riferito perché è proprio su questa base che si fonda la relazione stabilita con Michele Psello. Van Riel riporta in apparato la lezione di Psello in II.5. La buona congettura di Psello ricorre nel Patmiaco, come risulta dallo stesso apparato critico[56].

Riprendo brevemente i termini della questione. Il passo di Procl. *In Ti.* III 335 Van Riel (= II 247 Diehl), presenta una descrizione del simbolo dell'anima del mondo, descrizione riferita da Porfirio come propria degli Egizi, nella forma della lettera *chi* all'interno di un cerchio:

> αὐτὸ δὲ τὸ ἐκ τῆς προσβολῆς γενόμενον σχῆμα τὸ Χ πολλὴν μὲν ἔχει καὶ πρὸς τὸ πᾶν οἰκειότητα καὶ πρὸς τὴν ψυχήν. Καὶ ὡς ὅ γε Πορφύριος ἱστορεῖ, παρὰ τοῖς Αἰγυπτίοις ἦν τις τοιοῦτος χαρακτὴρ σύμβολον φέρων τῆς κοσμικῆς ψυχῆς, τῷ Χ κύκλον περιβαλών· ἐσήμαινον γὰρ ἴσως διὰ μὲν τῶν εὐθειῶν τὴν δυοειδῆ πρόοδον αὐτῆς, διὰ δὲ τοῦ κύκλου τὴν μονοειδῆ ζωὴν καὶ τὴν κατὰ κύκλον νοερὸν ἐπιστροφήν[57].

[53]. Cf. anche Menchelli 2024.
[54]. Cf. anche Menchelli 2024.
[55]. L'edizione di Van Riel 2022 contribuisce a definire all'interno della tradizione di Proclo la posizione del patmiaco, che Van Riel colloca, come si è detto, accanto al Chigi R VIII 58. Il Patmiaco reca, come si è accennato, un ricco apparato scoliastico confrontabile con l'apparato scoliastico di M e di N oltre che del Chisianus (quest'ultimo con meno scolii, e più interventi di seconda mano). L'Anonimo Patmiaco non resta dunque isolato nella tradizione. Il Patmiaco è al tempo stesso portatore di buone lezioni singolari, frutto di congettura, perché il Chigi si mantiene fedele alla tradizione: è merito di Van Riel avere definito una volta per tutte l'esistenza della fonte φ, per cui si può affermare con sicurezza che quando il patmiaco reca una lezione superiore al resto della tradizione non si tratta di conservazione ma di innovazione, ovvero congettura, innovazione congetturale, cf. *infra*, p. 000.
[56]. Van Riel 2022.
[57]. "The X shape itself that results from the affixing [of the two strips = strisce] has the highest degree of appropriateness to the universe and to the soul. Porphyry in fact records that something with this character, ie. a X within a circle, was taken by the Egyptians as a symbol of the cosmic soul. Perhaps it signified the soul's bi-form procession through the straight lines, but through the circle it signified its uniform life and the reversion it has in virtue of the intellectual circle. Cf. Baltzly 2009. Ovvero: "La forma stessa del chi che risulta dalla apposizione delle due strisce ha il massimo grado di appropriatezza all'universo e all'anima". "Porfirio infatti riporta che presso gli Egizi c'era un segno simile dove il CHI era circondato da un cerchio che rappresentava (recante) il simbolo dell'anima del mondo. Forse, infatti, volevano indicare (indicavano) attraverso le linee diritte la processione duiforme di essa e attraverso il cerchio la vita uniforme e la conversione / reversione in virtù del cerchio intellettivo // la conversione intellettiva in virtù del cerchio / secondo il cerchio / circolare" (tr. mia). Una diversa traduzione del passo è stata fornita da Festugière 1966-1968: "Oui, et comme le rapporte Porphyre, il y a eu chez les égyptiens un signe pareil, où le X était

Già la traduzione del passo offerta da Festugière sembra leggere il femminile *noeran* per l'aggettivo *noeros*, riferito dunque a *epistrophe*, invece del maschile riferito a cerchio, *kyklos*.

Se consideriamo la tradizione manoscritta del libro III del commento di Proclo al *Timeo* (che conosce importanti variazioni rispetto ai libri I e II)[58] troviamo per il passo citato la lezione κατὰ κύκλον νοερὸν ἐπιστροφήν nei codici che sono testimoni primari del commento: il ms. Chigi R VIII 58 del secolo XII, il ms. Venezia, Marc. gr. 195, del secolo XIV, già utilizzato da Diehl, il ms. Paris, Par. gr. 1840 di Demetrio Mosco, del XV secolo. L'insieme della tradizione del commento ha dunque la lezione con l'aggettivo al maschile riferito a *kyklos*.

Tuttavia, come si è già accennato, il rotolo di Patmos, Eileton 897 presenta questa interessante variante κατὰ κύκλον νοερὰν ἐπιστροφήν, attribuendo l'aggettivo alla *epistrophe*, la conversione o *reversione* dell'anima. Il rotolo di Patmos è il solo, all'interno della tradizione manoscritta, portatore della variante al femminile riferita alla *epistrophe*. Proprio Michele Psello (II 5 O'Meara) è testimone, nella tradizione indiretta, della variante al femminile, *noera epistrophe*.

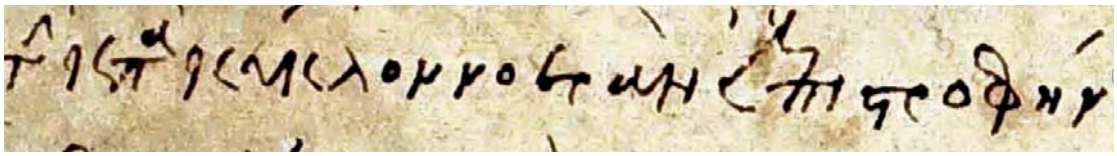

Fig.1: La lezione al femminile è comune a Michele Psello, *Philosophica minora* II 5 O'Meara (νοερὰ ἐπιστροφή) (e potrebbe essere una buona congettura di Psello). Biblioteca del Monastero di san Giovanni il Teologo

Un'altra variante significativa nel commento di Proclo al *Timeo* nel *Patmiacus* Eileton 897 si leggeva poco prima in Procl. *In Ti.* III 331.1–3 Van Riel (= II 244.17 Diehl): (…) ἡ μὲν γὰρ εὐθεῖα τὴν ἀπὸ τῶν κρειττόνων πρόοδον δηλοῖ τῆς ψυχικῆς ζωῆσ, ἡ δὲ εἰς κύκλον κατάκαμψις τὴν νοερὰν στροφήν[59].

Diehl in apparato scriveva: quindi ἐπιστροφήν? Eileton 897 ha qui proprio la variante ἐπιστροφήν, che è probabilmente preferibile.

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entouré d'un cercle, qui représentait le symbole de l'Âme du Monde. Peut-être en effet voulaient-ils indiquer par les droites la procession duiforme de l'Âme, par le cercle sa vie uniforme et sa conversion intellective circulaire".

[58]. I testimoni primari come si è detto sono, oltre al rotolo di Patmos, BAV Chigi R VIII 58 (Chis o H), il codice M, citato sopra (cf. supra, fino a 247 Diehl), mutilo in fine, e un manoscritto della fine del XV secolo, di mano di Demetrio Mosco, il Parisinus Graecus 1840 (P) (libri I-III, ancora una volta parzialmente mutilo nel libro III, del tutto privo di *scholia*).

[59]. "For the straight line makes clear the procession of psychic life from superior beings, while the fact that it has been bent round in a circle indicates the intellectual turning back" (Baltzly 2009). ["Ici, dans les exposés sur la ligne et les cercles, il fait voir le vital et l'intellectif de l'Âme, comment elle participe à la vie de l'Intellect, comment elle se retourne vers elle-même]". "Car la ligne désigne la procession de la vie de l'Âme à partir des entités supérieures, et le recourbement en cercle désigne la conversion intellective" (Festugière 1966–1968).

Se consideriamo lo *stemma codicum* tracciato da Van Riel (2022)[60], il Patmiaco Eileton 897 risale, come si è detto, alla fonte φ, così come il *Chisianus*. In considerazione dell'accordo del *Chisianus* con gli altri testimoni, ancora più evidente diviene il fatto che le lezioni singolari proprie del Patmiaco sono innovazioni (congetturali)[61].

Le due lezioni di Eileton 897, a III 331 Van Riel = 244 Diehl ἐπιστροφήν, che, come si è detto, è probabilmente preferibile, e a III 335 Van Riel (= II 247 Diehl) νοερὰν ἐπιστροφήν, sembrano essere due interpretazioni del testo, e non le troviamo in altri testimoni.

Per riassumere, l'Anonimo Patmiaco ha introdotto due correzioni nel testo[62]:

a. quando ha scritto *epistrophe* invece di *strophe* III 331 Van Riel (= II 244 Diehl), ottenendo un testo con *noeran epistrophen* come risultato della sua correzione,

b. quando ha scritto *noeran epistrophen* a III 335 Van Riel (= II 247 Diehl), eliminando la connessione tra *noeros* e *kyklos* e connettendo l'aggettivo *noeran* con la *epistrophe*, forse ricordando III 331 Van Riel (= II 244 Diehl).

Sono buone congetture? La valutazione non può che essere filosofica, possiamo limitarci a sottoporle[63], e a cercare di rendere quanto più possibile chiaro il contesto che le ha prodotte. Qualunque sia la scelta filosofica dell'editore, le due correzioni dell'Anonimo Patmiaco sono interessanti e plausibili.

Lo sono anche in merito alla ricostruzione della personalità dell'Anonimo di Patmos. Due filologi dell'XI secolo sarebbero intervenuti sul testo allo stesso modo, oppure un medesimo filologo si cela dietro questo intervento. Per altro l'Anonimo era intervenuto, come si è detto, nel medesimo senso non solo a III 335 Van Riel (= II 247 Diehl) ma anche poco più in alto a III 331 Van Riel (= II 244 Diehl), con una innovazione che consente di retrodatare una congettura proposta da Diehl in apparato (*epistrophe*)[64].

Ma come si è visto la propensione alla congettura non è la sola peculiarità dell'Anonimo di Patmos in relazione al testo, significativamente ritagliato a costruire una epitome d'autore.

[60]. Considerando soltanto i codici del III libro, dunque né il codice C né il codice N.
[61]. Forse sono coordinate. Proclo sta parlando delle due line e del cerchio (non dei cerchi, vedi 252 Diehl).
[62]. Altre correzioni meritano di fatto nuovi studi.
[63]. Possono essere buone correzioni perché: in 244 Proclo sta parlando della *noera epistrophe* ed *epistrophe* è più tecnico del semplice *strophe*, così *epistrophe* è stato suggerito anche da Diehl nel suo apparato critico, come riportato anche da van Riel; da 244 a 247 il tema non cambia. La discussione di Proclo è la stessa, c'è continuità nel tema trattato e la continuità fornisce la spiegazione per il secondo intervento di correzione; non c'è ragione di introdurre *kyklos noeros*, che è argomento discusso dopo, dopo alcune pagine.
[64]. Cf. Menchelli 2020.

Un ulteriore elemento in questo senso è la distinzione che l'Anonimo di Patmos opera riguardo ai lemmi, del tutto coerenti con il testo procliano, ma al tempo stesso posti in evidenza con l'indicazione dell'autore, Platone[65].

Gli interventi di correzione nell'Anonimo di Patmos

Con lo scritto pselliano corrispondente a Moore 1030, entriamo nell'officina di Michele Psello, leggiamo la sua narrazione in prima persona dell'attività di scrittura e commento, con le sue opere ancora su *eiletaria* ma destinate alla pubblicazione e alla divulgazione presso i propri studenti e sodali, e non solo[66].

L'operazione "d'autore" condotta in Patmos Eileton 897 può per altro essere colta nel manufatto come una operazione conclusa e al tempo stesso soggetta a correzione.

Un importante aspetto preso in esame da Van Riel è proprio quello delle correzioni[67], nonché della distinzione delle diverse mani correttrici, sia nel codice Chigi, R.VIII.58, H, sia nel *Patmiacus* Eileton 897, Y, ad esso apparentato.

Anche le correzioni di H^2 sono in effetti in relazione con il *Patmiacus* stesso secondo quanto risulta dallo studio di Van Riel[68]: talora H^2 è in accordo anche con gli interventi di Y.

Se si guarda all'attività di prima mano nel Patmiaco, l'intervento a III 42.10 Van Riel (= II 30–31 Diehl) αἱ πλευραί, con aggiunta dell'articolo *supra lineam*, è per esempio da attribuirsi alla mano stessa che ha scritto il testo.

Quanto si può sostenere è che:

a). diverse correzioni nella Epitome del *Patmiacus* sono della mano che scrive il testo, ovvero di prima mano; questa prima mano interviene tracciando due righe sopra il testo scorretto e aggiungendo per esempio sopra la correzione;

b). una mano correttrice diversa dalla prima viene individuata nella edizione Van Riel a proposito, per esempio, dell'articolo inserito a III 84 Van Riel (= II 61 Diehl);

[65]. Cf. Van Riel 2022: "The Platonic lemmas are mostly introduced by the combination of a dicolon and a paragraphos followed by pl (compendium for Platon)". Sui lemmi cf. anche Menchelli 2020. Ci si potrebbe chiedere se nel trasferire l'Epitome su codice, l'indicazione Platone avrebbe potuto favorire la sostituzione del lemma del commento con il ricorso al testo platonico e un brano tratto direttamente da Platone come avviene nelle opere di Psello conservate (per altro con un solo lungo brano iniziale, dunque diversamente soggette a questa sostituzione), oppure nelle copie dei commenti procliani eseguita da Pachimere.

[66]. Cf. anche Wolska and Conus per il commento. Per la discussione rimando a Menchelli 2024.

[67]. Cf. Van Riel 2022: "The text has been corrected both by the scribe himself and by a different hand".

[68]. H^2 legge lo stesso testo che è presente in Y, in almeno alcuni casi, quindi, esiste un terzo manoscritto ai quali ambedue attingono oppure, come secondo lo stemma Van Riel, H^2 interviene su H attingendo ancora alla fonte phi comune anche al Patmiaco.

c). una mano correttrice interessante scrive in inchiostro nerissimo e interviene, questa volta, in più punti del testo; a questa mano deve essere per esempio attribuita la lezione corretta θεωρίαν a pag. 250 Diehl, in luogo di θεωρείαν, che era innovazione dell'Anonimo di Patmos.

Le correzioni più importanti sono in ogni caso ad opera della mano 1, e avvengono sia ristabilendo il testo corretto, sia congetturando[69]. Gli stessi interventi di correzione della mano correttrice 1, la stessa del testo, più significativi di quelli delle altre mani, sembrano dunque essere interventi d'autore. Il rotolo non sarebbe tanto lo straordinario esempio di un esemplare di autore antico trascritto in tale forma libraria quanto forse un poco più "ordinario" esempio di "autore" bizantino che ne ha ricavato un'epitome.

Non era forse possibile intervenire con rasure in presenza di un supporto quale è la carta, più fragile, rispetto per esempio alla pergamena: queste correzioni hanno pertanto a mio avviso lo stesso valore delle rasure negli esemplari pergamenacei, per suggerire in combinazione con gli altri elementi rilevati che si tratti di un autografo.

Nella stessa direzione conduce il carattere di "unicità" del testimone[70], e la sua probabile conservazione in un luogo protetto. A tale proposito gli interventi di mano successiva sembrano indicare che una sua fruizione ne segnò in ogni caso la possibilità di salvataggio e trasmissione.

Postilla paleografica

In presenza dell'ipotesi di autografia diverrebbe prezioso il confronto con Psello lettore del *Timeo*. Se per tale dialogo l'esemplare platonico pselliano sembra corrispondere alla "fonte g" perduta, come si è detto sopra, nel caso del *Fedro*, il codice W, del secolo XI, è stato chiamato in causa in maniera convincente. A tale proposito si può richiamare il ricco apparato di note sempre del secolo XI che figurano sui margini del codice.

Tali note ripropongono appunto a margine passi del testo dei dialoghi, e si concentrano soltanto su alcuni di essi. La mano potrebbe essere compatibile con l'Anonimo di Patmos, data

[69]. Così nel caso dell'inserimento del termine *nous* in un altro punto chiave del testo (sul quale intenderei tornare).
[70]. Per quest'ultimo aspetto cf. Petrucci 1982, con indicazioni supplementari; per la discussione sul libro d'autore cf. anche Petrucci 1992, con ulteriore bibliografia. Il maestro di Psello, Giovanni Mauropode, si affidava nell'XI secolo alle strategie editoriali in auge per la produzione della propria raccolta d'autore, con il contributo di Isaia, che vuole celebrare l'iniziativa. Dai "brogliacci" d'autore si passa a un prodotto "di qualità" (che preserva peraltro tracce dell'articolazione delle opere), sorvegliato presumibilmente dall'autore, sicuramente dal suo segretario, secondo una dinamica autore-entourage d'autore. Con Patmos Eileton 897 siamo in presenza dello stadio precedente, quello della redazione dell'opera epitomata da parte dell'autore. Le raccolte delle opere di Psello includono opere antologizzate, veri e propri insiemi di citazioni, che Psello stesso identifica come opere d'autore, ed è proprio Psello a testimoniare la propria familiarità con la forma rotolo.

anche la diversa destinazione della sua attività, qui concentrata tra note di lettura e/o *marginalia* tratti dal testo. Forse proprio W è passato tra le mani di Michele Psello, la cui attenzione al testo platonico si manifesterebbe dunque dal ricorso diretto a Platone anche nei passi del *Fedro*, nonché, per il *Timeo*, dalla puntuale segnalazione dei passi platonici nel testo dell'epitome del commento procliano di Eileton 897.

Bibliografia

D. Baltzly (ed.), *Proclus. Commentary on Plato's* Timaeus, Vol. 4, Book 3, Part 2: *Proclus on the World Soul*, Cambridge, Cambridge University Press, 2009.

L. Benakis, *Neues zur Proklos-Tradition in Byzanz*, in G. Boss and G. Seel (eds.), *Proclus et son influence*, Zürich, Éditions du Grand Midi, 1987, pp. 247–59.

D. Bianconi, *Qualcosa di nuovo su Giovanni Catrario*, in «Medioevo Greco», 6 (2006), pp. 69–91.

D. Bianconi, *"Piccolo assaggio di abbondante fragranza". Giovanni Mauropode e il Vat. gr. 676*, in «Jahrbuch der österreichischen Byzantinistik», 61 (2011), pp. 89–103.

D. Bianconi – F. Ronconi (eds.), *La « collection philosophique » face à l'histoire*, Spoleto, CISAM, 2020.

J. Burnet (ed.), *Platonis Opera*, Vol. 2: *Tetralogiae III–IV*, Oxford, Oxford University Press, 1901.

A. Carlini, *I lemmi del commento di Proclo all'*Alcibiade I *e il codice W di Platone*, in «Studi Classici e Orientali», 10 (1961), pp. 179–87.

A. Carlini (ed.), *Platone.* Alcibiade, Alcibiade secondo, Ipparco, Rivali, Torino, Boringhieri, 1964.

A. Carlini, *Studi sulla tradizione antica e medievale del Fedone*, Roma, Edizioni dell'Ateneo, 1972.

G. Cavallo, *Stralci di storia di un gruppo di manoscritti greci del secolo IX*, in P. Chiesa – A. M. Fagnoni – R. E. Guglielmetti (ed.), Ingenio Facilis. *Per Giovanni Orlandi (1938–2007)*, Firenze, Edizioni del Galluzzo, 2017, pp. 3–63.

E. Diehl, Subsidia Procliana, in «Rheinisches Museum für Philologie», 54 (1899), pp. 197–200.

E. Diehl (ed.), *Procli Diadochi in Platonis* Timaeum *Commentaria*, 3 vols., Leipzig, Teubner, 1903−1906.

A. Diller, *The Scholia on Strabo*, in «Traditio», 10 (1954), pp. 29–50.

A.-J. Festugière, *Proclus, Commentaire sur le* Timée, 5 vols., Paris, Vrin, 1966–1968.

M. R. Formentin, *Uno scriptorium a palazzo Farnese?*, in «Scripta», 1 (2008), pp. 77–102.

M. R. Formentin, *Catalogus codicum graecorum Bibliothecae Nationalis Neapolitanae*, Vol. III, Roma, Istituto Poligrafico e Zecca dello Stato, 2015.

P. Golitsis, *Un commentaire perpétuel de Georges Pachymère à la* Physique *d'Aristote, faussement attribué à Michel Psellos*, in «Byzantinische Zeitschrift», 100 (2007), pp. 637–76.

P. Golitsis, *Georges Pachymère comme didascale. Essai pour une reconstitution de son enseignement philosophique*, in «Jahrbuch der Österreichischen Byzantinistik», 58 (2008), pp. 53–68.

P. Golitsis, *Copistes, élèves et érudits : la production de manuscrits philosophiques autour de Georges Pachymère*, in A. Bravo García – I. Pérez Martín (eds.), *The Legacy of Bernard de Montfaucon: Three Hundred Years of Studies on Greek Handwriting*, Turnhout, Brepols, 2010, pp. 157–70, pp. 757–68.

P. Golitsis, *Nicéphore Calliste Xanthopoulos, élève de Georges Pachymère*, in Marie Cronier and Brigitte Mondrain (eds.), *Le Livre manuscrit grec: écriture, matériaux, histoire*, Paris, ACHCByz, 2020, pp. 305–15.

D. Harlfinger, *Autographa aus der Palaiologenzeit*, in W. Seibt (ed.), *Geschichte und Kultur der Palaiologenzeit*, Wien, Verlag der Österreichischen Akademie der Wissenschaften, 1996, pp. 43–50.

G. Jonkers, *The textual tradition of Plato's* Timaeus *and* Critias, Amsterdam, Diss. Free Univ. *Amsterdam, 1989* (Leiden-Boston, Brill, 2017).

C. Luna, *Le texte du* Parménide *de Platon dans le commentaire de Proclus*, in L. Ferroni (ed.), Tempus quaerendi. *Nouvelles expériences philologiques dans le domaine de la pensée de l'Antiquité tardive*, Paris, Les Belles Lettres, 2019, pp. 309–410.

A.-Ph. Segonds – C. Luna (eds.), *Proclus, Commentaire sur le* Parmenide *de Platon : Introduction Generale*, Livre 1, Paris, Les Belles Lettres, 2007.

M. Jagoda Luzzatto, *Codici tardoantichi di Platone ed i cosiddetti* Scholia Arethae, in «Medioevo greco», 10 (2010), pp. 77–110.

D. Marcotte, *La « Collection Philosophique » : historiographie et histoire des textes*, «Scriptorium», 68 (2014), pp. 145–65.

M. Menchelli, *Platone e Prisciano. Le pericopi platoniche nel libro XVIII dell'*Ars *e il Platone dei grammatici*, in L. Martorelli (ed.), *Greco antico nell'Occidente carolingio. Frammenti di testi attici nell'*Ars *di Prisciano*, Hildesheim-Zürich-New York, Olms, 2014, pp. 205–47.

M. Menchelli, *Due citazioni dal Timeo in Michele Psello e il* Commento al Timeo *di Proclo. Prime osservazioni su* Philosophica minora *II 4-5*", in «Medioevo Greco», 16 (2016), pp. 185–98.

M. Menchelli, *Dal codice al rotolo. Considerazioni sul 'nuovo' rotolo di Patmos del commento di Proclo al* Timeo *platonico*, in G. Agosti – D. Bianconi (eds.), *Pratiche didattiche tra centro e periferia nel Mediterraneo tardoantico*, Spoleto, CISAM, 2019, pp. 319–46.

M. Menchelli, *Tradizione manoscritta e forme del libro nel commento di Proclo al* Timeo *platonico. Il III libro nel rotolo di Patmos, Biblioteca del Monastero di San Giovanni, Eileton 897 e nel codice di Firenze, BML, Plut. 28.20, tra Michele Psello e l'età dei Paleologi*", in M. Cronier – Brigitte Mondrain (eds.), *Le Livre manuscrit grecs : écritures, matériaux, histoire*, Paris, ACHCByz, 2020, pp. 857–74.

M. Menchelli, *Autorialità e scuola nell'Anonimo di Patmos, Eileton 897, commento di Proclo al* Timeo *platonico, e in Michele Psello (Moore 1030)*, in «Medioevo greco», 24 (2024), pp. 243–64.

B. Mondrain, *La réutilisation de parchemin ancien dans les livres à Constantinople au XIV^e^ et au XV^e^ siècle : quelques exemples, de la "Collection philosophique" aux folios palimpsestes du Parisinus gr. 1220,* in S. Lucà (ed.), *Libri palinsesti greci: conservazione, restauro digitale, studio,* Roma, Comitato Nazionale per le Celebrazioni del Millennio della Fondazione dell'Abbazia di S. Nilo a Grottaferrata, 2008, pp. 111–29.

P. Moore, Iter Psellianum. *A Detailed Listing of Manuscript Sources for All Works Attributed to Michael Psellus, Including a Comprehensive Bibliography*, Toronto, Pontifical Institute of Mediaeval Studies, 2005.

C. Moreschini, *I lemmi del commento di Proclo al* Parmenide *in rapporto alla tradizione manoscritta di Platone,* in «Annali della Scuola Normale Superiore di Pisa», 33 (1964), pp. 251–5.

C. Moreschini, *Ermia alessandrino nel Medioevo e nel Rinascimento,* in «Studi Classici e Orientali», 55 (2009), pp. 283–300.

D. J. Murphy, *The Manuscripts of Plato's* Charmides, in «Mnemosyne, 43 (1990), pp. 334–8.

D. J. Murphy, Parisinus gr. *1813 and its apographa in Plato's* Laches, in «Mnemosyne», 47 (1994), pp. 1–11.

D. J. O'Meara (ed.), *Michaelis Pselli Philosophica minora*, Vol. II, *Opuscula Psychologica, Theologica, Daemonologica,* Leipzig, Teubner, 1989.

I. Pérez Martín, *Estetica e ideologia nei manoscritti bizantini di Platone*, in «Rivista di Studi Bizantini e Neoellenici», 42 (2005), pp. 113–35.

I. Pérez Martín, *The Transmission of Some Writings by Psellos in Thirteenth Century Constantinople*, in A. Rigo – P. Ermilov – M. Trizio (eds.), Theologica minora*: The Minor Genres of Byzantine Theological Literature*, Turnhout, Brepols, 2013, pp. 159–74, pp. 180–5.

I. Pérez Martín, *The Transmission of Michael Psellos' Writings on* Meteorologica, in J. Signes Codoñer – I. Pérez Martín (eds.), *Textual Transmission in Byzantium: Between Textual Criticism and Quellenforschung*, Turnhout, Brepols, 2014, pp. 291–311.

A.-Ph. Segonds (ed.), *Proclus, Sur le* premier Alcibiade *de Platon*, t. I, Paris, Les Belles Lettres, 1985.

C. Steel (ed.), *Procli in Platonis* Parmenidem *Commentaria*, 3 vols., Oxford, Oxford University Press, 2007−2009.

G. Van Riel, *"Tel Glaucos de la mer, couvert de coquillages, d'algues et de pierre." Sur le correcteur anonyme du texte de l'*In Timaeum *de Proclus dans le ms Chis. R.VIII.58*, in L. Ferroni (ed.), Tempus quaerendi. *Nouvelles expériences philologiques dans le domaine de la pensée de l'Antiquité tardive*, Paris, Les Belles Lettres, 2019, pp. 275–307.

G. Van Riel (ed.), *Procli Diadochi in Platonis* Timaeum *Commentaria*, 5 vols., Oxford, Oxford University Press, 2022.

F. Vendruscolo, *Tra Barbaro e Parrasio: i manoscritti greci di Vittorio Falconio (*alias *Vettor Fausto)*, in «Italia Medioevale e Umanistica», 59 (2018), pp. 217–58.

L. G. Westerink (ed.), *Proclus, In Platonis* Alcibiadem, Amsterdam, 1954.

ABSTRACT

Mariella Menchelli, *Reading Proclus' Commentaries in Byzantium: Pachymeres, Katrarios (Napoli, BN, III.D.28, In Ti., and III.E.17, In Alc.) and the "New" Patmos Scroll*

The philosophical reception and reading of Proclus's commentaries on Plato's dialogues in Byzantium are closely linked to the transmission of Neoplatonic manuscripts. This chapter focuses on key witnesses from the early Palaeologan period, when these materials were copied and studied by the philosophical circles of Constantinople and Thessaloniki. The Naples collections, BN, provide crucial examples. Proclus's *Commentary on Plato's* First Alcibiades, Neap. BN III.E.17, is partly a *codex unicus*, copied by Georgius Pachymeres himself. Proclus's *Commentary on Plato's* Timaeus, Neap. BN III.D.28, was written by John Katrarios and belongs to the ϕ-branch of the textual tradition, shared with ms. Chigi R VIII 58 and the "new" Patmos Eileton 897. The Patmos scroll, a paper *volumen* from the eleventh century (the era of Psellos), is an exceptional witness discovered in the 1980s and is notable for its rich *apparatus* of mathematical *scholia*. The aim of this chapter is to explore these three witnesses – the *In Alc*. copied by Pachymeres, the In *Tim*. copied by Katrarios, and the Patmos scroll – to highlight their importance for the textual history and interpretation of Proclus's work in Byzantium.

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